

Coherent effects in scattering of particle wave packets

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References (see for more)

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Outline

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- 3 Effective susceptibilities: photon and electron dielectric susceptibilities, electron mass operator in the presence of the hadron wave packet
- 4 Explicit expressions for:
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 - Electron mass operator and the inclusive probability to record an electron in electron-by-hadron scattering
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Motivation

- In considering scattering of particles as plane waves, the disconnected contributions to the S -matrix are excluded. This results in concealing the effects appearing for real-world quantum states which are not the plane waves.
- There is a class of processes where the wave functions of particles interact coherently with other particles even at large energies [P.O. Kazinski, T.V. Solovyev, Eur. Phys. J. C **82**, 790 (2022)]. For coherent scattering, the scattering amplitudes stemming from different parts of the wave packet add up constructively as if the wave packet would be a charged fluid.
- Coherent scattering provides the tool to trace the dynamics of quantum states of particles and, in particular, to reveal the details of the collapse of the wave function exposed to a quantum measurement. This gives us the answers to the questions:
 - Where is the precise boundary between the system and the detector measuring and projecting the state of this system?
 - How long can we use the Schrödinger equation to describe the quantum dynamics and when should we apply the projectors corresponding to the measurements?
 - Whether does the measurement of the properties of one particle change instantaneously the properties of the second particle entangled with the first one so that the second particle emits photons?

Results

- The general theory of coherent processes in QFT with wave packets is developed. This theory can be regarded as the description of holography of quantum states.
- Scattering of electrons by a hadron wave packet at small angles is described.
- Scattering of photons by a photon wave packet and by an electron wave packet at small angles is described.
- The effective dielectric susceptibilities of a single photon and a single electron are introduced and calculated. The effective mass operator of the electron in the presence of the hadron wave packet is found.
- The theory of spontaneous and stimulated transition radiation from particle wave packets is developed.
- Radiation due to quantum measurement is described. For a free particle measured by the detector, the properties of this radiation are similar to the properties of transition radiation.

Main idea in terms of diagrams

Definition of the connected part of the S -matrix

$$S_{\beta\alpha} = S_{\beta\alpha}^c + \sum'_{\text{part}(\beta,\alpha)} (\pm 1) S_{\beta_1\alpha_1}^c \cdots S_{\beta_n\alpha_n}^c. \quad (1)$$

$S_{\beta\alpha}$ is the S -matrix.

$S_{\beta\alpha}^c$ is the connected part of the S -matrix.

Process $1 + 2 \rightarrow 1' + 2'$

The diagrammatic equation (2) illustrates the decomposition of the S-matrix into a disconnected part and a connected part. On the left, a square box labeled S has two incoming arrows at the bottom labeled p_1 and p_2 , and two outgoing arrows at the top labeled p'_1 and p'_2 . This is set equal to the sum of two diagrams. The first diagram on the right shows two separate vertical lines, each with an incoming arrow at the bottom and an outgoing arrow at the top, labeled p_1 and p_2 respectively. The second diagram on the right shows a circle labeled S^c with four arrows: two incoming at the bottom labeled p_1 and p_2 , and two outgoing at the top labeled p'_1 and p'_2 . The entire equation is labeled (2) on the far right.

- The stability of the vacuum and of the one-particle states is implied.

Differential cross-section for plane-wave states

$$d\sigma(\mathbf{p}'_1) \sim \int d\mathbf{p}'_2 \left| \begin{array}{c} \mathbf{p}'_1 \quad \mathbf{p}'_2 \\ \circlearrowleft S^c \\ \mathbf{p}_1 \quad \mathbf{p}_2 \end{array} \right|^2. \quad (3)$$

Inclusive probability to record particle 1' for the initial states of a general form

$$dP(\mathbf{p}'_1) \sim \int d\mathbf{p}'_2 \left| \begin{array}{c} \mathbf{p}'_1 \quad \mathbf{p}'_2 \\ \square S \\ \varphi(\mathbf{p}_1) \quad \psi(\mathbf{p}_2) \end{array} \right|^2 = \int d\mathbf{p}'_2 \left| \begin{array}{c} \mathbf{p}'_1 \quad \mathbf{p}'_2 \\ \uparrow \quad \uparrow \\ \varphi(\mathbf{p}_1) \quad \psi(\mathbf{p}_2) \end{array} + \begin{array}{c} \mathbf{p}'_1 \quad \mathbf{p}'_2 \\ \circlearrowleft S^c \\ \varphi(\mathbf{p}_1) \quad \psi(\mathbf{p}_2) \end{array} \right|^2. \quad (4)$$

Main idea in terms of diagrams

Inclusive probability to record particle 1' for the initial states of a general form

$$dP(\mathbf{p}'_1) = dP_0(\mathbf{p}'_1) + dP_c(\mathbf{p}'_1) + dP_{inc}(\mathbf{p}'_1). \quad (5)$$

$dP_0(\mathbf{p}'_1)$ is the probability to record particle 1' in the initial state.

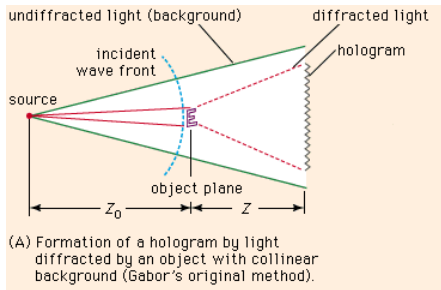
$dP_c(\mathbf{p}'_1)$ is the coherent (interference) contribution to the probability.

$dP_{inc}(\mathbf{p}'_1)$ is the incoherent contribution to the probability.

$$\begin{aligned}
 dP_0(\mathbf{p}'_1) &\sim \begin{array}{c} \varphi \longrightarrow \boxed{D} \longleftarrow \varphi \\ \psi \longrightarrow \quad \quad \quad \longleftarrow \psi \end{array}, & dP_c(\mathbf{p}'_1) &\sim \begin{array}{c} \varphi \longrightarrow \boxed{D} \longrightarrow \textcircled{S^c} \longleftarrow \varphi \\ \psi \longrightarrow \quad \quad \quad \longleftarrow \psi \end{array} + c.c., \\
 dP_{inc}(\mathbf{p}'_1) &\sim \begin{array}{c} \varphi \longrightarrow \textcircled{S^c} \longrightarrow \boxed{D} \longrightarrow \textcircled{S^c} \longleftarrow \varphi \\ \psi \longrightarrow \quad \quad \quad \longleftarrow \psi \end{array}.
 \end{aligned} \quad (6)$$

- In the coherent contribution, the state of the particle described by the wave function ψ is not changed by scattering.
- The incoherent contribution is the standard contribution to the differential cross-section for the initial particle states of a general form.

Main idea in terms of diagrams. Holography



Probability to record a photon

$$dP(\mathbf{k}') = \left| \begin{array}{c} \mathbf{k}' \\ \text{wavy line} \\ d(\mathbf{k}) \end{array} + \begin{array}{c} \mathbf{k}' \\ \text{circle with } \Pi \\ \text{wavy line} \\ d(\mathbf{k}) \end{array} \right|^2 = dP_0(\mathbf{k}') + dP_c(\mathbf{k}') + dP_{inc}(\mathbf{k}'). \quad (7)$$

$$dP_0(\mathbf{p}'_1) \sim d \text{ --- } \text{wavy line} \text{ --- } [D] \text{ --- } \text{wavy line} \text{ --- } d, \quad dP_c(\mathbf{p}'_1) \sim d \text{ --- } \text{wavy line} \text{ --- } [D] \text{ --- } [\Pi] \text{ --- } \text{wavy line} \text{ --- } d + c.c., \quad (8)$$

$$dP_{inc}(\mathbf{p}'_1) \sim d \text{ --- } \text{wavy line} \text{ --- } [\Pi] \text{ --- } [D] \text{ --- } [\Pi] \text{ --- } \text{wavy line} \text{ --- } d.$$

Main idea in terms of diagrams. Other processes

Inclusive probability to record a photon in spontaneous radiation

$$dP(\mathbf{k}') \sim \int d\mathbf{p}' \left| \begin{array}{c} \mathbf{p}' \\ \swarrow \\ \text{S}^c \\ \nearrow \\ \varphi(\mathbf{p}) \end{array} \right|^2 = \begin{array}{c} \\ \phantom{\text{S}^c} \\ \\ \phantom{\text{S}^c} \\ \\ \phantom{\text{S}^c} \\ \\ \phantom{\text{S}^c} \\ \\ \phantom{\text{S}^c} \\ \end{array} \quad (9)$$

Inclusive probability to record a photon in stimulated radiation

$$dP(\mathbf{k}'_1) \sim \int d\mathbf{p}' \left| \begin{array}{c} \mathbf{p}' \quad \mathbf{k}'_1 \\ \uparrow \quad \downarrow \\ \varphi(\mathbf{p}) \quad d(\mathbf{k}) \\ \dots \\ \text{S}^c \\ \dots \\ \varphi(\mathbf{p}) \quad d(\mathbf{k}) \end{array} \right|^2 = dP_0(\mathbf{k}'_1) + dP_c(\mathbf{k}'_1) + dP_{inc}(\mathbf{k}'_1). \quad (10)$$

$$dP_0(\mathbf{p}'_1) \sim \begin{array}{c} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{array}, \quad dP_c(\mathbf{p}'_1) \sim \begin{array}{c} \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \\ \dots \end{array} + c.c., \quad (11)$$

Main idea in terms of diagrams. Other processes

Inclusive probability to record a photon in spontaneous radiation from N charged particles

$$dP(\mathbf{k}') \sim \int d\mathbf{p}'_1 \cdots d\mathbf{p}'_N \left| \begin{array}{c} \begin{array}{c} \mathbf{p}'_1 \\ \uparrow \\ \varphi_1(\mathbf{p}_1) \end{array} \quad \cdots \quad \begin{array}{c} \mathbf{p}'_{N-1} \quad \mathbf{p}'_N \\ \uparrow \quad \swarrow \\ \varphi_{N-1}(\mathbf{p}_{N-1}) \quad \varphi_N(\mathbf{p}_N) \end{array} \\ \downarrow \\ \text{S}^c \\ \downarrow \\ \mathbf{k}' \end{array} \right|^2 = NdP_{inc}(\mathbf{k}') + N(N-1)dP_c(\mathbf{k}'). \quad (12)$$

$$dP_{inc}(\mathbf{k}') \sim \begin{array}{c} \varphi_1 \longrightarrow \quad \longleftarrow \varphi_1 \\ \vdots \\ \varphi_{N-1} \longrightarrow \quad \longleftarrow \varphi_{N-1} \\ \\ \varphi_N \longrightarrow \text{S}^c \quad \text{---} \text{D} \quad \text{---} \text{S}^c \longleftarrow \varphi_N \\ \downarrow \quad \quad \quad \downarrow \\ \text{---} \end{array} , \quad (13)$$

$$dP_c(\mathbf{k}') \sim \begin{array}{c} \varphi_1 \longrightarrow \quad \longleftarrow \varphi_1 \\ \vdots \\ \varphi_{N-2} \longrightarrow \quad \longleftarrow \varphi_{N-2} \\ \varphi_{N-1} \longrightarrow \quad \text{---} \text{S}^c \longleftarrow \varphi_{N-1} \\ \\ \varphi_N \longrightarrow \text{S}^c \quad \text{---} \text{D} \quad \text{---} \text{S}^c \\ \downarrow \quad \quad \quad \downarrow \\ \text{---} \end{array} .$$

Motivation

- The properties inherent to single elementary particles such as mass, spin, charges, magnetic and dipole moments, and others underlie our understanding of physics.
- One of such characteristics of particles is their dielectric susceptibility [P.O. Kazinski, T.V. Solovyev, Eur. Phys. J. C **82**, 790 (2022)]. Thus we can talk about a new property of elementary particles.
- The susceptibility specifies, in particular, the optical properties of a medium. Therefore, the color of a photon can be defined.

Definition of susceptibility

$$S = -\frac{1}{4} \int d^4x F_{\mu\nu} F^{\mu\nu} + S_{int},$$
$$S_{int} = \frac{1}{2} \int d^4x E_i \chi_{ij} E_j, \tag{14}$$

E_i is the electric field strength.

χ_{ij} is a nonlocal tensor operator in the spacetime (the susceptibility).

Effective photon susceptibility

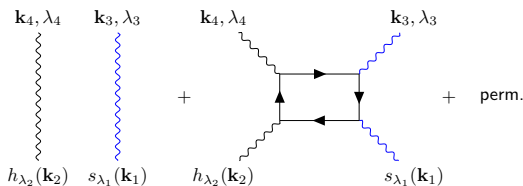


Figure: The diagrams describing photon by photon scattering in the leading orders of perturbation theory. The time axis is directed upwards. The blue lines correspond to a tested (soft) photon, whereas the black ones are for a probe (hard) photon.

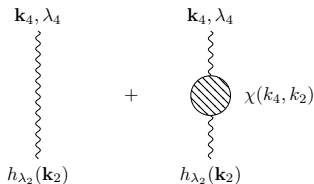


Figure: The diagrams describing photon scattering by the effective dispersive medium characterized by the susceptibility tensor $\chi(k_4, k_2)$ in the leading orders of perturbation theory.

Weyl symbol of susceptibility

$$\chi_{ij}(x; \mathbf{K}) = \frac{8\alpha^2}{\mathbf{K}^2} \left\{ (\psi_s^\dagger \psi_s) \delta_{ij}^\perp - i(\psi_a^\dagger \sigma_2 \psi_a) \varepsilon_{ijk} n_k - \right. \\ \left. - \frac{1}{2} [(\psi_g^\dagger \sigma_3 \psi_g) \sigma_3 - (\psi_g^\dagger \sigma_1 \psi_g) \sigma_1]_{ll'} (e_l)_i(\mathbf{K}) (e_{l'})_j(\mathbf{K}) \right\}. \quad (15)$$

$n_i := K_i/|\mathbf{K}|$. $\delta_{ij}^\perp := \delta_{ij} - n_i n_j$. $\mathbf{e}_l(\mathbf{K})$ are the transverse linear polarization vectors.

Notation

$$\psi_{s,\lambda}(x) := f_s^{1/2}(s) s_\lambda(x) = \int \frac{d\mathbf{k}_3 f_s^{1/2}(s)}{\sqrt{(2\pi)^3 2|\mathbf{k}_3|}} e^{i\mathbf{k}_3 \mathbf{x}} s_\lambda(\mathbf{k}_3; x^0), \\ \psi_{a,\lambda}(x) := f_a^{1/2}(s) s_\lambda(x) = \int \frac{d\mathbf{k}_3 f_a^{1/2}(s)}{\sqrt{(2\pi)^3 2|\mathbf{k}_3|}} e^{i\mathbf{k}_3 \mathbf{x}} s_\lambda(\mathbf{k}_3; x^0), \quad (16) \\ \psi_{g,\lambda}(x) := g^{1/2}(s) s_\lambda(x) = \int \frac{d\mathbf{k}_3 g^{1/2}(s)}{\sqrt{(2\pi)^3 2|\mathbf{k}_3|}} e^{i\mathbf{k}_3 \mathbf{x}} s_\lambda(\mathbf{k}_3; x^0),$$

$s = |\mathbf{k}_3| |\mathbf{K}| (\mathbf{n}_3 - \mathbf{n})^2$, $\mathbf{n}_3 = \mathbf{k}_3 / |\mathbf{k}_3|$.

$s_\lambda(\mathbf{k}_3; x^0)$ is the wave functions of a single photon in the interaction representation or the complex amplitude of the coherent state of tested photons.

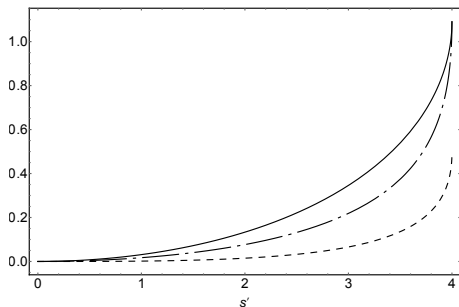


Figure: The dependence of $f_s(s)$, $f_a(s)$, and $g(s)$ on $s' = s/m^2$. The solid line is $f_s(s)$, the dashed line is $f_a(s)$, and the dashed dotted line is $g(s)$.

- f_s gives the contribution to the locally isotropic part of the susceptibility;
- f_a gives the contribution to the gyrotropic part of the susceptibility resulting in circular birefringence;
- g gives the contribution to the part of the susceptibility leading to linear birefringence.

Susceptibility of a photon

- For $s < 4m^2$, a single photon wave packet can be regarded as a transparent dispersive medium possessing linear and circular birefringences.
- The circular birefringence (gyrotropy) is suppressed at $s \ll 4m^2$ and is absent in the Heisenberg-Euler effective Lagrangian.
- The general formula (15) for susceptibility is applicable for both beams of photons in a coherent state and single photon states.

Estimate for the beam of tested photons

$$\chi_{ij} \sim \frac{2\alpha}{\pi} \frac{\mathbf{k}_3^2}{m^2} K_u^2, \quad K_u^2 := e^2 \mathbf{A}^2 / m^2 \sim \alpha n_s / (|\mathbf{k}_3| m^2). \quad (17)$$

K_u is the undulator strength parameter. n_s is the photon number density in the tested state s_α .

Estimate for the single photon wave packet

$$\chi_{ij} \sim 8\alpha^2 \frac{|\mathbf{k}_3| \sigma_s^3}{m^4} \lesssim 8\alpha^2 \frac{\mathbf{k}_3^4}{m^4}. \quad (18)$$

σ_s is the standard deviation of momenta in the wave packet of a soft photon s_α .

Inclusive probability to record a photon in light-by-light scattering

Inclusive probability to record a probe photon scattered by paraxial beam of tested ones for sufficiently small $|\Delta\mathbf{k}|$

$$dP_D = \frac{1}{2} \sum_{\lambda_4, \lambda'_4} D_{\lambda'_4 \lambda_4}^{(s)} \left\{ \rho(1 + \boldsymbol{\zeta}\boldsymbol{\sigma}) - 2\kappa(\xi_0 + \boldsymbol{\xi}\boldsymbol{\zeta}) \text{Im } \tilde{\rho} - 2\kappa[(\boldsymbol{\xi} + \xi_0\boldsymbol{\zeta}) \text{Im } \tilde{\rho} + (\boldsymbol{\xi} \times \boldsymbol{\zeta}) \text{Re } \tilde{\rho}] \boldsymbol{\sigma} \right\}_{\lambda_4 \lambda'_4} d\mathbf{k}_4. \quad (19)$$

$D^{(s)}$ is the projector to a certain spin state of the recorded photon.

$$\rho_{\beta\beta'} = \frac{(2\pi)^3}{V} \frac{(1 + \boldsymbol{\zeta}\boldsymbol{\sigma})_{\lambda_2 \lambda'_2}}{2} \rho(\mathbf{k}_2; \mathbf{k}'_2) - \text{initial state of the probe photon},$$

$$\rho := \rho(\mathbf{k}_4, \mathbf{k}_4), \quad \tilde{\rho} := \tilde{\rho}(\mathbf{k}_{4\parallel}; \mathbf{k}_4) = \int d\mathbf{k}_{4\perp} \rho(\mathbf{k}_{4\parallel}, \mathbf{k}_{4\perp}; \mathbf{k}'_4) \Big|_{\mathbf{k}'_4 = \mathbf{k}_4}, \quad (20)$$

$$\mathbf{k}_{4\parallel} := (\mathbf{n}_4 - \mathbf{n}_3) \frac{(\mathbf{k}_4(\mathbf{n}_4 - \mathbf{n}_3))}{(\mathbf{n}_4 - \mathbf{n}_3)^2}, \quad \kappa = \frac{\alpha^2}{2\pi^2 |\mathbf{k}_4| |\mathbf{n}_4 - \mathbf{n}_3|},$$

$$\xi_0^l = \int \frac{d\mathbf{k}_3}{|\mathbf{k}_3|} f_s(s) s^\dagger(\mathbf{k}_3) s(\mathbf{k}_3), \quad \xi_1^l = \int \frac{d\mathbf{k}_3}{2|\mathbf{k}_3|} g(s) s^\dagger(\mathbf{k}_3) \sigma_1 s(\mathbf{k}_3),$$

$$\xi_2^l = \int \frac{d\mathbf{k}_3}{|\mathbf{k}_3|} f_a(s) s^\dagger(\mathbf{k}_3) \sigma_2 s(\mathbf{k}_3), \quad \xi_3^l = - \int \frac{d\mathbf{k}_3}{2|\mathbf{k}_3|} g(s) s^\dagger(\mathbf{k}_3) \sigma_3 s(\mathbf{k}_3), \quad (21)$$

$s_l(\mathbf{k}_3)$ are given in the basis of linear polarization vectors.

Unpolarized probe photon, $\zeta = 0$

$$\zeta^0 = 1 \rightarrow \zeta'^0 = 1 - 2\kappa\xi_0 \frac{\text{Im} \tilde{\rho}}{\rho}, \quad \zeta = 0 \rightarrow \zeta' = -2\kappa\xi \frac{\text{Im} \tilde{\rho}}{\rho}. \quad (22)$$

- The hard probe photon being initially in the state with the Stokes vector $\zeta = 0$ becomes polarized with the Stokes vector proportional to the vector ξ .

The case $\text{Im} \tilde{\rho} = 0$

$$\zeta \rightarrow \zeta' = \zeta - 2\kappa(\xi \times \zeta) \frac{\text{Re} \tilde{\rho}}{\rho}. \quad (23)$$

- The Stokes vector ζ precesses around the vector ξ .
- The polarization degree of a hard probe photon, $|\zeta|$, is conserved up to the terms of higher order in the coupling constant.

Relative magnitude of the effect for tested beams of photons

$$\eta \sim \chi_{ij} |\mathbf{k}_4| L \sim \frac{\alpha}{\pi} \frac{\mathbf{k}_3^2}{m^2} K_u^2 |\mathbf{k}_4| L = 2.31 \times 10^{-8} K_u^2 \frac{|\mathbf{k}_4|}{m} \frac{L}{\mu\text{m}} \frac{\mathbf{k}_3^2}{\text{eV}^2}. \quad (24)$$

L is the length of the path traveled by the probe photon wave packet in the tested one.

Relative magnitude of the effect for a single tested photon

$$\eta \sim 6.60 \times 10^{-7} \frac{s \sigma_s^2}{m^4}. \quad (25)$$

$$L \sim 1/\sigma_s.$$

$$s = |\mathbf{k}_3| |\mathbf{k}_4| (\mathbf{n}_4 - \mathbf{n}_{30})^2.$$

Effective electron susceptibility

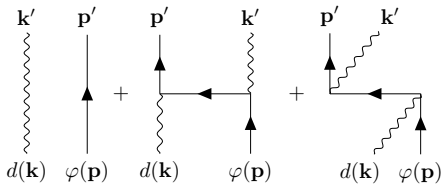


Figure: The diagrams describing the Compton process in the leading orders of the perturbation theory (the time axis is directed upwards).

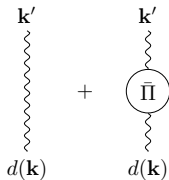


Figure: The diagrams describing photon scattering by the effective potential determined by the polarization operator in the leading orders of perturbation theory.

Susceptibility of a photon

- Coherent scattering of photons by an electron wave packet is the same as scattering of photons by a fluid with plasma dielectric permittivity.

Density matrix at the instant of time x^0

$$\rho(\mathbf{p}, \mathbf{p}'; x^0) := e^{-ip_0 x^0} \rho(\mathbf{p}, \mathbf{p}') e^{ip'_0 x^0}. \quad (26)$$

Relativistic density matrix in the coordinate representation

$$\rho(\mathbf{x}, \mathbf{y}; x^0) := \int \frac{d\mathbf{p} d\mathbf{p}' m}{(2\pi)^3 \sqrt{E(\mathbf{p})E(\mathbf{p}')}} e^{i\mathbf{p}\mathbf{x} - i\mathbf{p}'\mathbf{y}} \rho(\mathbf{p}, \mathbf{p}'; x^0). \quad (27)$$

Weyl symbol of the dielectric susceptibility of the electron wave packet in the small recoil limit

$$\chi_{ij}(x; \mathbf{K}) = -\frac{4\pi\alpha\rho(\mathbf{x}, \mathbf{x}; x^0)}{mK_0^2} \delta_{ij}. \quad (28)$$

$K_0 = |\mathbf{K}|$.

Electron mass operator in the presence of a hadron wave packet

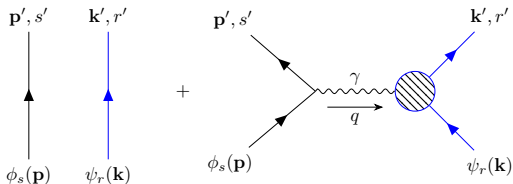


Figure: The diagrams describing electron hadron scattering in the leading orders of the perturbation theory (the time axis is directed upwards). The blue lines correspond to hadrons, whereas the black ones are for electrons.

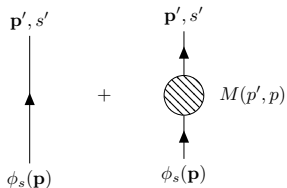
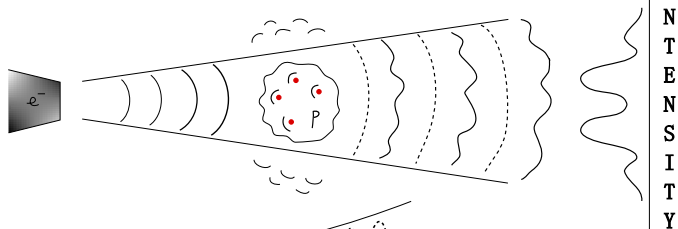


Figure: The diagrams describing electron scattering by the effective potential determined by the mass operator in the leading orders of perturbation theory.

Holography of a hadron wave function



INTENSITY BY RUTHERFORD'S FORMULA

Rutherford formula

$$d\sigma = \left(\frac{Z\alpha}{mv^2} \right)^2 \frac{\sin\theta d\theta d\varphi}{\sin^4(\theta/2)}. \quad (29)$$

θ and φ are the spherical coordinates.

$\alpha \approx 1/137$ is the fine structure constant. Z is the charge number. m is the electron mass. v is the electron velocity.

Results

- In the domain where the wave packet of the initial electron interferes with the scattered wave, the coherent scattering dominates and the hadron wave packet can be regarded as a charged fluid.
- Out of this domain, the standard incoherent contribution dominates. The hadron wave packet can be considered a gas of point charged particles scattering incoherently the incoming electrons. For sufficiently narrow in momentum space electron wave packets, the scattering probabilities are summed up rather than the scattering amplitudes.

Results

- The effective mass operator of the electron determining the coherent scattering is

$$M(q) = e\gamma^\mu A_\mu^{eff}(q), \quad j_\mu^{eff}(q) := -q^2 A_\mu^{eff}(q),$$

$$j_\mu^{eff}(x) := \int \frac{dq}{(2\pi)^4} e^{iqx} j_\mu^{eff}(q) = eM \int \frac{d\mathbf{k}d\mathbf{k}'}{(2\pi)^3} \frac{\rho_{rr'}^n(\mathbf{k}, \mathbf{k}')}{\sqrt{k_0 k'_0}} e^{i(k'-k)x} \bar{u}^{r'}(\mathbf{k}') \Gamma_\mu u^r(\mathbf{k}).$$
(30)

Γ^μ is a nonlocal electromagnetic vertex.

- The inclusive probability to record an electron in electron-by-hadron scattering contains the infrared divergence at the order α^2 of the perturbation theory. It resides in the modulus squared of the standard connected part of the S -matrix. It stems from the fact that the electron wave packet is nonzero for the momentum of the electron recorded by the detector, i.e., for such a momentum that the transferred momentum vanishes. The incoherent contribution to the spin density matrix of the recorded electron reads

$$w_{s'_1 s'_2}^{(4)} = \frac{Z^2 e^4 \delta_{s'_1 s'_2}}{2(2\pi)^4 \beta_e'^2} \int \frac{d\mathbf{q}_\perp d\tilde{\mathbf{q}}_\perp d\mathbf{k}'}{(\mathbf{q}_\perp^2 - i0)(\tilde{\mathbf{q}}_\perp^2 + i0)} \rho_e(\mathbf{p}' + \mathbf{q}_\perp, \mathbf{p}' + \tilde{\mathbf{q}}_\perp) \rho_n(\mathbf{k}' - \mathbf{q}_\perp, \mathbf{k}' - \tilde{\mathbf{q}}_\perp).$$
(31)

$\mathbf{q}_\perp = \mathbf{k}'_\perp - \mathbf{k}_\perp$, $\tilde{\mathbf{q}}_\perp = \tilde{\mathbf{k}}'_\perp - \tilde{\mathbf{k}}_\perp$ are the transferred momenta transverse to β_e' .

Initial state

$$\hat{R} = \hat{R}_{ph} \otimes \hat{R}_e \otimes |0\rangle_{e+} \langle 0|_{e+}. \quad (32)$$

- The measurement is performed at the instant of time $t_0 < t_{out}$. As a result, one of the electrons is detected in one of the states distinguished by the projector D_e in the one-particle Hilbert space of electron states. The projector in the Fock space is $\hat{\Pi}_{D_e}$. At the instant of time $t = t_{out}$, a single photon is recorded in one of the states singled out by the projector D in the one-particle Hilbert space of photon states. The corresponding projector in the Fock space is $\hat{\Pi}_D$. The probability of such a chain of events is

$$P(\hat{\Pi}_D \leftarrow \hat{\Pi}_{D_e}) = \text{Sp}(\hat{\Pi}_D \hat{U}_{t_{out}, t_0} \hat{\Pi}_{D_e} \hat{U}_{t_0, t_{in}} \hat{R} \hat{U}_{t_{in}, t_0} \hat{\Pi}_{D_e} \hat{U}_{t_0, t_{out}}). \quad (33)$$

\hat{U}_{t_2, t_1} is the evolution operator of QED, and all the operators are given in the Schrödinger representation.

Conditional probability

$$P(\hat{\Pi}_D | \hat{\Pi}_{D_e}) = P(\hat{\Pi}_D \leftarrow \hat{\Pi}_{D_e}) / P(\hat{\Pi}_{D_e}). \quad (34)$$

Probability to detect the electron at the instant of time t_0 in the states distinguished by the projector D_e

$$P(\hat{\Pi}_{D_e}) = \text{Sp}(\hat{\Pi}_{D_e} \hat{U}_{t_0, t_{in}} \hat{R} \hat{U}_{t_{in}, t_0}). \quad (35)$$

Amplitude of stimulated radiation from a single free Dirac particle due to measurement of its state

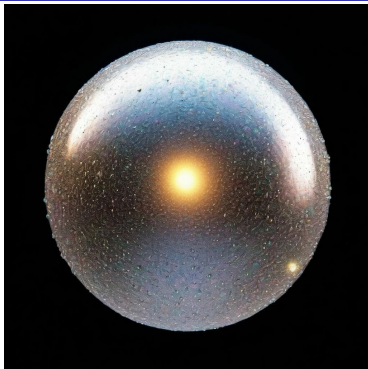
$$\begin{aligned}
 A^{\bar{\gamma}} = & -ie \left[\int_0^{\infty} dx^0 \int d\mathbf{x} \bar{\varphi}(x) \Gamma^i \varphi(x) \frac{f_{(\lambda)i}^*(\mathbf{k}) e^{ik_\mu x^\mu}}{\sqrt{2V k_0}} + \right. \\
 & \left. + \int_{-\infty}^0 dx^0 \int d\mathbf{x} \frac{\bar{\varphi}(x) \Gamma^i \psi(x)}{\langle \varphi | \psi \rangle} \frac{f_{(\lambda)i}^*(\mathbf{k}) e^{ik_\mu x^\mu}}{\sqrt{2V k_0}} \right].
 \end{aligned} \tag{36}$$

$\psi(x)$ is the free evolving state of the Dirac particle.

$\varphi(x)$ is the free evolving state coinciding at $t = 0$ with the state which the detector projects to.

- The first term in this expression describes the radiation from the classical current of a Dirac particle after the reduction of the wave function of this particle.
- The second term in this expression defines the normalized amplitude of photon radiation due to transition from the state ψ to the state φ during the time interval $t \in (-\infty, 0]$.

Conclusion: Photon and electron



Results

- The wave packet of any particle interacting with electromagnetic field possesses a dielectric permittivity that is revealed in coherent scattering of a photon by the wave packet of this particle.
- The photon wave packet has a dielectric permittivity of a transparent medium possessing frequency and spatial dispersions and linear and circular birefringences. The susceptibility of a beam of photons is proportional to the particle number density of photons at a given point.
- The wave packet of an electron has the same dielectric permittivity as a plasma. In this sense, it is metallic in color.