$\mathcal{N} = 2$ supersymmetric higher spins in the harmonic approach

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Outline

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Supersymmetry and higher spins

- Supersymmetric higher-spin theories provide a bridge between superstring theory and low-energy (super)gauge theories.
- Free massless bosonic and fermionic higher spin field theories: Fronsdal, 1978; Fang, Fronsdal, 1978.
- The natural tools to deal with supersymmetric theories are off-shell superfield methods. In the superfield approach the supersymmetry is closed on the off-shell supermultiplets and so is automatically manifest.
- The component approach to $4D$, $\mathcal{N}=1$ supersymmetric free massless higher spin models: Courtright, 1979; Vasiliev, 1980.
- The complete off-shell $\mathcal{N}=1$ superfield Lagrangian formulation of $\mathcal{N}=1, 4D$ free higher spins: Kuzenko et al, 1993, 1994.

- An off-shell superfield Lagrangian formulation for higher-spin **extended** supersymmetric theories, with all supersymmetries manifest, was unknown for long even for free theories.
- This gap was filled in I. Buchbinder, E. Ivanov, N. Zaigraev, JHEP 12 (2021) 016. An off-shell manifestly $\mathcal{N}=2$ supersymmetric unconstrained formulation of $4D, \mathcal{N}=2$ super Fronsdal theory for integer spins was constructed in the harmonic superspace approach.
- Manifestly $\mathcal{N}=2$ supersymmetric off-shell cubic couplings of $4D, \mathcal{N}=2$ to the matter hypermultiplets were further constructed in I. Buchbinder, E. Ivanov, N. Zaigraev, 2022, 2023.
- Quite recently, we generalized HSS non-conformal construction to the case of $\mathcal{N}=2$ superconformal multiplets and their hypermultiplet coupling (arXiv:2404.19016 [hep-th], JHEP 08 (2024) 120).
- Our papers opened a new domain of applications of the harmonic superspace formalism, that time in $\mathcal{N}=2$ higher-spin theories.

Harmonic superspace

- In $4D$, the only self-consistent off-shell superfield formalism for $\mathcal{N}=2$ (and $\mathcal{N}=3$) theories is the harmonic superspace approach (Galperin, Ivanov, Kalitzin, Ogievetsky, Sokatchev, 1984, 1985).

- Harmonic $\mathcal{N}=2$ superspace:

$$Z = (x^m\,,\ \theta^\alpha_i\,,\ \bar\theta^{\dot\alpha j}, u^{\pm i}), \quad u^{\pm i} \in SU(2)/U(1),\ u^{+i}u^-_i = 1.$$

- Analytic harmonic $\mathcal{N}=2$ superspace:

$$\zeta_A = (x^m_A, \theta^{+\alpha}, \bar\theta^{+\dot\alpha}, u^{\pm i}),\ \theta^{+\alpha,\dot\alpha} := \theta^{\alpha,\dot\alpha i}u^+_i,\ x^m_A := x^m - 2i\theta^{(i}\sigma^m\bar\theta^{j)}u^+_i u^+_j$$

- All basic $\mathcal{N}=2$ superfields are analytic:

$$\underline{\text{SYM}}: \quad V^{++}(\zeta_A)\,,\ \ \underline{\text{matter hypermultiplets}}:\ q^+(\zeta_A)\,,\ \bar q^+(\zeta_A)$$

$$\underline{\text{supergravity}}: \quad H^{++m}(\zeta_A)\,, H^{++\alpha+}(\zeta_A)\,, H^{++5}(\zeta_A)\,, \hat\alpha = (\alpha, \dot\alpha)$$

$\mathcal{N} = 2$ spin 1 multiplet

- An instructive example is Abelian $\mathcal{N} = 2$ gauge theory,

$$V^{++}(\zeta_A)\,, \quad \delta V^{++} = D^{++}\Lambda(\zeta_A)\,,\; D^{++} = \partial^{++} - 4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}\partial_{\alpha\dot{\alpha}}\,.$$

- Wess-Zumino gauge ($8 + 8$ off-shell degrees of freedom):

$$V^{++}(\zeta_A) = (\theta^+)^2\phi + (\bar{\theta}^+)^2\bar{\phi} + 2i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}}A_{\alpha\dot{\alpha}}$$
$$+ (\bar{\theta}^+)^2\theta^{+\alpha}\psi^i_\alpha u^-_i + (\theta^+)^2\bar{\theta}^+_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 D^{(ik)}u^-_i u^-_k\,.$$

- Invariant action:

$$S \sim \int d^{12}Z\,\left(V^{++}V^{--}\right),\; D^{++}V^{--} - D^{--}V^{++} = 0\,,\; \delta V^{--} = D^{--}\Lambda\,,$$
$$[D^{++}, D^{--}] = D^0, \quad D^0\,V^{\pm\pm} = \pm 2\,V^{\pm\pm}\,.$$

$\mathcal{N}=2$ spin 2: linearized $\mathcal{N}=2$ supergravity

- Analogs of $V^{++}(\zeta_A)$ are the following set of analytic gauge potentials:

$$\Big(h^{++m}(\zeta_A)\,,\ h^{++5}(\zeta_A)\,,\ h^{++\hat{\mu}+}(\zeta_A)\Big), \quad \hat{\mu}=(\mu\,,\dot{\mu})\,,$$
$$\delta_\lambda h^{++m} = D^{++}\lambda^m + 2i\big(\lambda^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\sigma^m_{\alpha\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big)\,,$$
$$\delta_\lambda h^{++5} = D^{++}\lambda^5 - 2i\big(\lambda^{+\alpha}\theta^+_\alpha - \bar{\theta}^+_{\dot{\alpha}}\bar{\lambda}^{+\dot{\alpha}}\big), \delta_\lambda h^{++\hat{\mu}+} = D^{++}\lambda^{+\hat{\mu}}\,.$$

- Wess-Zumino gauge:

$$h^{++m} = -2i\theta^+\sigma^a\bar{\theta}^+\Phi^m_a + \big[(\bar{\theta}^+)^2\theta^+\psi^{m\,i}u^-_i + c.c.\big] + \ldots$$
$$h^{++5} = -2i\theta^+\sigma^a\bar{\theta}^+C_a + \ldots\,, \quad h^{++\mu+} = \ldots$$

- The residual gauge freedom:

$$\lambda^m \ \Rightarrow\ a^m(x)\,,\ \lambda^5 \ \Rightarrow\ b(x)\,,\ \lambda^{\mu+} \ \Rightarrow\ \epsilon^{\mu i}(x)u^+_i + \theta^{+\nu}l_{(\nu}{}^{\mu)}(x)\,.$$

- The physical fields are $\Phi^m_a, \psi^{m\,i}_\mu, C_a$ ($(\mathbf{2},\mathbf{3/2},\mathbf{3/2},\mathbf{1})$ on shell). In the "physical" gauge:

$$\Phi^m_a \sim \Phi_{\beta\dot{\beta}\alpha\dot{\alpha}} \Rightarrow \Phi_{(\beta\alpha)(\dot{\beta}\dot{\alpha})} + \varepsilon_{\alpha\beta}\varepsilon_{\dot{\alpha}\dot{\beta}}\Phi\,.$$

$\mathcal{N} = 2$ spin 3 and higher spins

- The spin 3 triad of analytic gauge superfields is introduced as :

$$\{h^{++(\alpha\beta)(\dot\alpha\dot\beta)}(\zeta)\,,\ h^{++\alpha\dot\alpha}(\zeta),\ h^{++(\alpha\beta)\dot\alpha+}(\zeta),\ h^{++(\dot\alpha\dot\beta)\alpha+}(\zeta)\}\,,$$

$$\delta h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = D^{++}\lambda^{(\alpha\beta)(\dot\alpha\dot\beta)} + 2i\big[\lambda^{+(\alpha\beta)(\dot\alpha}\bar\theta^{+\dot\beta)} + \theta^{+(\alpha}\bar\lambda^{+\beta)(\dot\alpha\dot\beta)}\big],$$

$$\delta h^{++\alpha\dot\alpha} = D^{++}\lambda^{\alpha\dot\alpha} - 2i\big[\lambda^{+(\alpha\beta)\dot\alpha}\theta^{+}_{\beta} + \bar\lambda^{+(\dot\alpha\dot\beta)\alpha}\bar\theta^{+}_{\dot\beta}\big],$$

$$\delta h^{++(\alpha\beta)\dot\alpha+} = D^{++}\lambda^{+(\alpha\beta)\dot\alpha}\,,\ \delta h^{++(\dot\alpha\dot\beta)\alpha+} = D^{++}\lambda^{+(\dot\alpha\dot\beta)\alpha}\,.$$

- The bosonic physical fields in WZ gauge are collected in

$$h^{++(\alpha\beta)(\dot\alpha\dot\beta)} = -2i\theta^{+\rho}\bar\theta^{+\dot\rho}\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho} + \dots\ \ h^{++\alpha\dot\alpha} = -2i\theta^{+\rho}\bar\theta^{+\dot\rho}C^{\alpha\dot\alpha}_{\rho\dot\rho} + \dots$$

- The physical gauge fields are $\Phi^{(\alpha\beta)(\dot\alpha\dot\beta)}_{\rho\dot\rho}$ (spin 3 gauge field), $C^{\alpha\dot\alpha}_{\rho\dot\rho}$ (spin 2 gauge field) and $\psi^{(\alpha\beta)(\dot\alpha\dot\beta)i}_{\gamma}$ (spin 5/2 gauge field). The rest of fields are auxiliary. On shell, $\mathbf{(3, 5/2, 5/2, 2)}$.

- The general case with the maximal spin **s** is spanned by the analytic gauge potentials

$$h^{++\alpha(s-1)\dot{\alpha}(s-1)}(\zeta), h^{++\alpha(s-2)\dot{\alpha}(s-2)}(\zeta), h^{++\alpha(s-1)\dot{\alpha}(s-2)+}(\zeta), h^{++\dot{\alpha}(s-1)\alpha(s-2)+}(\zeta),$$

where $\alpha(s) := (\alpha_1 \ldots \alpha_s), \dot{\alpha}(s) := (\dot{\alpha}_1 \ldots \dot{\alpha}_s)$.

- The relevant gauge transformations can also be defined and shown to leave, in the WZ-like gauge, the physical field multiplet $(\mathbf{s}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1/2}, \mathbf{s}-\mathbf{1})$.

- The on-shell spin contents of $\mathcal{N}=2$ higher-spin multiplets;

$\underline{spin\ 1}:\ 1,\ (1/2)^2,\ (0)^2$

$\underline{spin\ 2}:\ 2,\ (3/2)^2,\ 1$

$\underline{spin\ 3}:\ 3,\ (5/2)^2,\ 2$

....

$\underline{spin\ s}:\ s,\ (s-1/2)^2,\ s-1$

- Each spin enters the direct sum of these multiplets twice, in accord with the general Vasiliev theory of $4D$ higher spins. The off-shell contents of the spin **s** multiplet: $8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_B + 8[\mathbf{s}^2 + (\mathbf{s}-\mathbf{1})^2]_F$.

Hypermultiplet couplings

- The construction of interactions in the theory of higher spins is a very important (albeit difficult) task.
- Supersymmetric $\mathcal{N} = 1$ generalizations of the bosonic cubic vertices with matter were explored in terms of $\mathcal{N} = 1$ superfields by Gates, Koutrolikos, Kuzenko, I. Buchbinder, E. Buchbinder and many others.
- In JHEP 05 (2022) 104 we have constructed the off-shell manifestly $\mathcal{N} = 2$ supersymmetric cubic couplings $(\frac{1}{2}, \frac{1}{2}, \mathbf{s})$ of an arbitrary higher integer superspin $\mathbf{s}$ gauge $\mathcal{N} = 2$ multiplet to the hypermultiplet matter in $4D, \mathcal{N} = 2$ harmonic superspace.
- In our approach $\mathcal{N} = 2$ supersymmetry of cubic vertices is always manifest and off-shell, in contrast, e.g., to the non-manifest light-cone formulations (Metsaev, 1905.11357, 1909.05241).

- The starting point is the $\mathcal{N}=2$ hypermultiplet off-shell free action:

$$S = \int d\zeta^{(-4)}\,\mathcal{L}^{+4}_{free} = -\int d\zeta^{(-4)}\,\frac{1}{2} q^{+a}\mathcal{D}^{++}q^{+}_{a}\,, a = 1, 2.$$

- Analytic gauge potentials for any spin **s** with the correct transformation rules can be recovered by proper gauge-covariantization of the harmonic derivative $\mathcal{D}^{++}$. The simplest option is gauging of $U(1)$,

$$\delta q^{+a} = -\lambda_0 J q^{+a}, \quad J q^{+a} = i(\tau_3)^a_{\ b} q^{+b}\,,$$
$$\mathcal{D}^{++} \Rightarrow \mathcal{D}^{++} + \hat{\mathcal{H}}^{++}_{(1)}\,, \quad \hat{\mathcal{H}}^{++}_{(1)} = h^{++}J\,,$$
$$\delta_\lambda \hat{\mathcal{H}}^{++}_{(1)} = [\mathcal{D}^{++}, \hat{\Lambda}]\,, \quad \hat{\Lambda} = \lambda J \Rightarrow \delta_\lambda h^{++} = \mathcal{D}^{++}\lambda\,.$$

- In $\mathcal{N}=2$ supergravity, that is for $\mathbf{s}=2$,

$$S_{(2)} = -\int d\zeta^{(-4)}\,\frac{1}{2} q^{+a}(\mathcal{D}^{++} + \mathcal{H}_{(2)})q^{+}_{a}\,, \quad \delta\mathcal{H}_{(2)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(2)}],$$
$$\mathcal{H}_{(2)} = h^{++M}(\zeta)\partial_M,\ \hat{\Lambda}_{(2)} = \lambda^M(\zeta)\partial_M,\ M := (\alpha\dot{\beta}, 5, \hat{\mu}+).$$

- For higher **s** all goes analogously. For $\mathbf{s}=3$

$$S_{(3)} = -\int d\zeta^{(-4)}\,\frac{1}{2} q^{+a}(\mathcal{D}^{++} + \mathcal{H}_{(3)}J)q^{+}_{a}\,,$$
$$\delta\mathcal{H}_{(3)} = [\mathcal{D}^{++}, \hat{\Lambda}_{(3)}], \quad \mathcal{H}_{(3)} = h^{++\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}, \quad \hat{\Lambda}_{(3)} = \lambda^{\alpha\dot{\alpha}\,M}(\zeta)\partial_M\partial_{\alpha\dot{\alpha}}$$

Superconformal couplings

- Free conformal higher-spin actions in $4D$ Minkowski space were pioneered by Fradkin & Tseytlin, 1985; Fradkin & Linetsky, 1989, 1991. Since then, a lot of works on (super)conformal higher spins followed (e.g., Segal, 2003, Kuzenko *et al*, 2017, 2023 ......).
- (Super)conformal higher-spin theories are considered as a basis for all other types of higher-spin models. Non-conformal ones follow from the superconformal ones through couplings to the superfield compensators.
- In (Buchbinder, Ivanov, Zaigraev, arXiv:2404.19016 [hep-th]), we extended the off-shell $\mathcal{N}=2, 4D$ higher spins and their hypermultiplet couplings to the superconformal case. Rigid $\mathcal{N}=2, 4D$ superconformal symmetry plays a crucial role in fixing the structure of the theory. Similar results were recently obtained by Kuzenko, Raptakis, arXiv: 2407.21573 [hep-th], in projective $\mathcal{N}=2, 4D$ superspace.
- $\mathcal{N}=2, 4D$ SCA preserves harmonic analyticity and is a closure of the rigid $\mathcal{N}=2$ supersymmetry and special conformal symmetry

$$\delta_\epsilon \theta^{+\hat{\alpha}} = \epsilon^{\hat{\alpha}i} u^+_i\,, \;\; \delta_\epsilon x^{\alpha\dot{\alpha}} = -4i\left(\epsilon^{\alpha i}\bar{\theta}^{+\dot{\alpha}} + \theta^{+\alpha}\bar{\epsilon}^{\dot{\alpha}i}\right) u^-_i\;, \hat{\alpha} = (\alpha, \dot{\alpha})\,,$$

$$\delta_k \theta^{+\alpha} = x^{\alpha\dot{\beta}} k_{\beta\dot{\beta}} \theta^{\hat{\beta}}\,, \;\; \delta_k x^{\alpha\dot{\alpha}} = x^{\rho\dot{\alpha}} k_{\rho\dot{\rho}} x^{\dot{\rho}\alpha}\,, \;\; \delta_k u^{+i} = (4i\theta^{+\alpha}\bar{\theta}^{+\dot{\alpha}} k_{\alpha\dot{\alpha}}) u^{-i}\,.$$

- What about the conformal properties of various analytic higher-spin potentials? No problems with the spin **1** potential V^{++}:

$$\delta_{sc} V^{++} = -\hat{\Lambda}_{sc} V^{++}\,, \quad \hat{\Lambda}_{sc} := \lambda^{\alpha\dot{\alpha}}_{sc}\partial_{\alpha\dot{\alpha}} + \lambda^{\hat{\alpha}+}_{sc}\partial_{\hat{\alpha}+} + \lambda^{++}_{sc}\partial^{--}$$

- The cubic vertex $\sim q^{+a}V^{++}Jq^{+}_{a}$ is invariant up to total derivative if

$$\delta_{sk} q^{+a} = -\hat{\Lambda}_{sc} q^{+a} - \frac{1}{2}\Omega q^{+a}\,, \quad \Omega := (-1)^{P(M)}\partial_M \lambda^M$$

Moreover, this vertex is invariant under arbitrary analytic superdiffeomorphisms, $\Lambda_{sk} \to \Lambda(\zeta)$.

- Situation gets more complicated for **s** ≥ 2. Requiring $\mathcal{N}=2$ gauge potentials for **s** $=2$ to be closed under $\mathcal{N}=2$ SCA necessarily leads to

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)}\,,$$
$$\hat{\mathcal{H}}^{++}_{(s=2)} := h^{++M}\partial_M = h^{++\alpha\dot{\alpha}}\partial_{\alpha\dot{\alpha}} + h^{++\alpha+}\partial^{-}_{\alpha} + h^{++\dot{\alpha}+}\partial^{-}_{\dot{\alpha}} + h^{(+4)}\partial^{--}$$
$$\delta_{k_{\alpha\dot{\alpha}}} h^{(+4)} = -\hat{\Lambda} h^{(+4)} + 4ih^{++\alpha+}\bar{\theta}^{+\dot{\alpha}}k_{\alpha\dot{\alpha}} + 4i\theta^{+\alpha}h^{++\dot{\alpha}+}k_{\alpha\dot{\alpha}}$$

It is impossible to avoid introducing the extra potential $h^{(+4)}$ for ensuring conformal covariance. The extended set of potentials embodies $\mathcal{N}=2$ **Weyl multiplet** ($\mathcal{N}=2$ conformal SG gauge multiplet).

- For $\mathbf{s} \geq 3$ the gauge-covariantization of the free q^{+a} action requires adding the gauge superfield differential operators of rank $\mathbf{s}-\mathbf{1}$ in ∂_M,

$$\mathcal{D}^{++} \to \mathcal{D}^{++} + \kappa_s \hat{\mathcal{H}}^{++}_{(s)} (J)^{P(s)}, \quad P(s) = \frac{1+(-1)^{s-1}}{2}$$

- For $\mathbf{s} = 3$:

$$\hat{\mathcal{H}}_{(s=3)} = h^{++MN}\partial_N \partial_M + h^{++}, \quad h^{++MN} = (-1)^{P(M)P(N)} h^{++NM}$$

- $\mathcal{N}=2$ SCA mixes different entries of h^{++MN}, so we need to take into account all these entries, as distinct from non-conformal case where it was enough to consider, e.g., $h^{++\alpha\dot{\alpha}M}$.
- The spin **3** gauge transformations of q^{+a} and h^{++MN} leaving invariant the action $\sim q^{+a}(D^{++} + \kappa_3 \hat{\mathcal{H}}_{(s=3)})q^+_a$ are

$$\delta_\lambda^{(s=3)} q^{+a} = -\frac{\kappa_3}{2}\{\hat{\Lambda}^M + \frac{1}{2}\Omega^M, \partial_M\}_{AGB} J q^{+a} \equiv -\kappa_3 \hat{\mathcal{U}}_{(s=3)} J q^{+a},$$

$$\delta_\lambda^{(s=3)} \hat{\mathcal{H}}^{++}_{(s=3)} = \left[\mathcal{D}^{++}, \hat{\mathcal{U}}_{(s=3)}\right],$$

$$\hat{\Lambda}^M := \sum_{N \leq M} \lambda^{MN}\partial_N, \ \Omega^M := \sum_{N\leq M} (-1)^{[P(N)+1]P(M)} \partial_N \lambda^{NM},$$

$$\{F_1, F_2\}_{AGB} = [F_1, F_2], \quad \{B_1, B_2\}_{AGB} = \{B_1, B_2\}.$$

- All the potentials except $h^{++\alpha\dot{\alpha}M}$ can be put equal to zero using the original extensive gauge freedom:

$$S^{(s=3)}_{int|fixed} = -\frac{\kappa_3}{2}\int d\zeta^{(-4)}\, q^{+a} h^{++\alpha\dot{\alpha}M}\partial_M\partial_{\alpha\dot{\alpha}} J q^+_a\,. \qquad (1)$$

- In such a gauge one is led to accompany the superconformal transformations by the proper compensating gauge transformations in order to preserve the gauge, so the final SC transformations are nonlinear in $h^{++M\alpha\dot{\alpha}}$.
- Using the linearized gauge transformations of $h^{++\alpha\dot{\alpha}M}$

$$\begin{aligned}
&\delta_\lambda h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} = \mathcal{D}^{++}\lambda^{(\alpha\beta)(\dot{\alpha}\dot{\beta})} + 4i\lambda^{+(\alpha\beta)(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})} + 4i\theta^{+(\alpha}\bar{\lambda}^{+\beta)(\dot{\alpha}\dot{\beta})},\\
&\delta_\lambda h^{++(\alpha\beta)\dot{\alpha}+} = \mathcal{D}^{++}\lambda^{+(\alpha\beta)\dot{\alpha}} - \lambda^{++(\alpha\dot{\alpha}}\theta^{+\beta)},\\
&\delta_\lambda h^{++(\dot{\alpha}\dot{\beta})\alpha+} = \mathcal{D}^{++}\lambda^{+(\dot{\alpha}\dot{\beta})\alpha} - \lambda^{++\alpha(\dot{\alpha}}\bar{\theta}^{+\dot{\beta})},\\
&\delta_\lambda h^{(+4)\alpha\dot{\alpha}} = \mathcal{D}^{++}\lambda^{++\alpha\dot{\alpha}} - 4i\bar{\theta}^{+\dot{\alpha}}\lambda^{+\alpha++} + 4i\theta^{+\alpha}\lambda^{+\dot{\alpha}++},
\end{aligned}$$

we can find WZ gauge for the spin 3 gauge supermultiplet

$$\begin{aligned}
h^{++(\alpha\beta)(\dot{\alpha}\dot{\beta})} &= -4i\theta^{+\rho}\bar{\theta}^{+\dot{\rho}}\Phi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})}_{\rho\dot{\rho}} + (\bar{\theta}^+)^2\theta^+\psi^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i\\
&\quad + (\theta^+)^2\bar{\theta}^+\bar{\psi}^{(\alpha\beta)(\dot{\alpha}\dot{\beta})i}u^-_i + (\theta^+)^2(\bar{\theta}^+)^2 V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})ij}u^-_i u^-_j\,,\\
h^{++(\alpha\beta)\dot{\alpha}+} &= (\theta^+)^2\bar{\theta}^+_{\dot{\nu}} P^{(\alpha\beta)(\dot{\alpha}\dot{\nu})} + (\bar{\theta}^+)^2\theta^+_\nu T^{(\alpha\beta\nu)\dot{\alpha}} + (\theta^+)^4\chi^{(\alpha\beta)\dot{\alpha}i}u^-_i\,,\\
h^{(+4)\alpha\dot{\alpha}} &= (\theta^+)^2(\bar{\theta}^+)^2 D^{\alpha\dot{\alpha}}.
\end{aligned}$$

- In the bosonic sector: the spin $\mathbf{s} = 3$ gauge field, $SU(2)$ triplet of conformal gravitons, singlet conformal graviton, spin 1 gauge field and non-standard field which gauges self-dual two-form symmetry:

$$\Phi^{(\alpha\beta\rho)(\dot{\alpha}\dot{\beta}\dot{\rho})}, \; V^{(\alpha\beta)(\dot{\alpha}\dot{\beta})(ij)}, \; P^{(\alpha\beta)(\dot{\alpha}\dot{\nu})}, \; D^{\alpha\dot{\alpha}}, \; T^{(\alpha\beta\gamma)\dot{\alpha}}$$

In the fermionic sector: conformal spin $5/2$ and spin $3/2$ gauge fields:

$$\psi^{(\alpha\beta\rho)(\dot{\alpha}\dot{\beta})i}, \quad \chi^{(\alpha\beta)\dot{\alpha}i}$$

- They carry total of $40 + 40$ off-shell degrees. Starting from $\mathbf{s} = 3$, all the component fields are gauge fields, no auxiliary fields are present.

- The sum of conformal spin 2 and spin 3 actions

$$S = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \kappa_2 \hat{\mathcal{H}}^{++}_{(s=2)} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\right) q^{+}_{a}$$

is invariant with respect to the (properly modified) spin $\mathbf{3}$ transformations to the leading order in κ_3 and to any order in κ_2. Thus the cubic vertex $(\mathbf{3}, \frac{1}{2}, \frac{1}{2})$ is invariant under the gauge transformations of conformal $\mathcal{N} = 2$ SG and we obtain the superconformal vertex of the spin $\mathbf{3}$ supermultiplet on *generic* $\mathcal{N} = 2$ Weyl SG background.

- The whole consideration can be generalized to the general integer higher-spin s case: $8(2s-1)_B + 8(2s-1)_F$ d.o.f. off shell.

Fully consistent higher-spin hypermultiplet coupling

- The superconformal cubic vertices $(\mathbf{s}, \frac{1}{2}, \frac{1}{2})$ are consistent to the leading order in the higher-spin analogs of Einstein constant. These can be made invariant with respect to gauge transformations also in the next orders.
- Come back to the simplest case of the spin **3** in curved superspace:

$$S_{(s=3)} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathfrak{D}^{++} + \kappa_3 \hat{\mathcal{H}}^{++}_{(s=3)} J\right) q^{+}_{a}.$$

It is gauge invariant to the leading order in κ_3. In the next order we have the following spin **3** gauge transformation of the hypermultiplet:

$$\delta^{(s=3)}_{\lambda}\left(-\frac{\kappa_3}{2} q^{+a}\hat{\mathcal{H}}^{++}_{(s=3)} J q^{+}_{a}\right) = -\frac{\kappa_3^2}{4} q^{+a}\left[\hat{\mathcal{H}}^{++}_{(s=3)}, \left\{\hat{\Lambda}^{M} + \frac{1}{2}\Omega^{M}, \partial_{M}\right\}_{AGB}\right] q^{+}_{a}.$$

We gained the differential operator of the third order in superspace derivatives. It can be compensated (modulo a total derivative) by the proper gauge transformation of the spin **4** superconformal multiplet

$$\kappa_4 \delta^{(s=3)}_{\lambda}\hat{\mathcal{H}}^{++}_{(s=4)} = -\frac{\kappa_3^2}{4}\left[\hat{\mathcal{H}}^{++}_{(s=3)}, \left\{\hat{\Lambda}^{M} + \frac{1}{2}\Omega^{M}, \partial_{M}\right\}_{AGB}\right].$$

- So the action

$$S_{s=3,4} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathfrak{D}^{++} + \kappa_3\hat{\mathcal{H}}^{++}_{(s=3)}J + \kappa_4\hat{\mathcal{H}}^{++}_{(s=4)}\right) q^{+}_{a}$$

respects the spin $\mathbf{s} = 3$ gauge invariance to κ_3^2 order.

- However, the action we started with is not invariant in the $\kappa_3\kappa_4$ order. Then the procedure just described can be continued step by step.
- To summarize the procedure, we introduce an analytic differential operator with all integer higher spins:

$$\hat{\mathcal{H}}^{++} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{H}}^{++}_{(s)}(J)^{P(s)}.$$

- The action of the infinite tower of integer $\mathcal{N} = 2$ superconformal higher spins interacting with the hypermultiplet in an arbitrary $\mathcal{N} = 2$ conformal supergravity background then reads:

$$S_{full} = -\frac{1}{2}\int d\zeta^{(-4)}\, q^{+a}\left(\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}\right) q^{+}_{a}. \qquad (2)$$

- Ascribing the proper gauge transformation to $\hat{\mathcal{H}}^{++}$, one can achieve gauge invariance to any order in the couplings constants. The total hypermultiplet gauge transformation reads

$$\delta_\lambda q^{+a} = -\hat{\mathcal{U}}_{hyp} q^{+a} = -\sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s \, (J)^{P(s)} \, q^{+a},$$

$$\hat{\mathcal{U}}_s q^{+a} := \sum_{k=s,s-2,\dots} \hat{\mathcal{U}}_s^{(k)} q^{+a}$$

This transformation acts linearly on the hypermultiplet superfield.

- For the set of gauge superfields we obtain the transformation law:

$$\delta_\lambda \hat{\mathcal{H}}^{++} = \left[\mathcal{D}^{++} + \hat{\mathcal{H}}^{++}, \hat{\mathcal{U}}_{gauge}\right], \qquad \hat{\mathcal{U}}_{gauge} := \sum_{s=1}^{\infty} \kappa_s \hat{\mathcal{U}}_s .$$

It mixes different spins, so it is a non-Abelian deformation of the spin $\mathbf{s}$ transformation laws. In the lowest order, it becomes Abelian and reproduces the sum of transformations of all integer spins $\mathbf{s} \geq 1$.

- The invariance under $\mathcal{N} = 2$ conformal supergravity transformations is automatic. So we have constructed the fully consistent gauge-invariant and conformally invariant interaction of hypermultiplet with an infinite tower of $\mathcal{N} = 2$ higher spins in an arbitrary $\mathcal{N} = 2$ conformal supergravity background.

Summary and outlook

The theory of $\mathcal{N} = 2$ supersymmetric higher spins $s \geq 3$ opens a new promising direction of applications of the harmonic superspace approach which earlier proved to be indispensable for description of more conventional $\mathcal{N} = 2$ theories with maximal spins $s \leq 2$. Once again, the basic property underlying these new higher-spin theories is the harmonic Grassmann analyticity (all basic gauge potentials are unconstrained analytic superfields involving an infinite number of degrees of freedom off shell before fixing WZ-type gauges).

Under way:

- The linearized actions of conformal higher-spin $\mathcal{N} = 2$ multiplets ($\mathcal{N} = 2$ analogs of the square of Weyl tensor)?

- Quantization, induced actions,...

- $\mathcal{N} = 2$ supersymmetric half-integer spins?

- An extension to AdS background? Superconformal compensators? The $\mathcal{N} = 2$ AdS_4 supergroup $OSp(2|4; R) \subset SU(2, 2|2)$, so the conformal invariance already implies AdS_4 invariance. It remains to pick up the appropriate super-compensator.

- From the linearized theory to its full nonlinear version? At present, the latter is known only for $s \leq 2$ ($\mathcal{N} = 2$ super Yang - Mills and $\mathcal{N} = 2$ supergravities). This problem seemingly requires accounting for **ALL** higher $\mathcal{N} = 2$ superspins simultaneously. New supergeometries?

THANK YOU FOR ATTENTION!