Non-linear electrodynamics of exotic branes

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The fundamental string

Polyakov action [Deser, Zumino (1976); Brink, DiVecchia, Howe (1976), Polyakov (1981)]

$$S_{F1} = \frac{1}{2\pi\alpha'} \int d^2\sigma \left(\sqrt{h}\, h^{ab} G_{\mu\nu} + \epsilon^{ab} B_{\mu\nu} \right) \partial_a X^\mu \partial_b X^\nu . \tag{1}$$

The string interacts with (and sources)

- the metric $G_{\mu\nu}$;
- antisymmetric gauge-field $B_2 = \frac{1}{2} B_{\mu\nu} dX^\mu \wedge dX^\nu$ with $H_3 = dB_2$.

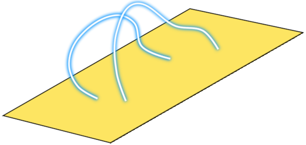
The closed string has more massless excitations $\Longrightarrow$ Ramond-Ramond supergravity fields

$$\begin{aligned} \text{IIA}: &\quad C_1\,, \quad C_3\,; \\ \text{IIB}: &\quad C_0\,, \quad C_2\,, \quad C_4\,. \end{aligned} \tag{2}$$

These interact with D-branes [Polchinski (1995)]

String theory is a theory of not only strings

D-branes



- String perturbation theory:

  D(irichlet)-branes are surfaces where open strings end

- Full theory:

  D-branes are dynamical objects of tension $T_{Dp} \sim \frac{1}{g_s}$

Dirac-Born-Infeld action for a Dp-brane:

$$S_{Dp} = \underbrace{T_{Dp} \int d^{p+1}\sigma e^{\varphi} \sqrt{\left| G_{ab} + B_{ab} + \frac{1}{2\pi\alpha'} F_{ab} \right|}}_{\text{kinetic (DBI) part}} + \underbrace{T_{D_p} \int C_{p+1} + \dots}_{\text{Wess-Zumino part}} \tag{3}$$

E. S. Fradkin, A. A. Tseytlin, “Non-linear electrodynamics from quantized strings” (1985)
E. S. Fradkin, A. A. Tseytlin, “Fields as excitations of quantized coordinates” (1985)

Main idea: integrate out string body coordinates keeping quantized coordinates of the string ends

Magnetic monopole

The NS-NS 2-form B_2 can be dualized: full non-linear

$$* d * H_3 + \cdots = 0 \quad \Longrightarrow \quad dH_7 = 0, \tag{4}$$

introducing the magnetic NS-NS 6-form potential $B_{\mu_1 \ldots \mu_6}$ interacting with a 5-brane (NS5)

Wess-Zumino coupling

$$\begin{aligned} S^{WZ}_{F1} &= T_{F1} \int_{\Sigma_2} B_2, && \text{electic coupling to } B_2 \\ S^{WZ}_{NS5} &= T_{NS5} \int_{\Sigma_6} B_6 + \ldots, && \text{electric coupling to } B_6, \\ T_{NS5} &\sim \frac{1}{g_s^2} \end{aligned} \tag{5}$$

Type IIB theory: the NS5-brane is S-dual to the D5-brane

[Eyras, Janssen, Lozano (1998)]

Magnetic graviton

The same can be done with the vielbein e_μ^α in the KK setup only linearly
$x^\mu = (x^m, z)$

$$\begin{aligned} f_2{}^z &= \frac{1}{2} f_{mn}{}^z dx^m \wedge dx^n, \\ f_2{}^z &\longrightarrow f_8{}^z = *f_2{}^z \longleftarrow A_{7,1} \end{aligned} \tag{6}$$

introducing the magnetic NS-NS (7,1)-potential $A_{m_1 \dots m_6 z, z}$ interacting with a 5-brane (KK5)

$$\begin{aligned} S^{WZ}_{KK5} &= T_{KK5} \int_{\Sigma_6} A_{7,1} + \dots, \quad \text{magnetic coupling to gravity,} \\ T_{KK5} &\sim \frac{1}{g_s^2} \end{aligned} \tag{7}$$

the KK5A-monopole is T-dual to the NS5B-brane [Eyras, Janssen, Lozano (1998)]

Full non-linear theory of the magnetic graviton is not known and probably not needed

[Hohm,Samtleben (2018)]

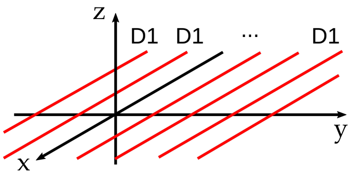
T-duality

E. S. Fradkin, A. A. Tseytlin, "Quantum Equivalence of Dual Field Theories," (1984)
T. H. Buscher, "A Symmetry of the String Background Field Equations," (1987)

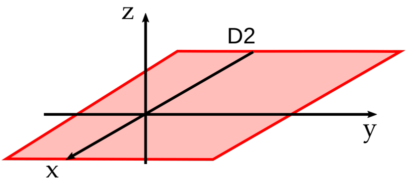
- Full quantum symmetry of the sigma-model [Fradkin, Tseytlin (1984)]
- Symmetry of string background field equations [Buscher (1987)]
- Maps (smeared) branes into each other

$$
\begin{aligned}
&\mathrm{D}p \longleftrightarrow \mathrm{D}(p \pm 1)\\
&\mathrm{NS5} \longleftrightarrow \mathrm{KK5} \longleftrightarrow 5_2{}^2 \longleftrightarrow 5_2{}^3 \longleftrightarrow 5_2{}^4
\end{aligned}
\tag{8}
$$

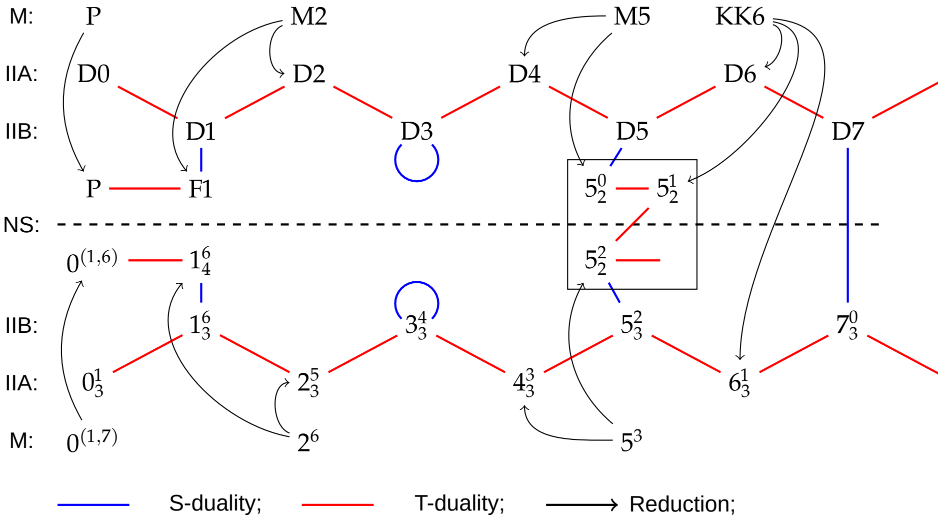
[Shelton, Taylor, Wecht (2005)]



$\overset{T_y}{\longleftrightarrow}$

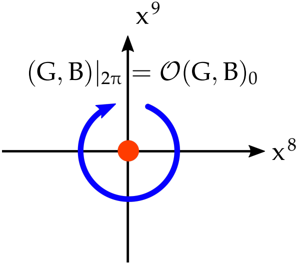


Branes of co-dim ≥ 2



[de Boer, Obers, Pioline, Shigemori, ...]

Exotic NS 5^2_2-brane



- 5 spatial volume directions
- $T \sim g_s^{-2}$;
- 2 special cycles
- described by a T-fold

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
NS5 (5^0_2) :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\bullet$
KK5 (5^1_2) :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\bullet$	$\odot$
5^2_2 :	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\bullet$	$\bullet$	$\odot$	$\odot$

(9)

$$\begin{bmatrix} G + BG^{-1}G & BG^{-1} \\ G^{-1}B & G^{-1} \end{bmatrix}(2\pi) = \underbrace{\begin{bmatrix} 1 & \beta^{mn} \\ 0 & 1 \end{bmatrix}}_{\mathcal{O}} \begin{bmatrix} G + BG^{-1}G & BG^{-1} \\ G^{-1}B & G^{-1} \end{bmatrix}(0) \tag{10}$$

Non-geometric 5^2_2 background

- KK5 monopole $H(r) = 1 + \frac{h}{r}$, $r^2 = \rho^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$

$$
\begin{aligned}
ds^2 &= H(r)^{-1}\Big(dz + A_i dx^i\Big)^2 + H(r)\delta_{ij}dx^i dx^j + ds^2_{(1,5)},\\
B &= 0, \quad 2\partial_{[i}A_{j]} = \varepsilon_{ijk}\partial_k H(r)
\end{aligned} \tag{11}
$$

- 5^2_2-brane (smeared), $H(\rho) = 1 + h\log\frac{\mu}{\rho}$, $\quad \rho^2 = (x^1)^2 + (x^2)^2$

$$
\begin{aligned}
ds^2 &= \frac{H}{H^2 + h^2\theta^2}\Big((dx^4)^2 + (dx^3)^2\Big) + H\Big((dx^1)^2 + (dx^2)^2\Big) + ds^2_{(1,5)},\\
B &= \frac{h\theta}{H^2 + h^2\theta^2} dx^4 \wedge dx^3
\end{aligned} \tag{12}
$$

- Monodromy $\theta \to \theta + 2\pi$

[Shelton, Taylor, Wecht (2005), deBoer, Shigemori (2012)]

Objects of string theory

1. NS branes :
   - F1 string, $T \sim g_s^0$, — el. interacts with $B_{\mu\nu}$
   - NS5 brane, $T \sim g_s^{-2}$ — magn. interacts with $B_{\mu\nu}$
   - KK5 monopole, $T \sim g_s^{-2}$ — magn. interacts with $G_{z\mu}$
2. Dp-branes (R-R sector)
   - Dp-brane, $T \sim g_s^{-1}$ — el. interacts with $C_{\mu_1,\dots,\mu_{p+1}}$,
3. Exotic branes, co-dim.≤ 2
   - NS sector: 5_2^2, 5_2^3, 5_2^4 branes, $T \sim g_s^{-2}$ — interact with T-duals of $B_{\mu\nu}$ and $G_{z\mu}$
   - R sector: P-branes, — interact with U-duals of $C_{(p)}$

"Exotic" branes are natural objects of string theory
appearing on equal footing with Dp and $\mathrm{NS5}$ branes.

The statements

- We construct a single action describing 5_2^p branes living in a doubled space-time;
- A particular brane is chosen by a projection to the ordinary space-time, equiv. orientation in the doubled space-time;

[Blair,EtM(2017), Bergshoeff,Kleinschmidt,EtM,Riccioni(2019), Molina,EtM(2022)]

- Exotic branes, i.e. $p \geq 1$, generate backgrounds depending on dual coordinates $\tilde{x}_\mu$;
- These solve equations of double field theory

[Bakhmatov,Kleinschmidt,EtM (2016)]

- Dependence on dual coordinates has been shown to result from the string world-sheet instanton corrections [Harvey, Jensen (2005), Kimura,Sasaki,Shiozawa (2018)]

- Similar backgrounds have been found for exotic branes of M-theory

[Bakhmatov,Berman,Kleinschmidt,Otsuki,EtM (2017), Berman,EtM,Otsuki(2018)]

String on a torus $\mathbb{T}^D$

- Momentum and winding modes of the closed string are combined

$$\mathcal{P}^M = \begin{bmatrix} p^m \\ w_n \end{bmatrix} \Longrightarrow \mathbb{X}^M = \begin{bmatrix} X_R^m + X_L^m \\ X_R^m - X_L^m \end{bmatrix} \text{ coordinates [Tseytlin (1990)]} \tag{13}$$

- Mass spectrum is invariant under $O(D, D)$;

$$M^2 = \mathcal{P}^M \mathcal{H}_{MN} \mathcal{P}^N + \dots,$$

generalized metric:

$$\mathcal{H}_{MN} = \begin{bmatrix} g - Bg^{-1}B & Bg^{-1} \\ g^{-1}B & g^{-1} \end{bmatrix}, \quad \mathcal{H} \to \mathcal{O}^T \mathcal{H} \mathcal{O}, \quad \mathcal{O} \in O(D, D) \tag{14}$$

- The invariant dilaton

$$d = \varphi + \frac{1}{4} \log g \tag{15}$$

Doubled space-time

- Momentum and winding modes of the closed string are combined

$$\mathcal{P}^M = \begin{bmatrix} p^m \\ w_n \end{bmatrix} \Longrightarrow \mathbb{X}^M = \begin{bmatrix} x^m \\ \tilde{x}_m \end{bmatrix} \text{ coordinates} \tag{16}$$

- Local $O(10,10)$-covariant geometry

$$\begin{aligned} L_\Lambda T^M &= \text{Shift}[T] + GL(n)\text{-rotation}[T] && \text{Riemannian geometry} \\ \mathcal{L}_\Lambda T^M &= \text{Shift}[T] + O(10,10)\text{-rotation}[T]. && \text{Generalized geometry} \end{aligned} \tag{17}$$

- Consistency condition: fields depend on either x^m or its dual $\tilde{x}_m$

[Hohm, Hull, Zwiebach]

T-duality frames

	world-volume							transverse directions							
	x^0	x^1	x^2	x^3	x^4	x^5		y^1	y^2	y^3	y^4	$\tilde{y}_1$	$\tilde{y}_2$	$\tilde{y}_3$	$\tilde{y}_4$
NS5	×	×	×	×	×	×	|	•	•	•	•	k	k	k	k
KK5	×	×	×	×	×	×	|	•	•	•	k	k	k	k	•
Q	×	×	×	×	×	×	|	•	•	k	k	k	k	•	•
R	×	×	×	×	×	×	|	•	k	k	k	k	•	•	•
R′	×	×	×	×	×	×	|	k	k	k	k	•	•	•	•

(18)

• — localization direction
k — Killing direction

$$\eta_{MN} k_a^M k_b^N = 0, \quad \eta_{MN} = \begin{bmatrix} 0 & 1_{4\times 4} \\ 1_{4\times 4} & 0 \end{bmatrix} \tag{19}$$

Killing vectors $k_a^M = (k_a^m; \tilde{k}_{am})$ satisfying the constraint and will define the embedding

Covariant potentials

NS sector [Bergshoeff, Kleinschmidt, Nutma, Peñas, Riccioni, ...]

$$
\left.\begin{aligned}
5_2{}^0 : \quad & B_{\mu_1\dots\mu_6}\varepsilon^{\mu_1\cdots\mu_6\mu\nu\rho\sigma} = D^{\mu\nu\rho\sigma}\\
5_2{}^1 : \quad & B_{\mu_1\dots\mu_7,\sigma}\varepsilon^{\mu_1\cdots\mu_7\mu\nu\rho} = D^{\mu\nu\rho}{}_{\sigma}\\
5_2{}^2 : \quad & B_{\mu_1\dots\mu_8,\rho\sigma}\varepsilon^{\mu_1\cdots\mu_8\mu\nu} = D^{\mu\nu}{}_{\rho\sigma}\\
5_2{}^3 : \quad & B_{\mu_1\dots\mu_9,\nu\rho\sigma}\varepsilon^{\mu_1\cdots\mu_9\mu} = D^{\mu}{}_{\nu\rho\sigma}\\
5_2{}^4 : \quad & B_{\mu_1\dots\mu_1 0,\mu\nu\rho\sigma}\varepsilon^{\mu_1\cdots\mu_1 0} = D_{\mu\nu\rho\sigma}
\end{aligned}\right\} D^{MNKL}, \quad M,N = 1,\dots,20, \tag{20}
$$

R-R sector

$$
|\mathcal{C}\rangle = \sum_p \frac{1}{2^p p!}\mathcal{C}_{\mu_1\dots\mu_p}\Gamma^{\mu_1\cdots\mu_p}|0\rangle \tag{21}
$$

Clifford algebra of O(10,10)

$$
\{\Gamma_\mu, \Gamma^\nu\} = \delta_\mu{}^\nu, \quad \Gamma_\mu|0\rangle = 0. \tag{22}
$$

The invariant action

$$
\begin{aligned}
S = &-\mathcal{T}_5 \int d^6\sigma e^{-2d}\hat{\tau}\sqrt{\det h}\sqrt{-\det\left\|\mathcal{H}_{MN}\hat{\partial}_a Y^M \hat{\partial}_b Y^N - \hat{\tau}^{-1} e^d (\det h)^{-\frac{1}{4}}\langle\lambda_{br}|\mathcal{G}_{ab}\rangle\right\|}\\
&-\mathcal{T}_5 \int D^{M_1\dots M_4} T_{M_1\dots M_{10}} \hat{\partial} Y^{M_5}\wedge\dots\wedge\hat{\partial}Y^{M_{10}} + S_{WZ}\left[|\lambda_{br}\rangle, \mathcal{G}\right]\\
\hat{\tau} = &\sqrt{1 + e^{2d}(\det h)^{-\frac{1}{2}}\langle\lambda_{br}|\mathcal{C}\rangle^2},
\end{aligned}
\tag{23}
$$

Information of the orientation

1. $k_a{}^M k_b{}^N \eta_{MN} = 0$ bosonic Killing vectors
2. $k_a{}^M \Gamma_M |\lambda_{br}\rangle = 0$ spinorial brane charge
3. $T^{M_1\dots M_{10}} = \epsilon^{a_1\dots a_{10}} k_{a_1}^{M_1}\dots k_{a_{10}}^{M_{10}}$ bosonic brane charge

Background

DFT-monopole interacting with background fields

$$S_{\rm Full} = S_{\rm DFT}[\mathcal{H}_{MN}, d] + S_{5-\rm brane}[Y^M] \tag{24}$$

Solution dual to the NS5-brane background (localized KK5-monopole):

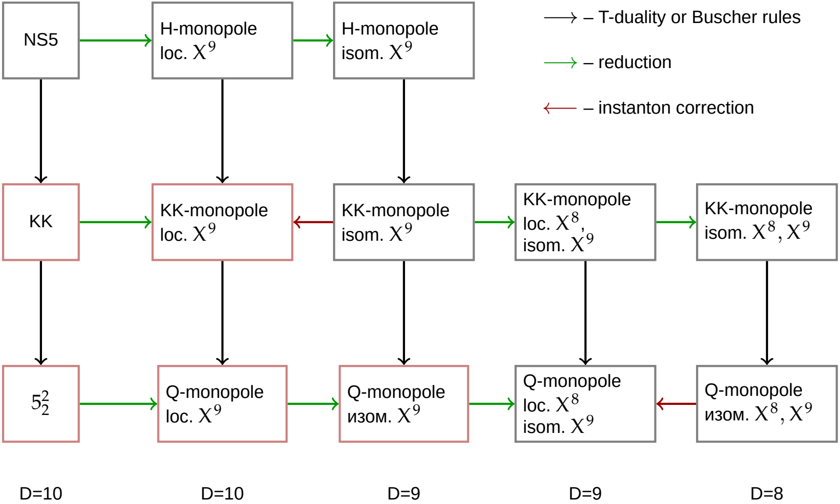
$$ds^2 = H(r)^{-1}\left(dz^2 + A_i dx^i\right)^2 + H(r)\,\delta_{ij}dx^i dx^j + ds^2_{(1,5)}, \quad B = 0 \tag{25}$$

Dependence on $\tilde{z}$ comes from counting worldsheet instanton contributions

[Tong (2002), Harvey, Jenssen (2005)]

$$H(\tilde{z}, r) = 1 + \frac{h}{\tilde{z}^2 + r^2} \Longrightarrow 1 + \frac{h}{2r}\left(1 + \sum_{k=1}^{\infty} e^{-kr-ik\tilde{z}} + \sum_{k=1}^{\infty} e^{-kr+ik\tilde{z}}\right) \tag{26}$$

Согласованность с общей картиной



[Jensen, Harvey, Gregory, Moore, Kimura, Sasaki]

T-duality frames: D-branes

The same is true for Dp-brane backgrounds

	0	1	2	3	4	5	6	7	8	9	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{4}$	$\tilde{5}$	$\tilde{6}$	$\tilde{7}$	$\tilde{8}$	$\tilde{9}$
D0	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×	×	×
D1	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×	×
D2	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×	×
D3	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×	×
D4	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×	×
D5	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×	×
D6	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×	×
D7	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×	×
D8	×	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●	×
D9	×	×	×	×	×	×	×	×	×	×	●	●	●	●	●	●	●	●	●

Open questions

- Full gauge invariant action for the KK5-monopole: gauge parameters depending on $\tilde{z}$
- Consistent treatment of the background/effectiver action correspondence for exotic branes (the magnetic graviton problem)
- Instanton correction and T-covariant description for D-branes

Спасибо за внимание!