Nonperturbative kinetic approach in the strong field QCD

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<u>1. Introduction</u>

1.1. <u>Topic:</u> Preequilibruium stage of QGP evolution in the conditions of ultrarelativistic heavy ion collisions

Problems:

- explanation of large multiplicity
- angular distribution of quarks on final stage of evolution QGP

1.2. <u>Flux tube model:</u> A. Casher, H. Neuberger, and S, Nussinov. PRD **20**, 179 (1979)

Basic idea: vacuum creation of quarks in strong gluon fields, i.e. it is nonlinear strong nonequilibrium effect, $w \sim \exp(-E_c/E),\ E_c = m_q/g^2,$

1. Introduction

Nonperturbative kinetic approach: S.A. Smolyansky, G. Roepke, S.M. Schmidt et al., hep-ph/9712377; GSI-Preprint-97-92 (Dormstadt, 1997); S.M. Schmidt, D. Blaschke, G. Roepke, S.A.Smolyansky, V.D. Toneev. Int. J. Mod. Phys. E7, 709 (1997).

Review: D.V. Vinnik, V.A. Mizerny, A.V. Prozorkevich, S.A. Smolyansky, V.D. Toneev. Phys. Atom. Nucl. **64**, 775 (2001),

1.3. Basic problem: Research of the dynamical QCD foundation for the selfconsistent nonperturbative kinetic approach in the problem of quark vacuum production in strong gluon fields.

2. Oscillator-type equation

2.1. Statement of problem: Let us consider firstly the quark sector of the $SU(N)$ QCD limited by interaction quarks with the quasiclassical internal (plasma) gluon ($B_a^\mu(x)$) and electromagnetic ($A^\mu(x)$) fields. The Dirac equation

$(i\gamma D - m)\psi = 0,\ D^\mu\psi_k = \left[(\partial^\mu - Q_i A^\mu)\delta_{ki} - gB_a^\mu t_{ki}^a\right]\psi_i,$ where Q_k is the electric charge of the quark with flavor "k", t_{ki}^a are the Gell-Mann matrixes.

Basic model assumption:

$$A^\mu(x) = (0,0,0,A^{(3)}(t) = A(t)), B_a^\mu(x) = (0,0,0,B_a^{(3)}(t) = B_a(t)).\ (1)$$

with the conditions of vacuum instability $A_{in} \neq A_{out},\ \ B_{in}^a \neq B_{out}^a$.

2. Oscillator-type equation

2.2. Quadrical Equation: The Dirac Eq. is defined in the matrix space $Cl_{(N)} \oplus G_{(N)}$ framed from the Clifford space of the γ – matrixes $(Cl_{(N)})$ and the Gell-Mann space $G_{(N)}$ of the matrixes t^{a}_{ki}.

The limitation (1) brings to an important simplification of the structure of the quadrical Eq. contained the unique combination of the γ – matrixes $\gamma^0\gamma^3$ permitting the spectral Eq. $\gamma^0\gamma^3 R_r = \lambda_r R_r$ with the condition

$$R_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}, R_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}; \;\; R_r^{+} R_{r\prime} = 2\delta_{rr\prime}\,; r, r' = 1{,}2. \text{ for } \lambda_1 = \lambda_2 = 1,$$

2. Oscillator-type equation

Such situation occurs in SFQED (A.A. Grib, S.G. Mamaev, V.M. Mostepanenko. Vacuum Quantum Effects in Strong Fields. Friedmann Laboratory Pub, 1994)

As a result, solution of the quadrical Eq. can seek in the form $\psi_{kr} = R_r \varphi_{kr}(\vec{p}, t) e^{i\vec{p}\vec{x}}$ It brings to the system of N limiting ODEs of the oscillator type with respect to the set of the quark functions with different flavors:

$$\ddot{\varphi}_i(\vec{p}, t) + \left\{ \left[\varepsilon_{i\perp}^2 + \left(p^{(3)} - Q_i A \right)^2 + \frac{1}{6} g^2 B^2 + i Q_i \dot{A} \right] \delta_{ik} + g \left[2B^a p^{(3)} + \frac{1}{2} g B^b B^c d^{cba} - i\dot{B}^a - 2Q_i A B^a \right] t_{ik}^a \right\} \varphi_k(\vec{p}, t) = 0, \qquad (2)$$

where $\varepsilon_{i\perp} = \left(m_i^2 + p_\perp^2 \right)^{1/2}$ and $p_\perp$ are perpendicular energy and momentum,

$B^2 = B^a B^a$.

2. Oscillator-type equation

2.3. Collective excitation energy of the q-subsystem

Asymptotic of the quadrical Eq. at $t \to -\infty$ is $\varphi_k^{(\pm)}(\vec{p},t) = \varphi_k^{(\pm)}(\vec{p})e^{\pm i\Omega_{(-)}t}$, where $\Omega_{(-)}$ is the collective energy of the q-subsystem in the in-state. Substitution in Eq. (2) brings to the system of linear homogeneous algebraic Eqs. ($\dot{A}_{in} = \dot{B}_{in}^{a} = 0$):

$$\{[\Omega_{(-)}^2 - \varepsilon_{i\perp}^2 - \left(p^{(3)} - Q_i A_{in}\right)^2 - \tfrac{1}{6} g^2 B_{in}^2]\delta_{ik} - g\,[B_{in}^a p^{(3)} + \tfrac{1}{2} g B_{in}^b B_{in}^a d^{cba} - 2Q_i B_{in}^a A_{in}]t_{ik}^a\}\varphi_k^{in(\pm)}(\vec{p}) = Q_{ik}^{in}\varphi_k^{in(\pm)}(\vec{p}) = 0 \quad (3)$$

with the solvability condition $\det\left\|Q_{ik}^{in}\right\|_N = 0.$ (4)

It is the algebraic Eq. with respect to collective energy $\Omega_{(-)}$.

2. Oscillator-type equation

Now one can to generalize Eqs. (3), (4) by analogy with SFQED ($\varepsilon_{QED}^{in} \to \varepsilon_{QED}(t)$), at the arbitrary time by the substitution $A_{in} \to A(t), B_{in}^{a} \to B^{a}(t)$ and $\Omega_{(-)} \to \Omega(t)$, which is defined by the generalization of Eq. (4), $\det\|Q_{ik}(t)\|_N = 0.$ (5)

It leads to the oscillator type Eq. for the quark subsystem with flavor "k" ,

$$\ddot{\varphi}_k^{(\pm)}(\vec{p},t) + \Omega_{(\alpha)}^2(\vec{p},t) \cdot \varphi_k^{(\pm)}(\vec{p},t) = 0, \qquad (6)$$

where $\Omega_{(\alpha)} = Q'_{(\alpha)} + iQ''_{(\alpha)}$ is the α – root of the Eq. (5), $\alpha = 1,2,\dots,N$ (auxiliary problem),

2. Oscillator-type equation

2.4. Quasi-Abelian approximation (QAA)

There is an important particular case corresponding to neglect by reciprocal effect of quarks with two different flavors $i \neq k$ in the Gell-Mann matrix. The quadrical Eq. (2) will be then $\ddot{\varphi}_k^{(\pm)} + [\varepsilon_k^2(\vec{p},t) + i(Q_k\dot{A} - g\dot{B}^a t_{kk}^a)] \cdot \varphi_k^{(\pm)} = 0$, (7)

where $\varepsilon_k\ (\vec{p},t) = \left\{\varepsilon_{k\perp}^2 + \left[p^{(3)} - Q_k A(t) + gB^a(t)t_{kk}^a\right]^2\right\}^{1/2}$ (8)

is the “quasienergy”.

In consequence of the introduced approximation, electromagnetic and gluon fields appears in Eqs. (7), (8) on equal term (but their matter Eqs. are different: the gluon Eq. preserves noncommutativity of the flavor states of quarks).

2. Oscillator-type equation

The equal right presence electromagnetic and gluon field in the oscillator type Eq. (7) allows to use the existed in SFQED derivation methods of the nonperturbative KEs for description of vacuum creation of particles:

A.A. Grib, S.G. Mamaev, V.M. Mostepanenko. Vacuum Quantum Effects in Strong Fields. Friedmann Laboratory Pub, 1994.

I. Bialynicki-Birula, P. Gornicki, J. Rafelski. PRD **44**, 1825 (1991).

A.M. Fedotov, E.G. Gelfer, K.Yu. Korolev, S.A. Smolyansky. PRD **83**, 025011 (2011).

I.A. Aleksandrov, V.V. Dmitriev, D.G. Sevastyanov, S.A. Smolyansky. Eur. Phys. J., **229**, 3469 (2020).

D.B. Blaschke, V.V. Dmitriev, G. Roepke, S.A. Smolyansky PRD **84**, 085028 (2011).

3. Application to the heavy ion collisions

back rection in the conditions of preequilibrium evolution of QGP (quark sector)

Actuality: H. Taya, T. Nishimura, A. Ohnishi. PRC, **110**. 014901 (2024).

3.1 Model: Preequilibrium stage of evolution of QGP continues with the moment of deconfinement of the quark degrees of freedom of the colliding nuclear matter up to attainment of a local equilibrium state when the hadronization process is beginning. The preequilibrium stage is accompanied by vacuum quark production that is defined some important characteristics of created hadron matter. Apparently, a strong gluon fields are generated by the ultrarelativistic counter propagating flows of the valent quarks.

Such picture means that

- in the first approximation, it can limited by SU(2) model of the quark matter generated from vacuum;
- it can ignore by the electromagnetic processes;
- the uncompensated electric charge of the q-subsystem is conserved.

Additionally it is assumed that the system is spatially homogeneous

3. Application to the heavy ion collisions

3.2. System of self-consistent Eqs. of the quark subsytem:

I. The basic KE for description of the vacuum quark production under action of a strong gluon fields in the framework of QAA model of SU(2) QCD is:

$$\dot{f}_k(\vec{p},t) = \frac{1}{2}\lambda_k(\vec{p},t)\int_{t_0}^{t} dt'\,\lambda_k(\vec{p},t')\left[1-2f_k(\vec{p},t') - f_k^{(0)}(\vec{p}\,)\right]\cos 2\theta_k(\vec{p},t,t'), \quad \text{(I)}$$

where for $a=3$, $\lambda_k(\vec{p},t) = \pm\frac{g\dot{B}^{(3)}\varepsilon_{\perp k}}{2\varepsilon_k^2(\vec{p},t)}$, where (+) ~ u – quark , (-) ~ d – quark;

$$\varepsilon_k(\vec{p},t) = \left[\varepsilon_{\perp k}^2(p_\perp) + \left(p^{(3)} - \frac{1}{2}gB^{(3)}\right)^2\right]^{1/2}, \ \theta_k(\vec{p},t,t') = \int_{t'}^{t} d\tau\,\varepsilon_k(\vec{p},\tau).$$

The equivalent system of ODEs:

$$\dot{f}_k = \frac{1}{2}\lambda_k u_k, \quad \dot{u}_k = \lambda_k\left(1-2f_k - f_k^{(0)}(\vec{p}\,)\right) - 2\varepsilon_k v_k, \quad \dot{v}_k = 2\varepsilon_k u_k. \qquad \text{(I')}$$

3. Application to the heavy ion collisions

II. Conservation of the uncompensated electric charge:

$$\rho(t) = |e| \int [dp][f_u(\vec{p},t) - f_d(\vec{p},t)] = \frac{2|e|Z}{3v_0},$$

where Z and v_0 are the proton number and the volume of a heavy ion.

III. Eq. for the quasiclassical gluon fields:

$$D_\mu G_a^{\mu\nu} = -j_a^\nu, \qquad G_a^{\mu\nu} = \partial^\mu B_a^\nu - \partial^\nu B_a^\mu + g f_{abc} B_b^\mu B_c^\nu,$$

f_{abc} are the structural constants of the SU(N) group and

$$j_a^\mu = 2g t_{ik}^4 \langle \bar{\psi}_k \gamma^\mu \psi_i \rangle$$

is the quark current in the color space.

3. Application to the heavy ion collisions

Adaptation to SU(2) variant of the theory for the single-component field $B^a(t)$ brings to the Maxwell-type Eq.

$$\ddot{B}^{(3)} = -2gk \sum_k \sigma_{kk}^{(3)} \langle \bar{\psi}_k \gamma^3 \psi_k \rangle = j_{cond} + j_{pol}, \qquad \text{(III)}$$

$$j_{cond}(t) = -4g \sum_k \sigma_{kk}^{(3)} \int [dp] \frac{p^{(3)} + \frac{1}{2} g B^{(3)}}{\varepsilon_k(\vec{p},t)} f_k(\vec{p},t),$$

$$j_{pol}(t) = 2g \sum_k \sigma_{kk}^{(3)} \int [dp] \frac{\varepsilon_{\perp k}}{\varepsilon_k(\vec{p},t)} u_k(\vec{p},t),$$

where $u_k(\vec{p},t)$ is the current polarization function.

Thus, limitation $B_a^\mu(x) \to B^a(t)$ leads to its Abelian structure.

3. Application to the heavy ion collisions

3.3. Integrals of motion:

$$\left(1 - 2f_k - f_k^{(0)}\right)^2 + u_k^2 + v_k^2 = const,\ \ k = 1,2 \qquad \text{(IV)}$$

3.4. Scenario:

the ultrarelativistic counter-propagated flows of the valent quark matter are generated a strong internal (plasma) color field (Eq. (III)); this field is the source of the quark creation from vacuum (Eqs. (I,I')): their creation brings to decrease of the color plasma field and energy exhaustion of the flows of the valent quarks (back reaction process). Dissipative mechanism is not here taken into account.

3. Application to the heavy ion collisions

3.5. Initial conditions:

the initial number densities of the valent quarks and their momentum in the flowers are $n_u^{(0)} = (2Z+N)/v_0 = \int [dp] f_u$, $n_d^{(0)} = (Z+2N)/v_0 = \int [dp] f_d$,

$p_k^{(0)} = \frac{m_k}{M}\sqrt{E_0^2 - M^2}$, where M and E_0 are mass and energy of an ion in the flow.

The simplest form of the initial distribution functions of the valent quarks corresponding to monoenergetic flows is:

$$f_k^{(0)}(\vec{p}) = \frac{1}{2}(2\pi)^3 n_k^{(0)}\left[\delta\left(p_k^{(0)} - p^{(3)}\right) + \delta\left(p_k^{(0)} + p^{(3)}\right)\right]\delta(p_\perp).$$

It is assumed $\dot{B}^a_{(0)} = 0$ in the moment of de confinement. Then

$$u_k^{(0)} = v_k^{(0)} = 0$$

4. Conclusion

- Introduced model of the nonperturbative kinetic description of the vacuum quark production on the preequilibrium stage of nuclear matter evolution of the colliding ultrarelativistic heavy ions is based entirely on the QCD and contains clear system of the model assumptions (limitation by the quasiclassical spatially homogeneous linear polarized internal gluon and electromagnetic fields, QAA, the system of the initial conditions).
- The result is the selfconsistent closed system of the KEs for the quark subsystem and the field Eqs. for the internal plasma fields.
- Such approach permits different modernization and allows to investigate some important characteristics of the quark subsystem in the final quasiequilibrium state, where hadronization happens.

4. Conclusion

- In comparison with the well known kinetic methods of description of vacuum particle creation, presented approach contains some new elements:

- two-component character of the quark subsystem

- excitation mechanism of the strong gluon field by ultrarelativistic counter propagated quark flows (this mechanism is a component part of the back reaction problem);

- conservation of the uncompensated charge of the quark flows.

These features complicate the numerical analysis of the considered problem.

THANK YOU FOR YOUR ATTENTION!