On Z-dominance, shift symmetry and spin-locality in HS theory

A.V. Korybut
based on arXiv:2212.05006 (with V.Didenko)

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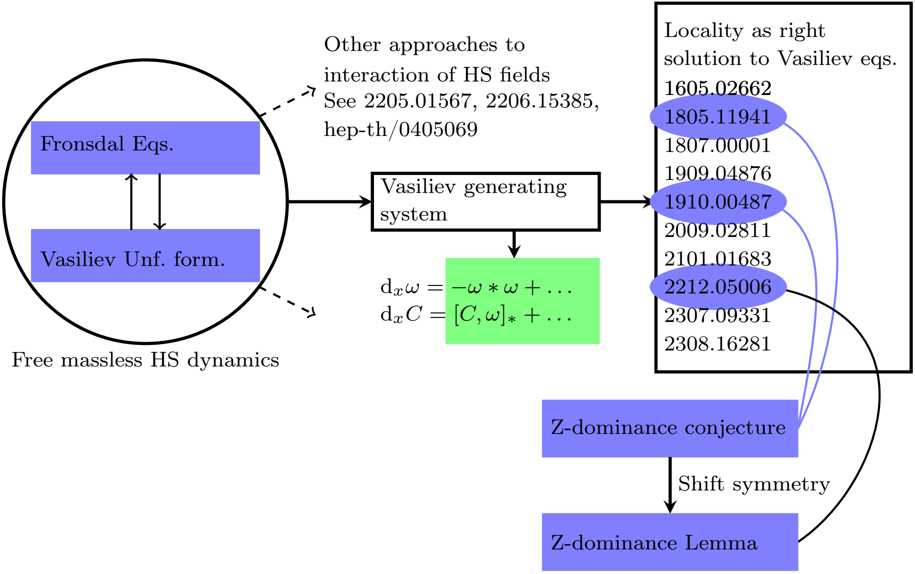
Other approaches to
interaction of HS fields
See 2205.01567, 2206.15385,
hep-th/0405069
Fronsdal Eqs.
Vasiliev Unf. form.
Free massless HS dynamics
Vasiliev generating system
$d_x\omega = -\omega * \omega + \dots$
$d_x C = [C, \omega]_* + \dots$
Locality as right
solution to Vasiliev eqs.
1605.02662
1805.11941
1807.00001
1909.04876
1910.00487
2009.02811
2101.01683
2212.05006
2307.09331
2308.16281
Z-dominance conjecture
Shift symmetry
Z-dominance Lemma

Fronsdal fields are packed in one-forms $dx^\mu \omega_{\mu\,\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x)$ and zero-forms $C_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x)$ which are themselves packed in generating functions ω and C

$$\omega(Y,x) = \sum_{n,m} dx^\mu \omega_{\mu\,\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x) y^{\alpha_1}\dots y^{\alpha_n}\bar y^{\dot\alpha_1}\dots\bar y^{\dot\alpha_m}; \;\; m+n = 2(s-1),$$

$$C(Y,x) = \sum_{n,m} C_{\alpha_1\dots\alpha_n,\dot\alpha_1\dots\dot\alpha_m}(x) y^{\alpha_1}\dots y^{\alpha_n}\bar y^{\dot\alpha_1}\dots\bar y^{\dot\alpha_m}; \;\; |m-n| = 2s.$$

Linear equations on zero-forms with stripped Ys read

$$D^L_{\beta\dot\beta} C_{\alpha\dots\alpha,\dot\alpha\dots\dot\alpha} \sim C_{\alpha\dots\alpha\beta,\dot\alpha\dots\dot\alpha\dot\beta} \tag{1}$$

Nonlinear terms in interraction

$$d_x C + [\omega, C]_* = \Upsilon(\omega, C, C) + \dots$$

$$\Upsilon(\omega,C,C)_{\gamma\dots\gamma,\dot\gamma\dots\dot\gamma} = \#\left(1 + \#\epsilon^{\alpha\beta} + \dots\right)\left(1 + i\#\bar\epsilon^{\dot\alpha\dot\beta} + \dots\right) C_{\alpha\dots\alpha,\dot\alpha\dots\dot\alpha} C_{\beta\dots\beta,\dot\beta\dots\dot\beta}$$

Finite amount of contraction in either dotted or undotted indicies = Spin-Locality. For lowest order verticies spin-locality implies space-time locality however higher order correction might drastically change (1). Nonetheless there is additional stronger requirement called projective-compactness which implies spacetime locality in any order (M.A.Vasiliev 2208.02004).

Spin-locality and Z-dominance

Solving equations of the Vasiliev generating system, i.e. equations of the form

$$\mathrm{d}_Z \Phi_n = J(\Phi_{n-1}, \Phi_{n-2}, \dots)$$

one can systematically obtain higher order corrections to interaction vertices which acquire the following form (schematically)

$$\Upsilon(\omega, C, \dots, C) = \sum \int_0^1 d\mathcal{T} \dots e^{i\mathcal{T} z_\alpha y^\alpha + \dots} \omega(\mathcal{T}z + \dots) C(\mathcal{T}z + \dots) \dots C(\mathcal{T}z + \dots) =$$
$$= \int d\mathcal{T} \delta(\mathcal{T}) e^{i\mathcal{T} z_\alpha y^\alpha + \dots} \dots \omega(\mathcal{T}z + \dots) C(\mathcal{T}z + \dots) \dots C(\mathcal{T}z + \dots).$$

If equations were solved "properly"one obtains Z-dominated nonlocality

$$\Upsilon(\omega, C, \dots, C) =$$
$$= \sum \int_0^1 d\mathcal{T} \dots e^{i\mathcal{T} z_\alpha y^\alpha + \underbrace{i\mathcal{T} \partial_{m\alpha} \partial_n{}^\alpha}_{} + \dots} \omega(\mathcal{T}z + \dots) C(\mathcal{T}z + \dots) \dots C(\mathcal{T}z + \dots)$$

Spin-locality and Z-dominance

However Z-dominated nonlocalities might not lead to spin-local vertices

$$\sum \int_0^1 d\mathcal{T} \ldots e^{i\mathcal{T} z_\alpha y^\alpha + \underbrace{i\mathcal{T}\partial_{m\alpha}\partial_n{}^\alpha} + \ldots} \omega(\mathcal{T}z + \ldots) C(\mathcal{T}z + \ldots) \ldots C(\mathcal{T}z + \ldots) =$$

$$= \int d\mathcal{T}\delta(\mathcal{T})\, \omega C^n + \int d\mathcal{T}\delta(\mathcal{T})\, (\partial_m \partial_n) \omega C^n + \int d\mathcal{T}\delta(\mathcal{T})\, (\partial_m \partial_n)^2 \omega C^n + \ldots$$

All (anti)holomorphic vertices of the third order in C with Z-dominated nonlocalities were found in 2009.02811 (V.Didenko, O.Gelfond, AK, M.Vasiliev). Despite the possible obstruction manifestly spin-local vertices were indeed extracted from them in 2101.01683 (O.Gelfond, AK). Perhaps some sort of symmetry is responsible for truncation!

Combined all the contribution to the holomorphic third order vertex can written in the remarkable factorized form

$$\Upsilon(y,t,p_j) = \int \mathscr{D}\rho\, \Pi^\alpha(y,t,p_j|\,\rho) \circledast \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T}) z_\alpha\, e^{i\mathcal{T} z_\alpha (y-P_z)^\alpha} \times$$

$$\times\, \omega(\mathsf{y}_\omega) C(\mathsf{y}_1)\ldots C(\mathsf{y}_3)\Big|_{\mathsf{y}=0}.$$

Here

$$t = -i\frac{\partial}{\partial \mathsf{y}_\omega},\;\; p_i = -i\frac{\partial}{\partial \mathsf{y}_i},\; P_z(t,p_j|\,\rho) = \rho^z_t t + \rho^z_1 p_1 + \ldots \rho^z_3 p_3\,, P_y = \ldots, P_t = \ldots.$$

$$\Pi^\alpha(y,t,p_j|\,\rho) := \sum_k \mathcal{R}_k(\rho)\pi^\alpha_k(y,t,p_j)\exp\Big\{-iy_\alpha {P_y}^\alpha - it_\alpha P^\alpha_t\Big\},$$

$$\int \mathscr{D}\rho := \int \mathrm{d}^n \rho\,.$$

«Half-product»

$$\phi(y) \circledast \Gamma(z,y) := \frac{1}{(2\pi)^2}\int \mathrm{d}^2u\, \mathrm{d}^2v\, e^{iuv}\, \phi(y+u)\Gamma(z-v,y)\,.$$

Function Π^α does not contain any nonlocalities! π^α is a polynomial in each argument.

Z-dominated nonlocality comes after half-product computation

$$\int \mathscr{D}\rho\, \Pi^{\alpha}(y,t,p_j|\,\rho) \circledast \int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T}) z_{\alpha}\, e^{i\mathcal{T} z_{\alpha}(y-P_z)^{\alpha}} =$$

$$= \int_0^1 d\mathcal{T} \int \mathscr{D}\rho \ldots \exp\left\{ i\mathcal{T} z_{\alpha}(y-P_z)^{\alpha} - \underbrace{i\mathcal{T} P_{z\alpha} P_y{}^{\alpha}} + i(1-\mathcal{T}) y^{\alpha} P_{y\alpha} + \ldots \right\}$$

Generating system guarantees that $\Upsilon(y,t,p_j)$ is z-independent

$$\frac{\partial}{\partial z^{\alpha}} \left(\int \mathscr{D}\rho\, \Pi^{\beta}(y,t,p_j|\,\rho) \circledast \int_0^1 \mathrm{d}\mathcal{T}\, \frac{1-\mathcal{T}}{\mathcal{T}} \mathcal{T} z_{\beta}\, e^{i\mathcal{T} z_{\alpha}(y-P_z)^{\alpha}} \right) = 0$$

In terms of half-product this condition can be rewritten as

$$\int \mathscr{D}\rho\, \Pi^{\beta}(y,t,p_j|\,\rho)(y-P_z)_{\beta} \circledast \int_0^1 \mathrm{d}\mathcal{T}\, \mathcal{T} z_{\alpha} e^{i\mathcal{T} z_{\alpha}(y-P_z)^{\alpha}} = 0\,.$$

Since it is true for all z it should be true in particular for $z=0$ which turns into y-independence condition

$$\frac{\partial}{\partial y^{\alpha}} \int_0^1 \mathrm{d}\mathcal{T} \int \mathscr{D}\rho\, \mathcal{T}(y-P_z)^{\beta} \Pi_{\beta}\big((1-\mathcal{T})(y-P_z)+P_z, t, p_j|\,\rho\big) = 0\,.$$

Scenario for obtaining manifestly Z-independent expression

If one manages to rewrite left part of half-product in the form

$$\Pi^{\beta}(y,t,p_j|\,\rho) = (y-P_z)^{\beta}\Pi(y,t,p_j|\,\rho)\,,$$

then after simple algebra it is possible to integrate by parts and bring the expression to the form

$$\int \mathscr{D}\rho\,(y-P_z)^{\beta}\Pi(y,t,p_j|\,\rho) \circledast \int_0^1 \mathrm{d}\mathcal{T}\,\frac{1-\mathcal{T}}{\mathcal{T}}\mathcal{T} z_{\beta}\,e^{i\mathcal{T} z_{\alpha}(y-P_z)^{\alpha}} = \\ = i\int \mathscr{D}\rho\,\Pi(y,t,p_j|\,\rho)\,.$$

I.e. one needs to «divide» $\Pi^{\beta}(y,t,p_j|\,\rho)$ over $(y-P_z)^{\beta}$.

Division formula

Consider an identity which rests on Schouten identity and partial integration

$$\Pi_\sigma(y,t,p_j|\,\rho) \equiv (y-P_z)_\sigma \int_0^1 \mathrm{d}\xi \,\frac{\partial}{\partial (y-P_z)^\beta}\Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\,\rho\big)+ \\ + \frac{\partial}{\partial y^\sigma}\int_0^1 \mathrm{d}\xi\,(y-P_z)^\beta \Pi_\beta\big(\xi(y-P_z)+P_z,t,p_j|\,\rho\big)\,.$$

It turns out that

$$\int \mathscr{D}\rho\,(y-P_z)^\alpha \int_0^1 \mathrm{d}\xi\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\,\rho\big)\circledast\int_0^1 \mathrm{d}\mathcal{T}\,\frac{1-\mathcal{T}}{\mathcal{T}}\mathcal{T} z_\alpha\, e^{i\mathcal{T} z_\alpha (y-P_z)^\alpha} = \\ = \int \mathscr{D}\rho\,\Pi^\alpha(y,t,p_j|\,\rho)\circledast\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})z_\alpha\, e^{i\mathcal{T} z_\alpha (y-P_z)^\alpha}\,.$$

$$\int \mathscr{D}\rho\,\Pi^\alpha(y,t,p_j|\,\rho)\circledast\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})z_\alpha\, e^{i\mathcal{T} z_\alpha (y-P_z)^\alpha}\Big|_{z=0} = \\ = i\int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big(\xi(y-P_z)+P_z,t,p_j|\,\rho\big)$$

Shift symmetry

For C^2 holomorphic vertices one can see

$$\Upsilon_{C^2}(y,t,p_1+a,p_2-a) = e^{ia_\alpha (t+y)^\alpha}\Upsilon_{C^2}(y,t,p_1,p_2)\,.$$

Analogous property is valid for cubic vertices

$$\Upsilon_{C^3}(y,t,p_1+a,p_2-a,p_3+a) = e^{ia_\alpha (t+y)^\alpha}\Upsilon_{C^3}(y,t,p_1,p_2,p_3)\,.$$

To see that such property indeed take place note that under above transformation derivatives effectively transform as

$$P_z(t,p_1+a,p_2-a,p_3+a) = P_z(t,p_1,p_2,p_3)\,,$$

$$P_y(t,p_1+a,p_2-a,p_3+a) = a + P_y(t,p_1,p_2,p_3)\,,$$

$$P_t(p_1+a,p_2-a,p_3+a) = a + P_t(p_1,p_2,p_3)\,,$$

$$\pi(y,t,p_1+a,p_2-a,p_3+a) = \pi(y,t,p_1,p_2,p_3)\,.$$

From all above properties it follows that left part of half-product transforms as

$$\Pi_\sigma(y,p_1+a,p_2-a,p_3+a) = e^{ia_\alpha (t+y)^\alpha}\Pi_\sigma(y,p_1,p_2,p_3)\,,$$

which eventually leads to the shift symmetry of the cubic vertex.

Reminder equals zero

Recall the division formula (ξ was changed to $(1-\mathcal{T})$)

$$\Pi_\sigma(y,t,p_j\,|\,\rho) \equiv (y-P_z)_\sigma \int_0^1 \mathrm{d}\mathcal{T}\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j\,|\,\rho\big)+ \\ +\frac{\partial}{\partial y^\sigma}\int_0^1 \mathrm{d}\mathcal{T}\,(y-P_z)^\beta\Pi_\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j\,|\,\rho\big)\,.$$

and z-indepedence condition (y-independence)

$$\frac{\partial}{\partial y^\alpha}\int_0^1 \mathrm{d}\mathcal{T}\int \mathscr{D}\rho\,\mathcal{T}(y-P_z)^\beta\Pi_\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j\,|\,\rho\big)=0\,.$$

Introduce two auxilary functions

$$\mathscr{Z}(t,p_i) := \int \mathscr{D}\rho \int_0^1 \mathrm{d}\mathcal{T}\,\mathcal{T}(y-P_z)^\beta\Pi_\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_i\big)\,,$$

$$\mathscr{R}(y,t,p_i) := \int \mathscr{D}\rho \int_0^1 \mathrm{d}\mathcal{T}\,(y-P_z)^\beta\Pi_\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_i\big)\,.$$

which are different only in measure in $\mathcal{T}$.

Due to shift symmetry one can deduce that $\mathscr{R}$ and $\mathscr{Z}$ are related by differential equation

$$\left(1+(y+\Delta)^{\alpha}\frac{\partial}{\partial y^{\alpha}}\right)\mathscr{R}(y,t,p)=\mathscr{Z}(t,p),\quad \Delta_{\alpha}:=\left(t_{\alpha}+i(-1)^{j+1}\frac{\partial}{\partial p_{j}^{\alpha}}\right).$$

Generic solution has the form

$$\mathscr{R}(y,t,p)=\mathscr{R}_{0}(y,t,p)+\mathscr{Z}(t,p)\,.$$

Solutions to homogeneous equation $\mathscr{R}_0$ are pathological, i.e. they are either not compatible with Lorentz symmetry or non analytic $\Longrightarrow$

$$\mathscr{R}(y,t,p)=\mathscr{Z}(t,p)\,.$$

Spin locality

After integration over all ρs division formula turns into

$$\int \mathscr{D}\rho\, \Pi_\alpha(y,t,p_j|\,\rho) = \int \mathscr{D}\rho\,(y-\underline{P_z})_\alpha \int_0^1 \mathrm{d}\mathcal{T}\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j|\,\rho\big)\,,$$

where l.h.s. is local by assumption. R.h.s. is a combination of two distinctive parts

$$y_\alpha \int \mathscr{D}\rho \int_0^1 \mathrm{d}\mathcal{T}\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j|\,\rho\big) -$$
$$- \int \mathscr{D}\rho\, P_{z\alpha} \int_0^1 \mathrm{d}\mathcal{T}\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j|\,\rho\big)\,,$$

the first term here is simply $y_\alpha \Upsilon(y,t,p)$.

$$\int \mathscr{D}\rho\, y^\alpha \Pi_\alpha(y,t,p_j|\,\rho) = -\int \mathscr{D}\rho\, y^\alpha P_{z\alpha} \int_0^1 \mathrm{d}\mathcal{T}\,\frac{\partial}{\partial y^\beta}\Pi^\beta\big((1-\mathcal{T})(y-P_z)+P_z,t,p_j|\,\rho\big)\,.$$

Hence the only non local contribution might be proportional to y_α

Term that causes problems can be rewritten in the corresponding way and initial equations turns into

$$\begin{aligned}\int \mathscr{D}\rho\, \Pi_\alpha(y,t,p_j|\,\rho) = &-\int \mathscr{D}\rho\, P_{z\alpha}\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta \Pi^\beta(\mathcal{T}P_z,t,p_j|\,\rho)+\\ &+y_\alpha\int \mathscr{D}\rho \int_0^1 \mathrm{d}\xi\, \partial_\beta\Pi^\beta(\xi y,t,p_j|\,\rho)-\\ &-y_\alpha \int \mathscr{D}\rho\int_0^1 \mathrm{d}\xi\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta\Pi^\beta\big((1-\mathcal{T})\xi y+\mathcal{T}P_z,t,p_j|\,\rho\big)-\\ &-\int \mathscr{D}\rho\int_0^1 \mathrm{d}\xi\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})^2\big(y^\sigma P_{z\sigma}\big)\partial_\alpha\partial_\beta\Pi^\beta\big((1-\mathcal{T})\xi y+\mathcal{T}P_z,t,p_j|\,\rho\big)\,. \end{aligned} \tag{2}$$

Hence

$$\widetilde{\Upsilon}(y,t,p_j) = i\int \mathscr{D}\rho\int_0^1 \mathrm{d}\xi\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta\Pi^\beta\big((1-\mathcal{T})\xi y+\mathcal{T}P_z,t,p_j|\,\rho\big) \tag{3}$$

is spin local by virtue of spin locality of the others. However it is not exactly the vertex. Vertex is given by

$$\Upsilon(y,t,p_j) = i\int \mathscr{D}\rho\int_0^1 \mathrm{d}\mathcal{T}\,(1-\mathcal{T})\partial_\beta\Pi^\beta\big((1-\mathcal{T})y+\mathcal{T}P_z,t,p_j|\,\rho\big)\,. \tag{4}$$

Connection between $\widetilde{\Upsilon}$ and Υ

$$\widetilde{\Upsilon}(y,t,p_j) = \int_0^1 \mathrm{d}\xi\,\Upsilon(\xi y,t,p_j)\,,\quad \Upsilon(y,t,p_j) = \left(y^\alpha\frac{\partial}{\partial y^\alpha}+1\right)\widetilde{\Upsilon}(y,t,p_j)\,. \tag{5}$$

Conclusion (Results and open questions)

- Shift symmetry of the holomorphic vertices was discovered

$$\Upsilon^{\eta}_{C^n}(y,t,p_1+a,p_2-a,...) = e^{ia_\alpha(y+t)^\alpha}\Upsilon^{\eta}_{C^n}(y,t,p_1,p_2,...)\,.$$

  This property was also found in the vertices that comes from generating system developed by Didenko (2209.01966). Counterpart for mixed vertices is not developed yet

- Shift symmetry turns to be the missing assumption for the proof of the Z-dominance conjecture.
- Effective method to obtain manifestly spin local vertices form Z-dominated is still lacking
- What is the realtion between shift symmetry and compact spin locality?
- How shift symmetry restrics differential homotopy?