Covariant Cubic Vertices for Interacting Irreducible Massless and Massive Higher Spin Fields

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- I.L. Buchbinder, A.R, General Cubic Interacting Vertex for Massless Integer HS Fields, PLB (2021), [arXiv:2105.12030],
- I.L. Buchbinder, A.R, Covariant Cubic Interacting Vertices for Massless and Massive Integer Higher Spin Fields, Symmetry (2023) [arXiv:2212.07097] ,
- A.R., BRST–BV approach for interacting HS fields, TMPh (2023) [arXiv:2303.02870],
- A.R., P.Moshin, Gauge Invariant Lagrangian Formulations for Mixed Symmetry Higher Spin Bosonic Fields in AdS Spaces, Universe ()2023 [arXiv:2305.00142],
- I.L. Buchbinder, A.R, Consistent Lagrangians for irreducible interacting higher-spin fields with holonomic constraints [arXiv:2304.10358] PEPAN (2023)

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Motivations

Wigner-Bargmann classification (1939, 1948) of UIRs $ISO(1, d-1)$ is characterized by $[(d+1)/2]$ Casimirs (A.P. Isaev)
1. $P^2 = m^2, W^2 = -m^2 s(s+1)$ - massive Unitary irrep (UIR) with (half)integer spin;
2a. $P^2 = 0, W^2 = 0,\ W^\mu = \lambda P^\mu$ - massless helicity UIR;
2b. $P^2 = 0, W^2 = \mu^2$ - massless continuous spin UIR;

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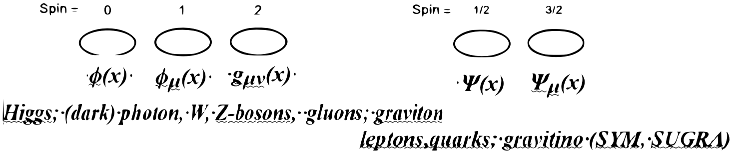
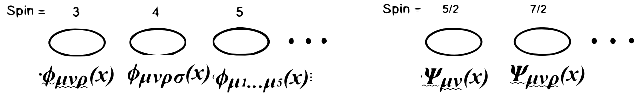
Lower Spin refers to consistent classical field theories ($s \leqslant 2$)



Higher Spin (HS) stands for problematic construction ($s > 2$)



Fronsdal '78 Fang-Fronsdal '79

BRST approach with complete Q for HS fields on $R^{1,d-1}$

We consider theory of higher spin (HS) ($m = 0$ & $m \neq 0$) fields as natural candidates for possible new particles: matter ($s = n + 1/2$); interactions ($s = n$); for DM & DE

HS fields related to (Super)SFT (E. Witten, 1986) and revealed at HE after Big Bang due to tensionless limit : $\Rightarrow$ for string BRST operator $\mathcal{Q}$ ($d = 26, 10$) for ($\alpha' \to \infty$): (G.Bonelli (2003), A. Sagnotti, M. Tsulaia, (2004)) now ($\forall d$).

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$\Longrightarrow \mathcal{Q} \overset{\alpha' \to \infty}{\longrightarrow} Q_c$:$\{\infty\}$`many HS fields` $\phi_\mu(x), .., \phi_{\mu(s)}(x)$ `in string spectra`

There are $100'$s results on free dynamics for HS fields, and $10'$s on cubic and $n-$ rtic interactions for HS fields on Minkowski and AdS spaces To derive it many approaches exist for free and interacting dynamics dividing on metric-like, frame-like (Lorentz frame) formulations; superfield approach
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[AKSZ model (Alexandrov,Kontsevich, Schwarz, Zaboronsky 1997)].

Known results on cubic vertices

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- *Cub. vertex in BRST approach with (in)complete $Q_{(c)}$ for irrep HS fields not found*

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- Cubic vertex for massless HS fields with helicities $(0, 0, s)$;
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Noether's procedure (G.Barnich, M.Henneaux 1998, A.R. $L > 1$ 2021):
Gauge theory of 1 -stage reducibility in de Witt condensed notations

$$S_0[A] - \texttt{classical action of fields } A^i, i = 1, ..., n,\ \varepsilon(A^i) = \varepsilon_i = 0,$$
$$\delta_0 S_0 = 0,\ \delta_0 A^i = R_0{}^i_{\alpha_0} \xi^{\alpha_0},\ \alpha_0 = 1, ..., m_0, \Longrightarrow$$
$$\overleftarrow{\partial}_i S_0 R_0{}^i_{\alpha_0} = 0 \quad \delta_0^{(0)} \xi^{\alpha_0} = Z_0{}^{\alpha_0}_{\alpha_1} \xi^{\alpha_1} : \alpha_1 = 1, ..., m_1 < m_0,$$

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Deformation of k copies of LF for fields $A^{i(p)}$, $p = 1, .., k$ with quadratic $\sum_p S_0^{(p)}[A^{(p)}]$ of free fields $A^{i(p)}$ (maybe $[i(p_1)] \neq [i(p_2)]$) with rank condition

$$\boxed{N = \sum_p (n^p - m^p) \quad \texttt{where} \quad \mathrm{rank}\| \overleftarrow{\partial}_i \overrightarrow{\partial}_j S_0^{(p)} \|_{\overleftarrow{\partial}_i S_0 = 0} = n^p - m^p}$$

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$$S_{int} = \sum_{p=1}^k S_0^{(p)}[A^{(p)}] + g^1 S_1 + g^2 S_2 + \dots g^r S_r,\ \boxed{deg_A S_r = r + 2},$$
$$\delta_{[l]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} + \dots + g^l \delta_1 A^{i(p)} = R_{[l]}{}^{i(p)}_{\alpha_0(t)}\xi^{\alpha_0(t)},\ \boxed{deg_A R_l{}^{i(p)}_{\alpha_0(t)} = l}$$
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$$R_0{}^{i(p)}_{\alpha_0(t)} \equiv R_0{}^i_{\alpha_0}\delta_t^p,\quad Z_0{}^{\alpha_0(p)}_{\alpha_1(t)} \equiv Z_0{}^{\alpha_0}_{\alpha_1}\delta_t^p.$$

Interaction vertices in the gauge theories: deformation procedure

Noether's identities as system in powers of g from $\delta_\Sigma S_{int} = 0$: $\boxed{\delta_\Sigma \equiv \sum_{l=0}^{\infty} \delta_l}$

$$g^1: \qquad \delta_0 S_1 + \delta_1 \overline{S}_0 = 0, \tag{1}$$

$$g^2: \qquad \delta_0 S_2 + \delta_1 S_1 + \delta_2 \overline{S}_0 = 0, \quad \overline{S}_0 \equiv \sum_{p=1}^{k} S_0^{(p)}$$

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$$g^2 : \qquad \Big(\delta_2^{(0)}\delta_0 + \delta_1^{(0)}\delta_1 + \delta_0^{(0)}\delta_2\Big) A^{i(p)}|_{\partial S_{[2]}=0} = 0, \tag{2}$$

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for the cubic vertex for GTh of the 1st reducibility stage

$$S_{int} = \sum_{p=1}^{3} S_0^{(p)}[A^{(p)}] + g^1 S_1, \ \boxed{deg_A S_1 = 3},$$

$$\delta_{[1]} A^{i(p)} = \delta_0 A^{i(p)} + g\delta_1 A^{i(p)} = R_{[1]}{}^{i(p)}_{\alpha_0(t)} \xi^{\alpha_0(t)}, \tag{3}$$

$$\delta_{[1]}^{(0)} \xi^{\alpha_0(p)} = \Big\{\delta_0^{(0)} + g\delta_1^{(0)}\Big\} \xi^{\alpha_0(p)} = Z_{[1]}{}^{\alpha_0(p)}_{\alpha_1(t)} \xi^{\alpha_1(t)}.$$

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$$g^2 : \qquad \Big(\delta_2^{(0)} \delta_0 + \delta_1^{(0)} \delta_1 + \delta_0^{(0)} \delta_2\Big) A^{i(p)}|_{\partial S_{[2]}=0} = 0, \qquad (2)$$

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$$\delta_{[1]}^{(0)} \xi^{\alpha_0(p)} = \Big\{\delta_0^{(0)} + g \delta_1^{(0)}\Big\} \xi^{\alpha_0(p)} = Z_{[1]}{}^{\alpha_0(p)}_{\alpha_1(t)} \xi^{\alpha_1(t)}.$$

The equations (1)–(2) pass to

$$g^1 : \qquad \delta_0 S_1 + \delta_1 \textstyle\sum_{p=1}^{3} S_0^{(p)} = 0, \qquad (4)$$

$$g^1 : \qquad \Big(\delta_1^{(0)} \delta_0 A^{i(p)} + \delta_0^{(0)} \delta_1 A^{i(p)}\Big)|_{\partial S_{[1]}=0} = 0, \ p = 1, 2, 3.$$

BRST-BFV approach with complete Q for HS fields

In BRST-BFV approach developed (S. Ouvry, J. Stern, A. Bengtsson, A. Pashnev, M. Tsulaia, I. Buchbinder, V. Krykhtin, A.R.)
instead of direct problem for generalized canonical quantization of ConstDS by the aim inverse problem - is an construction of GI LF for HS fields with (m, s)

$$\boxed{\begin{array}{c}\texttt{irrep conditions}\\ \text{ISO(1,d-1), SO(2,d-1)}\end{array}} \xrightarrow{\texttt{SFT}} \boxed{\begin{array}{c}\texttt{(super)algebra}\{o_I(x)\} : \mathcal{H}\\ [o_I, o_J\} = f^K_{IJ}(o)o_k + \Delta_{ab}(g_0)\end{array}}$$

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irrep conditions	SFT →	(super)algebra$\{o_I(x)\} : \mathcal{H}$
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conversion →
E. Fradkin, A.Pashnev, I.Buchbinder, A.R.

$$O_I = o_I + o'_I : \mathcal{H} \bigotimes \mathcal{H}'$$
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$\xrightarrow[\text{I.B.,E.Fradkin, G.V., M.Henneaux}]{\text{BFV}}$ BRST operator $\{O_I\}$:$Q'(x)$, $Q' = C^I O_I + \frac{1}{2} C^I C^J F^K_{IJ} \mathcal{P}_K + \text{more}$

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$\xrightarrow{\text{LF}}$

$$Q' = Q + (g_0 + h + \text{more})C_g + \ldots : Q^2|_{\sigma|\chi\rangle=0} = 0$$

mass-shell : $Q|\chi\rangle = 0, \text{gh}(|\chi\rangle) = 0 \Rightarrow$ action : $S = \int d\eta_0 \langle\chi|KQ|\chi\rangle$

spin: $(g_0 + \text{more})(|\chi\rangle, |\Lambda\rangle, \ldots) = -h(|\chi\rangle, |\Lambda\rangle, \ldots)$

gauge symmetry: $\delta|\chi\rangle = Q|\Lambda\rangle, \delta|\Lambda\rangle = Q|\Lambda^1\rangle, \ldots$

Q - for the 1-st class constraints $\{O_\alpha\} \subset \{O_I\}$ without invertable g_0. on 2-3 stages appears gauge and Stuekelberg fields

BRST approach with complete Q for HS fields on $R^{1,d-1}$

Talk devoted to (off-shell) covariant general Lagrangian (cubic: $g\phi_1^{\mu(s1)}\phi_2^{\nu(s2)}\phi_3^{\rho(s3)}$) vertices for irreducible HS fields on $R^{1,d-1}$ (AdS_d). We developed a concept of deformation Noether's procedure of free GTh on a base of BRST-BFV $\equiv$ BRST & BRST-BV approaches with complete BRST operator (J.Buchbinder, A.Pashnev, M.Tsulaia, V.Kryhktin, A.R. 1998-2023).

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$$\texttt{particle}\ (m,s):\quad \big(\partial^\nu\partial_\nu+m^2,\ \partial^{\mu_1},\ \underline{\eta^{\mu_1\mu_2}}\big)\phi_{\mu(s)}=(0,0,0)\qquad\Longleftrightarrow$$

$$\big(l_0,\ l_1,\ \underline{l_{11}},g_0-d/2\big)|\phi\rangle=(0,0,0,s)|\phi\rangle.$$

$\operatorname{diag}\eta^{\mu\nu}=(+,-,...,-)$, String-like vector $|\phi\rangle\in\mathcal{H}$, operators $l_0,\ l_1,\ l_{11},g_0$ are:

$$|\phi\rangle=\sum_{s\geq 0}\frac{\imath^s}{s!}\phi^{\mu(s)}\prod_{i=1}^{s}a^+_{\mu_i}|0\rangle,\quad [a_\nu,a^+_\mu]=-\eta_{\mu\nu},$$

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Talk devoted to (off-shell) covariant general Lagrangian (cubic: $g\phi_1^{\mu(s1)}\phi_2^{\nu(s2)}\phi_3^{\rho(s3)}$) vertices for irreducible HS fields on $R^{1,d-1}$ (AdS_d). We developed a concept of deformation Noether's procedure of free GTh on a base of BRST-BFV $\equiv$ BRST & BRST-BV approaches with complete BRST operator (J.Buchbinder, A.Pashnev, M.Tsulaia, V.Kryhktin, A.R. 1998-2023).

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$$\overset{complete}{Q}=\eta_0 l_0+\eta_1^+\check{l}_1+\check{l}_1^+\eta_1+\imath\eta_1^+\eta_1\mathcal{P}_0+\eta_{11}^+\widehat{L}_{11}+\widehat{L}_{11}^+\eta_{11}\equiv\overset{incomplete}{Q_c}+Tr,$$
$$Q_c^2=0,\quad Q^2|_{\sigma\left(|\chi\rangle_s,\,|\Lambda\rangle_s\right)=0}=0$$

$$\mathcal{S}_{0|s}[\phi,\phi_1,...]=\mathcal{S}_{0|s}[|\chi\rangle_s]=\int d\eta_{0\,s}\langle\chi|KQ|\chi\rangle_s,\quad \delta\Big(|\chi\rangle_s,\,|\Lambda\rangle_s\Big)=Q\Big(|\Lambda\rangle_s,|\Lambda^1\rangle_s\Big)$$

with trace $\widehat{L}_{11}$ (dual) $\widehat{L}_{11}^+$ selecting $ISO(1,d-1)$ irreps with integer s spin operator σ:

BRST approach with complete Q for free HS fields on $R^{1,d-1}$

$$Q = \eta_0 l_0 + \eta_1^+ \check{l}_1 + \check{l}_1^+ \eta_1 + \eta_{11}^+ \widehat{L}_{11} + \widehat{L}_{11}^+ \eta_{11} + \imath \eta_1^+ \eta_1 \mathcal{P}_0$$

$$\{\eta_0, \mathcal{P}_0\} = \imath, \quad \{\eta_1, \mathcal{P}_1^+\} = \{\eta_1^+, \mathcal{P}_1\} = \{\eta_{11}, \mathcal{P}_{11}^+\} = \{\eta_{11}^+, \mathcal{P}_{11}\} = 1.$$

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extended trace constraints

$$\big(\widehat{L}_{11},\, \widehat{L}_{11}^+\big) = \big(L_{11} + \eta_1 \mathcal{P}_1,\, L_{11}^+ + \mathcal{P}_1^+ \eta_1^+\big). \qquad (5)$$
$$L_{11} = l_{11} + (b^+ b + h) b - 1/2 d^2, \quad L_{11}^+ = l_{11}^+ + b^+ - 1/2 d^{+2}$$

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Algebra of l_0 ,$\check{l}_1 = l_1 + md$, $\check{l}_1^+ = l_1^+ + md^+, L_{11}, L_{11}^+, G_0$:

$$[l_0, \check{l}_1^{(+)}] = 0, \ [\check{l}_1, \check{l}_1^+] = l_0 \quad \text{and} \quad [L_{11}, L_{11}^+] = G_0, \ [G_0, L_{11}^+] = 2L_{11}^+,$$
$$[l_1, L_{11}^+] = -l_1^+, \quad [l_1, G_0] = l_1$$

with extended number particle operators

$$G_0 = g_0 + 2b^+ b + d^+ d + h, \quad (b,\, b^+, d, d^+) : \ [b,\, b^+] = [d, d^+] = 1$$

- auxiliary conversion oscillators generating auxiliary Fock space $\mathcal{H}'$. conversion parameter $h = h(s) = -s - \frac{d-6}{2}$. for Verma module for $so(1,2)$

BRST approach with complete Q for free HS fields on $R^{1,d-1}$

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- auxiliary conversion oscillators generating auxiliary Fock space $\mathcal{H}'$. conversion parameter $h = h(s) = -s - \frac{d-6}{2}$. for Verma module for $so(1,2)$ spin operator σ:

$$\sigma = G_0 + \eta_1^+ \mathcal{P}_1 - \eta_1 \mathcal{P}_1^+ + 2(\eta_{11}^+ \mathcal{P}_{11} - \eta_{11} \mathcal{P}_{11}^+), \quad \sigma(|\chi\rangle_s,\, |\Lambda\rangle_s,\, |\Lambda^1\rangle_s) = \vec{0}, \quad (6)$$

BRST approach with complete BRST operator to Lagrangians for HS fields on $R^{1,d-1}$

$$\sigma(|\chi\rangle_s,\ |\Lambda\rangle_s,\ |\Lambda^1\rangle_s) = (0,0,0), \tag{7}$$

with periodic $\mathbb{Z}_2$ ε and decreasing $\mathbb{Z}$ gh_H $(0,0)$, $(1,-1)$, $(0,-2)$ respectively.
All operators act in total Hilbert space $\mathcal{H}_{tot} = \mathcal{H} \otimes \mathcal{H}_{gh} \otimes \mathcal{H}'$ with inner pr.

$$\langle\chi|\psi\rangle = \int d^d x \langle 0|\chi^*\big(a, b; \eta_1, \mathcal{P}_1, \eta_{11}, \mathcal{P}_{11}\big)\psi\big(a^+, b^+; \eta_1^+, \mathcal{P}_1^+, \eta_{11}^+, \mathcal{P}_{11}^+\big)|0\rangle.$$

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$$\langle\chi|\psi\rangle = \int d^dx\langle 0|\chi^*(a,b;\eta_1,\mathcal{P}_1,\eta_{11},\mathcal{P}_{11})\psi(a^+,b^+;\eta_1^+,\mathcal{P}_1^+,\eta_{11}^+,\mathcal{P}_{11}^+)|0\rangle.$$

Operators Q,σ supercommute and Hermitian (e.g. I.L. Buchbinder, A. Pashnev, M. Tsulaia, PLB (2001), I. Buchbinder, A. R., NPB (2012) arXiv:1110.5044)

$$Q^2 = \eta_{11}^+\eta_{11}\sigma, \qquad [Q,\ \sigma\} = 0; \tag{8}$$

$$(Q^+,\sigma^+)K = K(Q,\sigma), \qquad K = \sum_{n=0}^{\infty}\frac{1}{n!}(b^+)^n|0\rangle\langle 0|b^n\prod_{i=0}^{n-1}(i+h(s)),$$

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Field $|\chi\rangle_s$, **gauge parameters** $|\Lambda\rangle_s$ $|\Lambda^1\rangle_s$ (as the result of spin condition):

$$|\chi\rangle_s = |\Phi\rangle_s + \eta_1^+\Big(\mathcal{P}_1^+|\phi_2\rangle_{s-2} + \mathcal{P}_{11}^+|\phi_{21}\rangle_{s-3} + \eta_{11}^+\mathcal{P}_1^+\mathcal{P}_{11}^+|\phi_{22}\rangle_{s-6}\Big) \tag{9}$$
$$+ \eta_{11}^+\Big(\mathcal{P}_1^+|\phi_{31}\rangle_{s-3} + \mathcal{P}_{11}^+|\phi_{32}\rangle_{s-4}\Big) + \eta_0\Big(\mathcal{P}_1^+|\phi_1\rangle_{s-1} + \mathcal{P}_{11}^+|\phi_{11}\rangle_{s-2}$$
$$+ \mathcal{P}_1^+\mathcal{P}_{11}^+\Big[\eta_1^+|\phi_{12}\rangle_{s-4} + \eta_{11}^+|\phi_{13}\rangle_{s-5}\Big]\Big),$$

$$|\Lambda\rangle_s = \mathcal{P}_1^+|\xi\rangle_{s-1} + \mathcal{P}_{11}^+|\xi_1\rangle_{s-2} + \mathcal{P}_1^+\mathcal{P}_{11}^+\Big(\eta_1^+|\xi_{11}\rangle_{s-4} + \eta_{11}^+|\xi_{12}\rangle_{s-5}\Big) + \eta_0\mathcal{P}_1^+\mathcal{P}_{11}^+|\xi_{01}\rangle_{s-3},$$

$$|\Lambda^1\rangle_s = \mathcal{P}_1^+\mathcal{P}_{11}^+|\xi^1\rangle_{s-3}.$$

with $|\phi_{...}\rangle_{...} \equiv |\phi(a^+, b^+, d^+)_{...}\rangle_{...}$: . $|\Phi\rangle_s|_{(b^+=d^+=0)} = |\phi\rangle_s$

$$|\Lambda\rangle_s = \mathcal{P}_1^+|\xi\rangle_{s-1} + \mathcal{P}_{11}^+|\xi_1\rangle_{s-2} + \mathcal{P}_1^+\mathcal{P}_{11}^+\Big(\eta_1^+|\xi_{11}\rangle_{s-4} + \eta_{11}^+|\xi_{12}\rangle_{s-5}\Big) + \eta_0\mathcal{P}_1^+\mathcal{P}_{11}^+|\xi_{01}\rangle_{s-3},$$
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with $|\phi_{...}\rangle_{...} \equiv |\phi(a^+,b^+,d^+)_{...}\rangle_{...}$: . $|\Phi\rangle_s|_{(b^+=d^+=0)} = |\phi\rangle_s$
After gauge-fixing procedure for $\mathcal{S}_{0|s}[|\chi\rangle_s]$ it follows LF in the single vector form with $s-1$ auxiliary fields

$$\mathcal{S}^m_{C|s}(\phi,...) = {}_s\langle\Phi|\left(l_0 - \check{l}_1^+\check{l}_1 - (\check{l}_1^+)^2\check{l}_{11} - \check{l}_{11}^+\check{l}_1^2 - \check{l}_{11}^+(l_0 + \check{l}_1\check{l}_1^+)\check{l}_{11}\right)|\Phi\rangle_s,$$
$$\delta|\Phi\rangle_s = \check{l}_1^+|\Xi\rangle_{s-1} \quad \text{and} \quad \check{l}_{11}\big(\check{l}_{11}|\Phi\rangle,\ |\Xi\rangle\big) = (0,0),$$

LF has smooth massless limit for $m = d^{(+)} = 0$ resulting to Fronsdal formulation (1978) in the form of single field $|\phi\rangle_s = |\Phi\rangle_s|_{d^+=0}$ with $(0,s)$.
From $\mathcal{S}^m_{C|s}$ it follows Singh-Hagen formulation for ungauge s traceless fields with physical $|\phi\rangle_s$. (2023)

BRST approach with incomplete Q_c for HS fields on $R^{1,d-1}$

For the approach with incomplete BRST operator Q_c with off-shell holonomic constraints (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev, Grigoriev,Tipunin, 2008) & R.Metsaev, PLB (2013) The Lagrangian formulation (LF) is irreducible GTh for HS field $(m = (\neq)0, s)$ includes trace condition $(l_{11}|\phi\rangle_s = 0)$ in the form of BRST-extended constraint $\mathcal{L}_{11}$ ($[Q_c, \mathcal{L}_{11}] = 0$, $[Q_c, \sigma_c] = 0$) imposed on $|\chi_c\rangle, |\Lambda^0_c\rangle$

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$$\boxed{\begin{array}{c} \mathcal{S}_{0|s}[|\chi_c\rangle] \ = \int d\eta_{0\,s}\langle\chi_c|Q_c|\chi_c\rangle_s, \quad \delta|\chi_c\rangle_s = Q_c|\Lambda_c\rangle_s, \\ \mathcal{L}_{11}\Big(|\chi_c\rangle, |\Lambda^0_c\rangle\Big) = \Big(l_{11} - 1/2d^2 + \eta_1 \mathrm{P}_1\Big)\Big(|\chi_c\rangle, |\Lambda^0_c\rangle\Big) = (0,0), \text{ с } \ l_{11} = 1/2a^\mu a_\mu \end{array}},$$

$$\big(|\chi_c\rangle_s, \ |\Lambda^0_c\rangle_s\big) \ = \Big(|\Phi\rangle_s - \mathcal{P}^+_1\big\{\eta_0|\Phi_1\rangle_{s-1} + \eta^+_1|\Phi_2\rangle_{s-2}\big\}, \ \ \mathcal{P}^+_1|\Xi\rangle_{s_i-1}\Big);$$

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$$\big(|\chi_c\rangle_s, \ |\Lambda^0_c\rangle_s\big) \ = \Big(|\Phi\rangle_s - \mathcal{P}^+_1\big\{\eta_0|\Phi_1\rangle_{s-1} + \eta^+_1|\Phi_2\rangle_{s-2}\big\}, \ \ \mathcal{P}^+_1|\Xi\rangle_{s_i-1}\Big);$$

An equivalence of the LFs with incomplete & complete BRST operators for any irrep with discrete spin on $\mathbb{R}^{1,d-1}$ is (cohomologically) established in A. R, JHEP (2018) 1803.04678, but for interacting theory of the same HS fields it has not yet been solved.

For the approach with incomplete BRST operator Q_c with off-shell holonomic constraints (Barnich, Grigoriev, Semikhatov, Tipunin 2004; Alkalaev, Grigoriev,Tipunin, 2008) & R.Metsaev, PLB (2013) The Lagrangian formulation (LF) is irreducible GTh for HS field $(m = (\neq)0, s)$ includes trace condition $(l_{11}|\phi\rangle_s = 0)$ in the form of BRST-extended constraint $\mathcal{L}_{11}$ $([Q_c, \mathcal{L}_{11}] = 0, \ \ [Q_c, \sigma_c] = 0)$ imposed on $|\chi_c\rangle, |\Lambda^0_c\rangle$

$$\boxed{\begin{array}{c} \mathcal{S}_{0|s}[|\chi_c\rangle] \ = \int d\eta_{0s}\langle\chi_c|Q_c|\chi_c\rangle_s, \quad \delta|\chi_c\rangle_s = Q_c|\Lambda_c\rangle_s, \\ \mathcal{L}_{11}\Big(|\chi_c\rangle, |\Lambda^0_c\rangle\Big) = \Big(l_{11} - 1/2d^2 + \eta_1\mathrm{P}_1\Big)\Big(|\chi_c\rangle, |\Lambda^0_c\rangle\Big) = (0,0), \text{ c } \ l_{11} = 1/2a^\mu a_\mu \end{array}},$$

$$\Big(|\chi_c\rangle_s, \ |\Lambda^0_c\rangle_s\Big) \ = \Big(|\Phi\rangle_s - \mathcal{P}^+_1\{\eta_0|\Phi_1\rangle_{s-1} + \eta^+_1|\Phi_2\rangle_{s-2}\}, \ \ \mathcal{P}^+_1|\Xi\rangle_{s_i-1}\Big);$$

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Aim is to present deformation procedure (DP):

1. of LF within BRST approach with complete Q for interacting TS HS fields with integer spins $s_1, s_2, ..., s_k$;

2. DP within recently proposed BRST-BV approach for minimal BRST-BV action, encoding gauge algebra; starting from free GTh

To develop DP we work with BRST & BRST-BV procedures with incomplete BRST Q_c with off-shell holonomic constraints

$$\texttt{BRST-BV} \; : S_{0|s}[|\chi_{g|c}\rangle] \; = \int d\eta_{0s} \langle \chi_{g|c}| Q_c |\chi_{g|c}\rangle_s$$

$$= \mathcal{S}_{0|s}[|\chi_c\rangle] + \int d\eta_{0s}\big(\langle \chi^*_c| \overrightarrow{s}_{0|c} |\chi\rangle_s + h.c.\big), \quad \mathcal{L}_{11}|\chi_{g|c}\rangle_s = 0,$$

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$$|\chi_{g|c}\rangle_s = |\chi_{\min|c}\rangle_s + |\chi^*_{\min|c}\rangle_s = \overbrace{|\chi_c\rangle_s + |C^0_c\rangle_s}^{2^2} + \overbrace{|\chi^*_c\rangle_s + |C^{0*}_c\rangle_s}^{2^2};$$

$$|C^0_c\rangle_s\mu_0 \equiv |\Lambda^0_c\rangle_s, \ |C^0_c\rangle_s = \mathcal{P}^+_1|C(a^+,d^+)\rangle_{s-1}, \ |C^{0*}_c\rangle_s = \eta_0\eta^+_1|C^*(a^+,d$$

$\underline{\texttt{Slavnov generator}}$: $\overrightarrow{s}_{0|c}(|\chi_c\rangle_s, |C^0_c\rangle_s) = Q_c(|C^0_c\rangle_s, \ 0),$

$\underline{\texttt{BRST-like transfs}}$: $\delta_B|\chi_{\min|c}\rangle_s = \mu\overrightarrow{s}_{0|c}|\chi_{\min|c}\rangle_s$ (& $\texttt{dual}$ $\overleftarrow{s}_{0|c}$).

BRST-BV approach with incomplete Q_c for HS fields on $R^{1,d-1}$

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$$\delta_B S_{0|s}[|\chi_{g|c}\rangle] = 0$$

.

We covariantize the cubic vertices $\big|V^{(3)}\rangle^{(m)_3}_{(s)_3} \in \mathcal{H}^{(1)} \otimes \mathcal{H}^{(2)} \otimes \mathcal{H}^{(3)}$ found in light-cone [R. Metsaev, 2006] with preserving the irreducibility for the fields on the interacting level for each copy of interacting HS fields. ($i = 1, 2, 3$ enumerating the copy of fields, masses $(m)_3 = (m_1, m_2, m_3)$ and spins $(s)_3 = (s_1, s_2, s_3)$)

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As compared to the covariant form of the vertices **for reducible UIRs** obtained with BRST approach with incomplete BRST operator Q_c [R. Metsaev PLB (2013)],

$$S_{[1]}[\chi_c^{(1)}, \chi^{(2)}, \chi^{(3)}] \;=\; \sum_{i=1}^{3} \mathcal{S}_{0|s_i} + g \int \prod_{e=1}^{3} d\eta_0^{(e)} \Big({}_{s_e}\langle \chi^{(e)} \big| V^{M(3)}\rangle_{(s)_3} + h.c. \Big),$$

$$\delta_{[1]} \big| \chi_c^{(i)}\rangle_{s_i} = Q_c^{(i)} \big| \Lambda_c^{(i)}\rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda_c^{(i+1)} \big|_{s_{i+2}} \langle \chi_c^{(i+2)} \big|$$

$$+ (i+1 \leftrightarrow i+2) \Big) \big| V^{M(3)}\rangle_{(s)_3}, \qquad \boxed{\Big(\sum_{i=1}^{3} Q^{i}{}_{c},\ \mathcal{L}_{11}^{(i)}\Big) \big| V^{M(3)}\rangle_{(s)_3} = 0.}$$

Inclusion into the system a $(l_{11}|\Phi\rangle{=}0)$ equally with the rest differential constraints, in order to all irrep conditions extracting the particle $(m = 0, s)$; follow from $S_{[1]}$

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$$\delta_{[1]} | \chi_c^{(i)} \rangle_{s_i} = Q_c^{(i)} | \Lambda_c^{(i)} \rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda_c^{(i+1)} | {}_{s_{i+2}}\langle \chi_c^{(i+2)} |$$

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Inclusion into the system a $(l_{11}|\Phi\rangle{=}0)$ equally with the rest differential constraints, in order to all irrep conditions extracting the particle $(m = 0, s)$; follow from $S_{[1]}$

the interacting theory with complete Q leads to new contributions to the vertex with additional terms with fewer space-time derivatives of fields, also with multiple traces.

Dynamic of free field of helicity (spin) s is determined in the extended configuration spave with GI action by $\phi_{\mu(s)}$ and auxiliary fields $\phi_{1\mu(s-1)}, \ldots$. All of them are included in $|\chi\rangle_s$ described A. Pashnev, M. Tsulaia, MPLA (1998); I. Buchbinder, A. R., NPB 2012, [arXiv:1110.5044]

$$\mathcal{S}_{0|s}[\phi, \phi_1, \ldots] = \mathcal{S}_{0|s}[|\chi\rangle_s] = \int d\eta_{0\,s} \langle\chi|KQ|\chi\rangle_s,$$

$\mathcal{S}_{0|s}[|\chi\rangle_s]$ invariant w,r.t. reducible gauge transforms

$$\delta|\chi\rangle_s = Q|\Lambda\rangle_s, \;\; \delta|\Lambda\rangle_s = Q|\Lambda^1\rangle_s, \quad \delta|\Lambda^1\rangle_s = 0.$$

with $|\Lambda\rangle_s$, $|\Lambda^1\rangle_s$ gauge parameter vectors of 0- & 1-levels in Abelian gauge transforms .

Including interaction through systems of equations for cubic vertices

Cubic vertex for HS fields (s_1, s_2, s_3) within BRST approach includes 3 copies of vectors $|\chi^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)}\rangle_{s_i}$, $|\Lambda^{(i)1}\rangle_{s_i}$ with $|0\rangle^i$ and oscillators $a^{(i)\mu+}$..., $i = 1, 2, 3$.
Deformed action and gauge transformations

$$S_{[1]|(s)_3}[\chi^{(1)}, \chi^{(2)}, \chi^{(3)}] = \sum_{i=1}^{3} \mathcal{S}_{0|s_i} + g \int \prod_{e=1}^{3} d\eta_0^{(e)} \Big({}_{s_e}\langle \chi^{(e)} K^{(e)} \big| V^{(3)} \rangle_{(s)_3} + h.c. \Big),$$

$$\delta_{[1]} \big| \chi^{(i)} \rangle_{s_i} = Q^{(i)} \big| \Lambda^{(i)} \rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda^{(i+1)} K^{(i+1)} \big|_{s_{i+2}} \langle \chi^{(i+2)} K^{(i+2)} \big| + (i+1 \leftrightarrow i+2) \Big) \big| \widetilde{V}^{(3)} \rangle_{(s)_3},$$

$$\delta_{[1]} \big| \Lambda^{(i)} \rangle_{s_i} = Q^{(i)} \big| \Lambda^{(i)1} \rangle_{s_i} - g \int \prod_{e=1}^{2} d\eta_0^{(i+e)} \Big({}_{s_{i+1}}\langle \Lambda^{(i+1)1} K^{(i+1)} \big|_{s_{i+2}} \langle \chi^{(i+2)} K^{(i+2)} \big| + (i+1 \leftrightarrow i+2) \Big) \big| \widehat{V}^{(3)} \rangle_{(s)_3}$$

with unknown $\big|V^{(3)}\rangle_{(s)_3}$, $\big|\widetilde{V}^{(3)}\rangle_{(s)_3}$, $\big|\widehat{V}^{(3)}\rangle_{(s)_3}$. obeying x-locality

$$\big| V^{(3)} \rangle_{(s)_3} = \prod_{i=2}^{3} \delta^{(d)}(x_1 - x_i) V^{(3)} \prod_{j=1}^{3} \eta_0^{(j)} |0\rangle, \quad |0\rangle \equiv \otimes_{e=1}^{3} |0\rangle^e, \qquad [i+3 \simeq i]$$

Proposition 1 (generating equations for cubic vertices)

The Noether identities for the cubic deformation (Φ^3) of LF for the particles (m_i, s_i), $i = 1, 2, 3$

$$g^1 : \quad \delta_0 S_{1|(s)_3} + \delta_1 \sum_{i=1}^{3} \mathcal{S}_{0|s_i} = 0,$$
$$g^1 : \quad \Big(\delta_1^{(0)}\delta_0 \big|\chi^{(i)}\rangle_{s_i} + \delta_0^{(0)}\delta_1 \big|\chi^{(i)}\rangle_{s_i}\Big)|_{\partial S_{[1]}=0} = 0,$$

transforms to the local system of equations:

$$\mathcal{Q}(V^3, \widetilde{V}^3) = \sum_{k=1} Q^{(k)} \big|\widetilde{V}^{(3)}\rangle_{(s)_3} + Q^{(j)}\Big(\big|V^{(3)}\rangle_{(s)_3} - \big|\widetilde{V}^{(3)}\rangle_{(s)_3}\Big) = 0, \ j = 1, 2, 3.$$
$$\mathcal{Q}(\widetilde{V}^3, \widehat{V}^3) - Q^{(j+2)} |\widehat{V}^{(3)}\rangle_{(s)_3} = 0, j = 1, 2, 3.$$

which for coinciding vertexes $\widetilde{V}^{(3)} = \widehat{V}^{(3)} = V^{(3)}$ has universal form

$$\boxed{\big(Q^{tot}, \sigma^{(j)}\big)\big|V^{(3)}\rangle_{(s)_3} = \vec{0}.}, \qquad Q^{tot} = \sum_{k=1} Q^{(k)}$$

Thus. the vertex should be BRST-closed and composed from $a_\mu^{(i)+}, b^{(i)+}, d^{(i)+}, \eta^{(i)+}, \mathcal{P}^{(i)+}$ of spin (s_1, s_2, s_3)

BRST-BV approach with complete Q for free HS fields on $R^{1,d-1}$

To formulate BRST-BV action in minimal sector (AKSZ model) -> configuration space $M^{(s)}_{\min} = \{\Phi^A_{\min} = (A^i, C^{\alpha_0}, C^{\alpha_1})\}$ with classical A^i, 0th and 1-st-level ghost fields $C^{\alpha_0}, C^{\alpha_1}$, (in condensed notations) and their antifields $\{\Phi^*_{A\,\min} = (A^i, C^*_{\alpha_0}, C^*_{\alpha_1})\}$ organized into respective Fock space vectors $|C^0\rangle_s, |C^1\rangle_s$, $|\chi^*\rangle_s, |C^{*0}\rangle_s, |C^{*1}\rangle_s$

Proposition 2 (BRST-BV minimal action)

The BRST-BV minimal action for the particle (m, s) is given by functional on $\mathcal{H}_{g|tot} = \mathcal{H}_g \otimes \mathcal{H}_{gh} \otimes \mathcal{H}'$

$$S^{(s)}_{\min}[\Phi_{\min}, \Phi^*_{\min}] = S_{0|s}[|\chi_g\rangle_s] = \int d\eta_{0\,s}\langle\chi_g|KQ|\chi_g\rangle_s,$$

$$= \mathcal{S}_{0|s}[|\chi\rangle_s] + \int d\eta_0 \left\{ {}_s\langle\chi^*|KQ|C^0\rangle_s + {}_s\langle C^{*0}|KQ|C^1\rangle_s + h.c. \right\}$$

with Grassmann-even *generalized field-antifield vector* $|\chi_g\rangle_s$ and satisfies to the classical master equation in terms of Grassmann-odd Poisson (anti)bracket $(\bullet, \bullet)^{(s)}$:

$$\left(S_{0|s}[|\chi_g\rangle_s], S_{0|s}[|\chi_g\rangle_s]\right)^{(s)} = 2S_{0|s}[|\chi_g\rangle_s]\frac{\overleftarrow{\delta}}{\delta\Phi^A_{\min}}\frac{\overrightarrow{\delta}}{\delta\Phi^*_{A|\min}}S_{0|s}[|\chi_g\rangle_s] = 0$$

BRST-BV approach with complete Q for free HS fields on $R^{1,d-1}$

$$|\chi_g\rangle_s = |\chi_{\min}\rangle_s + |\chi^*_{\min}\rangle_s, \qquad |\chi^{(*)}_{\min}\rangle_s = |\chi^{(*)}\rangle_s + |C^{(*)0}\rangle_s + |C^{(*)1}\rangle_s \tag{10}$$

with 2^5 independent ghost monomials equal to number of field-antifield component vectors, by the rule $\eta_0^{n_0}\eta_I^{+n_I}\mathcal{P}_J^{+n_J}\phi^{\mu(s)} \mapsto \eta_0^{n_0+1\mathrm{mod}2}\mathcal{P}_I^{+n_I}\eta_J^{+n_J}\phi^*_{\mu(s)}$, $n's = 0, 1$ e.g. for

$$|\chi^*\rangle_s = \eta_0\Big\{|\Phi^*\rangle_s + \mathcal{P}_1^+\Big(\eta_1^+|\Phi^*_2\rangle_{s-2} + \eta_{11}^+|\Phi^*_{21}\rangle_{s-3} + \eta_1^+\eta_{11}^+\mathcal{P}_{11}^+|\Phi^*_{22}\rangle_{s-6}\Big)$$
$$+ \mathcal{P}_{11}^+\Big(\eta_1^+|\Phi^*_{31}\rangle_{s-3} + \eta_{11}^+|\Phi^*_{32}\rangle_{s-4}\Big)\Big\} + \eta_1^+|\Phi^*_1\rangle_{s-1} + \eta_{11}^+|\Phi^*_{11}\rangle_{s-2}$$
$$+ \eta_1^+\eta_{11}^+\Big[\mathcal{P}_1^+|\Phi^*_{12}\rangle_{s-4} + \mathcal{P}_{11}^+|\Phi^*_{13}\rangle_{s-5}\Big],,$$

with antifield vectors in power of a^+_μ, d^+, b^+: with $\phi^*_{\mu(s)}$ in $|\phi^*\rangle_s = |\Phi^*\rangle_s|_{(d^+,b^+)=0}$.

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with antifield vectors in power of a^+_μ, d^+, b^+: with $\phi^*_{\mu(s)}$ in $|\phi^*\rangle_s = |\Phi^*\rangle_s|_{(d^+,b^+)=0}$.
The fields, fields vectors, ghost oscillators $\eta^I, \mathcal{P}_I$ satisfy to the Grassmann parity and ghost numbers distributions, for $(gh_H + gh_L) = gh_{\text{tot}}$: $(\epsilon, gh_{tot})|\chi_g\rangle = (0,0)$
BRST-BV action invariant w.r.t *minimal Lagrangian BRST-like transformations* (with a Grassmann-odd constant parameter μ, $(gh_H, gh_L)\mu = (0,-1)$) for $|\chi_{\min}\rangle_s$

$$\delta_B|\chi_{\min}\rangle_s \;=\; \mu\frac{\overrightarrow{\delta}}{\delta\big({}_s\langle\chi^*_{\min}|K\big)}S_{0|s}[|\chi_g\rangle_s] \;=\; \mu Q\big(|C^0\rangle_s + |C^1\rangle\rangle_s\big),$$

with constant antifields (as well as for the duals $\langle\chi^{(*)}_{\min}|$)

Deformation BRST-BV procedure for interacting HS fields on $R^{1,d-1}$

$k = 1, ..., N^2 - 1$ (for $SU(N)$) $k \geq 3$ samples of LFs with vectors $|\chi^{(j)}\rangle_{s_j}$, ghost fields $|C^{(j)0}\rangle_{s_j}$, $|C^{(j)1}\rangle_{s_j}$ and respective antifield vectors combined in k-copies of *generalized field-antifield vectors* $|\chi_g^{(j)}\rangle_{s_j}$ of the form (10) with $|0\rangle^j$ and
$B_a^{(j)} = \{a_\mu^{(j)}, b^{(j)}, d^{(j)}, \eta^{(j)I}, \mathcal{P}_I^{(j)}\}$, $B_a^{(j)+}$ for $j = 1, ..., k$.

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$$S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k] \; = \; \sum_{j=1}^{k} S^{m_j}_{0|s_j}[\chi_g^{(j)}] + \sum_{f=1}^{e} g^f \, S^{(m)_k}_{f|(s)_k}[(\chi_g)_k]; \qquad (11)$$

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$$S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k] \;=\; \sum_{j=1}^{k} S^{m_j}_{0|s_j}[\chi_g^{(j)}] + \sum_{f=1}^{e} g^f\, S^{(m)_k}_{f|(s)_k}[(\chi_g)_k]; \qquad (11)$$

where

$$S^{(m)_k}_{1|(s)_k}[(\chi_g)_k] = \sum_{1\leq i_1<i_2<i_3\leq k} \int \prod_{j=1}^{3} d\eta_0^{(i_j)} \Big({}_{s_{i_j}}\langle \chi_g^{(i_j)} K^{(i_j)} \big| V^{(3)}\rangle^{(m)_{(i)_3}}_{(s)_{(i)_3}} + h.c. \Big); \qquad (12)$$

$$S^{(m)_k}_{2|(s)_k}[(\chi_g)_k] = \sum_{1\leq i_1<i_2<i_3<i_4\leq k} \int \prod_{j=1}^{4} d\eta_0^{(i_j)} \Big({}_{s_{i_j}}\langle \chi_g^{(i_j)} K^{(i_j)} \big| V^{(4)}\rangle^{(m)_{(i)_4}}_{(s)_{(i)_4}} + h.c. \Big),$$

$\dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots\ \dots$

$$S^{(m)_k}_{e|(s)_k}[(\chi_g)_k] = \sum_{1\leq i_1<i_2<\ldots<i_e\leq k} \int \prod_{j=1}^{e} d\eta_0^{(i_j)} \Big({}_{s_{i_j}}\langle \chi_g^{(i_j)} K^{(i_j)} \big| V^{(e)}\rangle^{(m)_{(i)_e}}_{(s)_{(i)_e}} + h.c. \Big),$$

Proposition 3 (BV generating equations for cubic vertices)

The preservation of the number of physical d.o.f. determined by LFs for free HS fields with (m_j, s_j) follows from the master-equation in total field-antifield space $\Pi T^* M^{(s)_k}_{\min}$ for $\big(\bullet,\bullet\big)^{(s)_k} = \sum_j \big(\bullet,\bullet\big)^{(s_j)}$:

$$\Big(S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k], S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k]\Big)^{(s)_k} = 2S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k]\frac{\overleftarrow{\delta}}{\delta\Phi^{Aa}_{\min}}\frac{\overrightarrow{\delta}}{\delta\Phi^{*}_{Aa|\min}}S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k] = 0$$

which for the cubic approximation

$$g^1 : \Big(S^{(m)_k}_{[0]|(s)_k}[(\chi_g)_k], S^{(m)_k}_{1|(s)_k}[(\chi_g)_k]\Big)^{(s)_k} := WS^{(m)_k}_{1|(s)_k}[(\chi_g)_k] = 0,\ W^2 = 0$$

transforms to the local system of equations coinciding with one for BRST approach

$$\boxed{\big(Q^{tot}, \sigma^{(j)}\big)\big|V^{(3)}\rangle_{(s)_3} = \vec{0}.}, \quad Q^{tot} = \sum_{k=1} Q^{(k)}$$

Proposition 3 (BV generating equations for cubic vertices)

The preservation of the number of physical d.o.f. determined by LFs for free HS fields with (m_j, s_j) follows from the master-equation in total field-antifield space $\Pi T^* M^{(s)_k}_{\min}$ for $(\bullet,\bullet)^{(s)_k} = \sum_j (\bullet,\bullet)^{(s_j)}$:

$$\left(S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k], S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k]\right)^{(s)_k} = 2S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k]\frac{\overleftarrow{\delta}}{\delta\Phi^{Aa}_{\min}}\frac{\overrightarrow{\delta}}{\delta\Phi^{*}_{Aa|\min}}S^{(m)_k}_{[e]|(s)_k}[(\chi_g)_k] = 0$$

which for the cubic approximation

$$g^1 : \left(S^{(m)_k}_{[0]|(s)_k}[(\chi_g)_k], S^{(m)_k}_{1|(s)_k}[(\chi_g)_k]\right)^{(s)_k} := WS^{(m)_k}_{1|(s)_k}[(\chi_g)_k] = 0, \ W^2 = 0$$

transforms to the local system of equations coinciding with one for BRST approach

$$\boxed{(Q^{tot}, \sigma^{(j)})|V^{(3)}\rangle_{(s)_3} = \vec{0}.}, \qquad Q^{tot} = \sum_{k=1} Q^{(k)}$$

The equations for quartic: $2WS^{(m)_k}_{2|(s)_k}[(\chi_g)_k] + \left(S^{(m)_k}_{1|(s)_k}[(\chi_g)_k], S^{(m)_k}_{1|(s)_k}[(\chi_g)_k]\right)^{(s)_k} = 0$ and higher vertices have more complicated form and its solutions may be non-local

Cubic deformation of BRST-BV action (in condensed notations),

$$\begin{aligned} S^{(m)_k}_{[1]|(s)_k}\big(\Phi^a_{\min}, \Phi^*_{a|\min}\big) &= \sum_j S^{(s_j)}_{\min}[\Phi^{(j)}_{\min}, \Phi^{*(j)}_{\min}] + g\Big(S^{(m)_k}_{1|(s)_k}\big(A^a\big) \\ &+ \sum_{b,c,d}^{k} \big[A^*_{ib} R^{ib}_{1|\alpha_0 d, jc} A^{jc} C^{\alpha_0 d} + \frac{1}{2} C^*_{\gamma_0 b} F^{\gamma_0 b}_{\alpha_0 c \beta_0 d} C^{\alpha_0 c} C^{\beta_0 d}\big]\Big) \end{aligned}$$

$(a, b, d = 1, ..., k)$ with structural function $F^{\gamma_0 b}_{\alpha_0 c \beta_0 d}$ leads to the closure the algebra of deformed gauge transformations at linear approximation in g

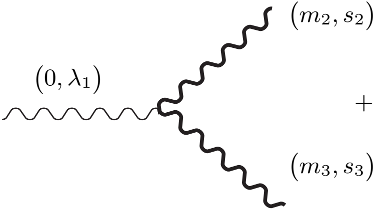
$$R^{ia}_{1|\alpha_0 b}(A) \frac{\overleftarrow{\delta}}{\delta A^{jc}} R^{j}_{0|\beta_0} - ((\alpha_0 b) \leftrightarrow (\beta_0 c)) = -R^{i}_{0|\gamma_0} F^{\gamma_0 a}_{\alpha_0 b \beta_0 c},$$

for $R^{ib}_{[1]|\alpha_0 c}(A) = R^{i}_{0|\alpha_0} \delta^b_c + g R^{ib}_{1|\alpha_0 c, ja} A^{ja}$.

General solution of BRST-BV equations for cubic vertices for HS fields of spins s_1, s_2, s_3

We derive 4 types of the cubic vertices in the approach with Q, in following different cases:

- $(0,\lambda_1) - (0,\lambda_2) - (0,\lambda_3)$ I. Buchbinder, A.R, PLB (2021), [arXiv:2105.12030], A.R. (TMPh) [arXiv:2303.02870];
- $(0,\lambda_1) - (0,\lambda_2) - (m_3, s_3)$
- $(0,\lambda_1) - (m_2, s_2) - (m_2, s_3)$ I. Buchbinder, A.R, [arXiv:2212.07097]
- $(0,\lambda_1) - (m_2, s_2) - (m_3, s_3)$

$|V^{(3)}\rangle^{(0,m_2,m_3)}_{(\lambda_1,s_2,s_3)} \equiv$  $+ \quad \ldots\ldots$

;

and derive from known (cubic) vertices for reducible reps of $ISO(1,d-1)$ (Metsaev 2013) the ones for irreps in I.L. Buchbinder, A.R, [arXiv:2212.07097]

General solution for the cubic vertices for HS fields of helicities s_1, s_2, s_3

we seek Q^{tot}-BRST - closed solution, $\left(Q^{tot}, \sigma^{(j)}\right)\big|V^{(3)}\rangle_{(s)_3} = \vec{0}$
$\big|V^{(3)}\rangle \sim \sum\prod_i^k U_{j_i}^{(s_i)}\mathcal{Z}\mathcal{L}_{k_i}^{(i)}$ of specific homogeneous in oscillators (linear in ∂_μ) operators

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(1), Q^{tot}-BRST- closed forms $\mathcal{L}_{k_i}^{(i)}$, $i = 1, 2, 3$, $k_i = 1, ..., s_i$ & $\mathcal{Z}$ constructed from Q_c^{tot}-BRST- closed $L^{(i)}$, Z in Siegel; R. Metsaev, (2013)

where $deg_{(a^+,\eta^+)}L^{(i)} = 1$: $L^{(1)}\phi^{(1)}_{\mu(s_1)}\phi^{(2)}_{\nu(s_2)}\phi^{(3)}_{\rho(s_3)} \mapsto \phi^{(1)}_{\mu(s_1-1)\mu}\partial^\mu\phi^{(2)}_{\nu(s_2)}\phi^{(3)}_{\rho(s_3)}$,
$deg_{(a^+,\eta^+)}Z = 3$: $Z\phi^{(1)}_{\mu(s_1)}\phi^{(2)}_{\nu(s_2)}\phi^{(3)}_{\rho(s_3)} \mapsto \phi^{(1)}_{\mu(s_1-1)\mu}\partial^\rho\phi^{(2)}_{\nu(s_2-1)}{}^{\mu}\phi^{(3)}_{\rho(s_3-1)\rho} + c.(1,2,3)$,

$$\mathcal{L}_{k_i}^{(i)} = (L^{(i)})^{k_i-2}\Big((L^{(i)})^2 - \frac{\imath k_i!}{2(k_i-2)!}\eta_{11}^{(i)+}\left[2\mathcal{P}_0^{(i+1)} + 2\mathcal{P}_0^{(i+2)} - \mathcal{P}_0^{(i)}\right]\Big),$$
$$L^{(i)} = (p_\mu^{(i+1)} - p_\mu^{(i+2)})a^{(i)\mu+} - \imath\big(\mathcal{P}_0^{(i+1)} - \mathcal{P}_0^{(i+2)}\big)\eta_1^{(i)+}, \quad p_\mu^{(i)} = -i\partial_\mu^{(i)}$$
$$Z = L_{11}^{(12)+}L^{(3)} + L_{11}^{(23)+}L^{(1)} + L_{11}^{(31)+}L^{(2)}.$$

General solution for the cubic vertices for HS fields of helicities s_1, s_2, s_3

we seek Q^{tot}-BRST - closed solution, $\left(Q^{tot}, \sigma^{(j)}\right)\big|V^{(3)}\rangle_{(s)_3} = \vec{0}$

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$$\mathcal{L}^{(i)}_{k_i} = (L^{(i)})^{k_i-2}\Big((L^{(i)})^2 - \frac{\imath k_i!}{2(k_i-2)!}\eta^{(i)+}_{11}\left[2\mathcal{P}^{(i+1)}_0 + 2\mathcal{P}^{(i+2)}_0 - \mathcal{P}^{(i)}_0\right]\Big),$$

$$L^{(i)} = (p^{(i+1)}_\mu - p^{(i+2)}_\mu)a^{(i)\mu+} - \imath\big(\mathcal{P}^{(i+1)}_0 - \mathcal{P}^{(i+2)}_0\big)\eta^{(i)+}_1,\quad p^{(i)}_\mu = -i\partial^{(i)}_\mu$$

$$Z = L^{(12)+}_{11}L^{(3)} + L^{(23)+}_{11}L^{(1)} + L^{(31)+}_{11}L^{(2)}.$$

to get them we have used momenta conservation law, $\sum_{i=1}^3 p^{(i)}_\mu = 0$, and trace non-invariance of $L^{(i)}$: $(\widehat{L}^{(i)}_{11}(L^{(i)})^2|0\rangle \neq 0$ and also $\widehat{L}^{(i)}_{11}(L^{(ii+1)+}_{11})^2|0\rangle \neq 0)$

$$L^{(ii+1)+}_{11} = a^{(i)\mu+}a^{(i+1)+}_\mu - \frac{1}{2}\mathcal{P}^{(i)+}_1\eta^{(i+1)+}_1 - \frac{1}{2}\mathcal{P}^{(i+1)+}_1\eta^{(i)+}_1.$$

General solution for the cubic vertices for HS fields of helicities s_1, s_2, s_3

we seek Q^{tot}-BRST - closed solution, $\left(Q^{tot}, \sigma^{(j)}\right)\left|V^{(3)}\right\rangle_{(s)_3} = \vec{0}$

$\left|V^{(3)}\right\rangle \sim \sum \prod_i^k U_{j_i}^{(s_i)} \mathcal{Z}\mathcal{L}_{k_i}^{(i)}$ of specific homogeneous in oscillators (linear in ∂_μ) operators

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$$L^{(i)} = (p_\mu^{(i+1)} - p_\mu^{(i+2)})a^{(i)\mu+} - \imath\big(\mathcal{P}_0^{(i+1)} - \mathcal{P}_0^{(i+2)}\big)\eta_1^{(i)+}, \quad p_\mu^{(i)} = -i\partial_\mu^{(i)}$$

$$Z = L_{11}^{(12)+}L^{(3)} + L_{11}^{(23)+}L^{(1)} + L_{11}^{(31)+}L^{(2)}.$$

to get them we have used momenta conservation law, $\sum_{i=1}^3 p_\mu^{(i)} = 0$, and trace non-invariance of $L^{(i)}$: $(\widehat{L}_{11}^{(i)}(L^{(i)})^2|0\rangle \neq 0$ and also $\widehat{L}_{11}^{(i)}(L_{11}^{(ii+1)+})^2|0\rangle \neq 0)$

$$L_{11}^{(ii+1)+} = a^{(i)\mu+}a_\mu^{(i+1)+} - \frac{1}{2}\mathcal{P}_1^{(i)+}\eta_1^{(i+1)+} - \frac{1}{2}\mathcal{P}_1^{(i+1)+}\eta_1^{(i)+}.$$

BRST-closed cubic on oscillators operators $\mathcal{Z}$

(2), one has new "Trace" 2-, 4- , ..., $[s_i/2]$ forms in powers in $\to$

$$U^{(s_i)}_{j_i}(\eta^{(i)+}_{11}, \mathcal{P}^{(i)+}_{11}) \ := \ (\widehat{L}^{(i)+}_{11})^{(j_i-2)}\{(\widehat{L}^{+(i)}_{11})^2 - j_i(j_i-1)\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}\}, \quad i=1,2,3.$$

$\mathcal{Z}_j$ (instead of Z^j in BRST approach with incomplete Q_c) is determined from the condition to be Q^{tot} BRST closed, e.g. for $j=1$

$$\mathcal{Z}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j} = Z\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j} - \sum_{i=1}^{3}k_i\frac{b^{(i)+}}{h^{(i)}}[[L^{(i)}_{11}, Z\}, L^{(i)}\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-\delta_{ij}}$$
$$+\sum_{i\neq e}^{3}k_ik_e\frac{b^{(i)+}b^{(e)+}}{h^{(i)}h^{(e)}}[L^{(e)}_{11}, [[L^{(i)}_{11}, Z\}, L^{(i)}\}L^{(e)}\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-\delta_{ij}-\delta_{ej}}$$
$$-\sum_{i\neq e\neq o}^{3}k_ik_ek_o\frac{b^{(i)+}b^{(e)+}b^{(o)+}}{h^{(i)}h^{(e)}h^{(o)}}[L^{(o)}_{11}, [L^{(e)}_{11}, [[L^{(i)}_{11}, Z\}, L^{(i)}\}L^{(e)}\}L^{(0)}\}\prod_{j=1}^{3}\mathcal{L}^{(j)}_{k_j-1}.$$

For $j>1$ expression for $\mathcal{Z}_j$ is deduced analogously
(see A.R. PEPAN (2022) arXiv:2205.00488).

General (covariant) and partial solutions for the cubic vertices

general solution for covariant cubic vertex preserving irreps of $ISO(1,d-1)$ for HS fields (s_1, s_2, s_3) (thus correct degrees of freedom) when passing to interacting theory

$$|V^{(3)}\rangle_{(s)_3} = |V^{M(3)}\rangle_{(s)_3} + \sum_{(j_1,j_2,j_3)>0}^{([s_1/2],[s_2/2],[s_3/2])} U^{(s_1)}_{j_1} U^{(s_2)}_{j_2} U^{(s_3)}_{j_3} |V^{M(3)}\rangle_{(s)_3 - 2(j)_3},$$

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$|V^{M(3)}\rangle_{(s)_3-2(j)_3}$ determined with modified forms respecting trace $\mathcal{L}^{(i)}_{k_i}$, $\mathcal{Z}_j$

$$\begin{aligned} |V^{M(3)}\rangle_{(s)_3-2(j)_3} &= \textstyle\sum_k \mathcal{Z}_{1/2\{(s-2J)-k\}} \prod_{i=1}^{3} \mathcal{L}^{(i)}_{s_i-2j_i-1/2(s-2J-k)}|0\rangle, \\ (s,J) &= \left(\textstyle\sum_i s_i,\ \sum_i j_i\right). \end{aligned}$$

and enumerated by naturals (k,j_1,j_2,j_3) satisfying to the equations

$$\boxed{s-2J-2s_{\min} \leq k \leq s-2J, \quad k = s-2J-2p,\ p\in\mathbb{N}_0,\ 0\leq j_i \leq [s_i/2].}$$

General (covariant) and partial solutions for the cubic vertices

general solution for covariant cubic vertex preserving irreps of $ISO(1, d-1)$ for HS fields (s_1, s_2, s_3) (thus correct degrees of freedom) when passing to interacting theory

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$$\begin{aligned} |V^{M(3)}\rangle_{(s)_3-2(j)_3} &= \textstyle\sum_k \mathcal{Z}_{1/2\{(s-2J)-k\}} \prod_{i=1}^3 \mathcal{L}^{(i)}_{s_i - 2j_i - 1/2(s-2J-k)} |0\rangle, \\ (s, J) &= \left(\textstyle\sum_i s_i,\ \sum_i j_i\right). \end{aligned}$$

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$$\boxed{s - 2J - 2s_{\min} \leq k \leq s - 2J, \quad k = s - 2J - 2p,\ p \in \mathbb{N}_0,\ 0 \leq j_i \leq [s_i/2].}$$

General vertex besides modified terms contains new ones. These are linear in trace $U^{(s_i)}_{1_i} = \widehat{L}^{(i)+}_{11}$ for each field copy
as differed from $V^{M(3)}\rangle_{(s)_3-2(j)_3}$ in vertex: $b^{(i)+}$, $\eta^{(i)+}_{11}\mathcal{P}^{(i)+}_{11}$, $i = 1, 2, 3$.

Cubic vertex for massless HS fields with helicities $(0,0,s)$

r-parameter family of vertices for $r=1,...,[s/2]$ with restoring the dimensional coupling constants t_r ($\dim t_r = s+d/2-3-2r$, in metric units providing a dimensionless of the classical action) and with $h^{(i)}(s_i) = -s\delta_{i3} - (d-6)/2$,

$$|V^{(3)|0}_{(0,0,s)} \;=\; \sum_{r\geq 0}^{[s/2]} t_r U^{(s)}_r \mathcal{L}^{(3)}_{s-2r} = \sum_{r\geq 0}^{[s/2]} t_r U^{(s)}_r \sum_{i=0}^{[(s-2r)/2]} (-1)^i (L^{(3)})^{s-2(r-i)} \times$$

$$\times (\hat{p}^{(3)})^{2i} \frac{(s-2r)!}{i!2^i(s-2r-2i)!} \frac{(b^{(3)+})^i}{C(i,h^{(3)})},\; C(i,h) \equiv \prod_{i=0}^{n-1}(i+h(s))$$

with following decomposition in powers of $\eta^{(3)+}_1$ for the operators

$$\mathcal{L}^{(3)}_k \;=\; \mathcal{L}^{(3)0}_k - \imath\widehat{\mathcal{P}}^{(3)}_0 \eta^{(3)+}_1 \big(\mathcal{L}^{(3)}_{k-1}\big)' \;\equiv\; \mathcal{L}^{(3)}_k\big|_{\eta^{(3)+}_1=0}$$

$$-\imath\widehat{\mathcal{P}}^{(3)}_0 \eta^{(3)+}_1 \sum_{i=0}^{[k/2]} (-1)^i \big(\widehat{p}^{(3)}_\mu a^{(i)\mu+}\big)^{k-1-2i} (\hat{p}^{(3)})^{2i} \frac{k!}{i!2^i(k-1-2i)!} \frac{(b^{(3)+})^i}{C(i,h^{(3)})}.$$

The field-antifield structure of interacting theory is determined by the generalized vectors

$$|\chi^{(j)}_g\rangle_0 = |\chi^{(j)}_{\min}\rangle_0 + |\chi^{*(j)}_{\min}\rangle_0 = \big(\phi^{(j)}(x) + \eta^{(j)}_0 \phi^{*(j)}(x)\big)|0\rangle, \quad |\chi^{(3)}_g\rangle_s = |\chi^{(3)}_{\min}\rangle_s + |\chi^{*(3)}_{\min}\rangle_s$$

Cubic vertex for massless HS fields with helicities $(0, 0, s)$

The minimal BRST-BV action $S^{(0)3}_{1|(s)_3}[(\chi_g)_3]$ (12) in the first order approximation in g

$$S^{(0)3}_{[1]|(s)_3}[(\chi_g)_3] \; = \; \int d^dx \sum_{i=1}^{2} \phi^{(i)} \Box \phi^{(i)} + \int d\eta_0 \Big[{}_s\langle \chi^{(3)} | K^{(3)} Q^{(3)} | \chi^{(3)} \rangle_s$$

$$+ \Big\{ {}_s\langle \chi^{*(3)} | K^{(3)} \overrightarrow{s}_0 | \chi^{(3)} \rangle_s + {}_s\langle C^{*0(3)} | K^{(3)} \overrightarrow{s}_0 | C^{0(3)} \rangle_s + h.c. \Big\} \Big] + g S^{(0)3}_{1|(s)_3}[(\chi_g)_3],$$

$$S^{(0)3}_{1|(s)_3} = \int \prod_{i=1}^{3} d\eta_0^{(i)} \Big(\Big\{ {}_s\langle \chi^{(3)} K \big|_0 \langle \phi^{(2)} \big|_0 \langle \phi^{(1)} \big| V^{(3)} \rangle_s^0 + \sum_{j=1}^{2} {}_0\langle \phi^{*(j)} \big| K \overrightarrow{s}_1 \big| \phi^{(j)} \rangle_0 \Big\} + h.c. \Big),$$

where, the first line corresponds to classical action for free fields, the second line contains antifield terms with generator of initial BRST-like transformations for classical $|\chi^{(3)}\rangle_s$ and ghost $|C^{(3)0}\rangle_s$ fields. In turn, the first term in $S^{(0)3}_{1|(s)_3}$ means for cubic interacting part of classical action $S^{(0)3}_{1|(s)_3}[(\chi)_3]$ and the second one for deformed generator of BRST-like transformations $\delta_{1|B}$ with generator $\overrightarrow{s}_1$:

$$\delta_{1|B} |\phi^{(j)}\rangle_0 \; = \; \mu \overrightarrow{s}_1 |\phi^{(j)}\rangle_0 \; = \; -g\mu \int \prod_{i=1}^{2} d\eta_0^{(j+i)} {}_s\langle C^{(3)0} K^{(3)} \big|_0 \langle \phi^{(j+1)} \big| V^{(3)} \rangle^0_{(0,0,s)},$$

with untouched transformations for $|\chi^{(3)}\rangle$: $\delta_{1|B} |\chi^{(3)}\rangle = 0$.

Cubic vertex for massless HS fields with helicities $(0,0,s)$

Omitting the details of partial GF procedure and resolving part of EoM

$$l_{11}|\Phi^{(3)}\rangle_s + |\Phi_2^{(3)}\rangle_{s-2} = 0, \quad l_{11}|\Phi_k^{(3)}\rangle_{s-k} = 0,\ k=1,2,\ l_{11}|C_{\Xi}^{0(3)}\rangle_{s-1} = 0,$$
$$\delta_{0|B}\Big(|\Phi\rangle_s, |\Phi_1^{(3)}\rangle_{s-1}, |\Phi_2^{(3)}\rangle_{s-2}, |C_{\Xi}^{0(3)}\rangle_{s-1}\Big) = \mu\left(l_1^+, l_0, l_1, 0\right)|C_{\Xi}^{0(3)}\rangle_{s-1}. \quad (13)$$

As the result, the interacting part of action $S^{(0)_3}_{[1]|(s)_3}$ will contain 2 terms with fields $\phi^{(3)\nu(s)}_{0,0}$ and $\phi^{(3)\nu(s-2)}_{2|0,0}$ without $b^{(3)+}$-generated fields, so that, it is written as

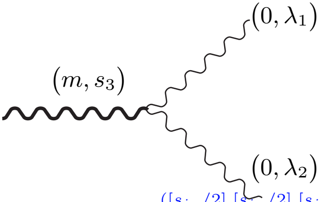
$$S^{(0)_3}_{1|(0,0,s)} = -2\int d^dx\Big[\sum_{r\geq 0}^{1} t_r \frac{(s-2r)!}{2^{2r}}\Big\{\sum_{u=0}^{s-2r}\frac{(-1)^u}{u!(s-2r-u)!} \quad (14)$$
$$\times\Big[\partial_{\nu_0}...\partial_{\nu_u}\phi^{(1)}\Big]\Big[\partial_{\nu_{u+1}}...\partial_{\nu_{s-2r}}\phi^{(2)}\Big]\Big\}\phi^{(3)\nu(s)}\prod_{p=1}^{r}\eta_{\nu_{s-2(r-p)-1}\nu_{s-2(r-p)}}$$
$$+t_0\Big\{\sum_{u=0}^{s-2}\frac{(-1)^u(s-2)!}{u!(s-2(r+1)-u)!}\Big(\partial_{\nu_0}...\partial_{\nu_u}\phi^{(1)}\Big)\Big(\partial_{\nu_{u+1}}...\partial_{\nu_{s-2}}\phi^{(2)}\Big)\Big\}\phi^{(3)\nu(s-2)}_{2|0,0}\Big],$$

Cubic vertex for massless HS fields with helicities $(0, 0, s)$

and generators of BRST-variations ($\overrightarrow{s}_{[1]}\phi^{(2)}(x_2) = -\overrightarrow{s}_{[1]}\phi^{(1)}(x_1)|_{[\phi^{(1)}(x_1)\to\phi^{(2)}(x_2)]}$)

$$\overrightarrow{s}_{[1]}\phi^{(1)}(x_1) = -gt_0 \int d^dx \Big\{ \sum_{u=0}^{s-1} \frac{(s-1)!}{u!(s-1-u)!} C_{\Xi}^{0(3)}{}^{\nu(s-1)}_{0,0}(x) \\ \times \Big(\partial_{\nu_{u+1}}...\partial_{\nu_{s-1}}\phi^{(2)}(x)\Big)\partial_{\nu_0}...\partial_{\nu_u} \Big\} \delta^{(d)}(x-x_1);$$

jointly with the free action (also with ones for the scalars), for free fields subject to the traceless constraints (13) may be served as interacting part of BRST-BV action for irreducible gauge theory in the triplet formulation for the fields in question the parity invariant vertex in multiplicative-like representation

$|V^{(3)}\rangle^{(0,0,m)}_{(\lambda_1,\lambda_2,s_3)} \quad \equiv$  $+ \quad \ldots\ldots$

;

$$\big|V^{(3)}\rangle^m_{(s)_{(i)_3}} = \big|V^{M(3)}\rangle^m_{(s)_{(i)_3}} + \sum_{(r_{i_1},r_{i_2},r_{i_3})>0}^{([s_{i_1}/2],[s_{i_2}/2],[s_{i_3}/2])} U^{(s_{i_1})}_{r_{i_1}} U^{(s_{i_2})}_{r_{i_2}} U^{(s_{i_3})}_{r_{i_3}} \big|V^{M(3)}\rangle^m_{(s)_{(i)_3}-2(r_i)_3},$$

$$V^{M(3)|m}_{(s)_{(i)_3}-2(r_i)_3} \quad = \quad \textstyle\sum_p \mathcal{L}^{(3)}_p \prod_{j=1}^3 \left(\mathcal{L}^{(i_j i_{j+1})+}_{11|1}\right)^{\tau_{i+2}},$$

is $(3+1)$-parametric family triple $(r_{i_j})_3$ respecting for number of traces and p for the minimal order of ∂_μ,

$$\tau_{i_j} = \frac{1}{2}\big(s_{(i)_3} - 2r_{(i)_3} - p\big) - s_{i_j}, \;\; j=1,2; \qquad \tau_k = \frac{1}{2}(s_{(i)_3} - 2r_{(i)_3} + p) - s + 2r_3,$$

$$\max\Big(0, (s-2r_3) - \sum^{2}(s_{i_j} - 2r_{i_j})\Big) \le p \le s - 2r_3 - \big|s_{i_1} - 2r_{i_1} - (s_{i_2} - 2r_{i_2})\big|,$$

General solution of BRST-BV equations for cubic vertices $(0,\lambda_1)$ – $(0,\lambda_2)$ –(m,s_3)

modified operators

$$
\begin{aligned}
\mathcal{L}_1^{(3)} &= L^{(3)} - [\widehat{L}_{11}^{(3)}, L^{(3)}\}\frac{b^{(3)+}}{h^{(3)}},\\
\widetilde{\mathcal{L}}_2^{(3)} &= (\mathcal{L}^{(3)})^2 - i\widehat{\mathcal{P}}_0^{(3)}\eta_{11}^{(3)+} - \widehat{l}_0^{(3)}\frac{b^{(3)+}}{h^{(3)}},\\
\widetilde{\mathcal{L}}_{2k}^{(3)} &= (\widetilde{\mathcal{L}}_2^{(3)})^k, \qquad \widetilde{\mathcal{L}}_{2k-1}^{(3)} = (\widetilde{\mathcal{L}}_2^{(3)})^{k-1}\mathcal{L}_1^{(3)},
\end{aligned}
$$

are invariant w.r.t. trace: $[\widehat{L}_{11}^{(3)}, \mathcal{L}_1^{(3)}\} = 0$, $\Rightarrow Q^{tot}$-closed.
One can check the Q^{tot}-closedness for the mixed-symmetry modified forms

$$
\begin{aligned}
\mathcal{L}_{11|1}^{(i_j i_{j+1})+} &= L_{11}^{(i_j i_{j+1})+} - \sum_{i_0} W_{(i_j i_{j+1})|0}^{(i_0)}\frac{b^{(i_0)+}}{h^{(i_0)}} + \frac{1}{2}\Big(\sum_{i_0\neq j_0}[\widehat{L}_{11}^{(j_0)}, W_{(i_j i_{j+1})|0}^{(i_0)}\}\\
&\times\frac{b^{(i_0)+}}{h^{(i_0)}}\frac{b^{(j_0)+}}{h^{(j_0)}} + \sum_{i_0}[\widehat{L}_{11}^{(i_0)}, W_{(i_j i_{j+1})|0}^{(i_0)}\}\frac{(b^{(i_0)+})^2}{h^{(i_0)}(h^{(i_0)}+1)}\Big),
\end{aligned}
$$

$|V^{(3)}\rangle \neq |V^{M(3)}\rangle$ not identical: $L_{11}^{(i)}\ |V^{M(3)}\rangle \neq 0$.

Correspondence

1) $|V^{M(3)}_{irrep}\rangle = |V^{M(3)}\rangle / L_{11}^{(i)} |V^{M(3)}\rangle$, 2) reducing $|V^{(3)}\rangle_{\mathcal{M}_{un} \to \mathcal{M}_c} = |\breve{V}^{(3)}\rangle$
Then $|V^{M(3)}_{irrep}\rangle = |\breve{V}^{(3)}\rangle$!

$$
|V^{M(3)}_{irrep}\rangle = \Bigg(1 - \sum_i \frac{1}{s_i - 2 + d/2} L_{11}^{(i)+} L_{11}^{(i)}
$$

$$
+\sum_{i_1}\Big[\prod_{k=1}^{2} \frac{1}{k(s_{i_1} - 1 - k + d/2)} (L_{11}^{(i_1)+})^2 (L_{11}^{(i_1)})^2 + \sum_{i_2>i_1}\prod_{k=1}^{2} \frac{1}{(s_{i_k} - 2 + d/2)} L_{11}^{(i_k)+} L_{11}^{(i_k)}\Big]
$$

$$
-\sum_{i_1}\Big[\prod_{k=1}^{3} \frac{1}{k(s_{i_1} - 1 - k + d/2)} (L_{11}^{(i_1)+})^3 (L_{11}^{(i_1)})^3 + \sum_{i_2>i_1}\prod_{k=1}^{2} \frac{1}{k(s_{i_1} - 1 - k + d/2)} \times
$$

$$
\times \frac{1}{(s_{i_2} - 2 + d/2)} (L_{11}^{(i_1)+})^2 L_{11}^{(i_2)+} (L_{11}^{(i_1)})^2 L_{11}^{(i_2)} + \prod_{k=1}^{3} \frac{1}{(s_k - 2 + d/2)} L_{11}^{(k)+} L_{11}^{(k)}\Big]\Bigg)|V^{M(3)}\rangle + \ldots
$$

$$
+ \ldots + (-1)^{\sum_i \left[\frac{s_i}{2}\right]} \prod_{i=1}^{3} \Big\{ \prod_{k=1}^{\left[\frac{s_i}{2}\right]} \frac{1}{k(s_i - 1 - k + d/2)} (L_{11}^{(i)+})^{\left[\frac{s_i}{2}\right]} (L_{11}^{(i)})^{\left[\frac{s_i}{2}\right]} \Big\} |V^{M(3)}\rangle.
$$

Conclusion

- BRST and BRST-BV approaches with complete BRST operator for irreducible interacting HS fields with integer spins in Minkowski spaces are developed;
- It is found general cubic interacting vertecies (off-shell) for irreducible interacting HS fields with integer helicities $\lambda_1, \lambda_2, \lambda_3$ on Minkowski $\mathbb{R}^{1,d-1}$ space;
- It constructed (off-shell) covariant cubic interaction vertex massless and massive irreducible HS fields with $(0, \lambda_1)$– $(0, \lambda_2)$– (m, s_3) with $(0, \lambda_1)$– (m, s_2)– (m, s_3) and with $(0, \lambda_1)$– (m_2, s_2)– (m_3, s_3) with some lower spin component examples within BRST approach. and reproduces new inputs into the vertex with traces and less numbers of space-time derivatives, including the terms without derivatives;
- It is suggested BRST-closed traceless cubic vertex in the BRST approach with incomplete BRST operator for irreducible interacting (massless and massive) HS fields. It appears by covariant analog of even-parity vertex suggested in the light-cone formalism [hep-th/0512342];
- it is found sufficient conditions for superalgebra of Q_c^{tot}, $\mathcal{L}_{11}^{(i)}$, $|V_c^{M(3)}\rangle$ $[Q_c^{tot}, \mathcal{L}_{11}^{(i)}] = 0$, $Q_c^{tot}|V_c^{M(3)}\rangle = 0$, $\mathcal{L}_{11}^{(i)}|V_c^{M(3)}\rangle = 0$ to preserve irreducibillity for interacting HS fields [I.L.Buchbinder, A.R, [arXiv:2304.10358] (2023)] in BRST-BFV approach.

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Thank you very much