Schwinger mechanism of magnon-antimagnon pair production on magnetic field inhomogeneities and the bosonic Klein effect

T. C. Adorno[a], S.P. Gavrilov[b], and D.M. Gitman[c,d]

[a]Adorno@xjtlu.edu.cn
[b]Herzen State Pedagogical University of Russia
[c]University of São Paulo, Brazil
[d]P.N. Lebedev Physical Institute, Moscow

Efim Fradkin Centennial Conference, Sept., 2-6, 2024

August 31, 2024

Contents

To adopt the x-case basics: critical potential steps [S.P. Gavrilov and D.M. Gitman, Phys. Rev. D 93, 045002 (2016); Eur. Phys. J. C **80**, 820 (2020)]

The *EFT* model describing low-energy dynamics of spin waves (magnons) can technically be identified with the scalar QED. In this model, the magnetic field $B^a(\mathbf{r})$ plays the role of the electric field potential, $A_0^a(\mathbf{r})$ (an x-step). We use this framework to study the magnon-antimagnon pair production on magnetic field inhomogeneities.

[Phys. Rev. B 110, 014410 (2024)]

Introduction

Vacuum instability due to the production of real particles under the action of a strong external field = the Schwinger effect; see Physics Subject Headings (new classification scheme is now being used by Phys. Rev. The effects of an intense electric field have been essential for a number of realistic models of high-energy physics, astrophysics, and physics of nanostructures; recent reviews [A. Fedotov et al, Phys. Rep. **1010**, 1-138 (2023)] For a number of recently created nanomaterials (graphene, topological insulators, Dirac and Weyl semimetals), the dynamics of massless electronic excitations at low energies is described by the Dirac model.
It turn out that the magnon EFT that describes antiferromagnets is relativistic-like. The concept of the Dirac materials can also be applied to materials with **Bose-Einstein statistics** for quasiparticle excitations. **The physics that corresponds to the production of near massless particles in these materials is different, but its typical manifestations at low energies have similar kinematics.**

EFT model

The *EFT* model describing low-energy dynamics of magnons can be represented as a relativistic model of a charged scalar field, the mass of which is determined by the sum of the potential of the easy axis and the ratio of the parameters of magnetization and condensation. The magnetic field gradient acts on magnons in the same way as a constant electric field acts on charged scalar particles, $\partial_x A_0 = \partial_x B$.
The system consists of localized spins which live on sites of a cubic-type lattice. These sites are numbering by the index n. The corresponding spin vector operators are denoted by $\hat{\mathbf{s}}^n$. The Hamiltonian (the "Bose–Hubbard model") reads:

$$\hat{H}_{\text{spin}} = -\sum_n \sum_{i=1}^{d} J\delta^{ab}\hat{s}^n_a \hat{s}^{n+\hat{\imath}}_b - \sum_n \left[\mu B^a(\mathbf{r}_n)\,\hat{s}^n_a + C^{ab}\hat{s}^n_a \hat{s}^n_b\right] \ ,$$

$([\hat{s}^n_a, \hat{s}^n_b] = i\epsilon^c_{ab}\ \hat{s}^n_c)$; $J > 0$ is the antiferromagnetic interaction coupling constant. $C^{ab}\hat{s}^n_a \hat{s}^n_b$, is the single-ion anisotropic interaction (a product of the quenching of the orbital moment by the crystalline field).

EFT model

In the leading order of the derivative expansion at low-energies the $SO(3)$ gauge invariant effective Lagrangian can be written as:

$$\mathcal{L} = \frac{f_t^2}{2}\left(D_0 n^a\right)^2 - \frac{f_s^2}{2}\left(\partial_i n^a\right)^2 + rC^{ab} n_a n_b \ ,$$

where the covariant derivative D_0 with the $SO(3)$ background gauge field is defined as:

$$D_0 n^a = \partial_0 n^a - \epsilon^a_{bc}\ n^b \mu B^a,\ \partial_0 = \frac{\partial}{\partial t}$$

and low-energy parameters f_t, f_s, and r can be determined from the underlying lattice model by the matching condition.

EFT model

Suppose that our spin system possesses a potential with an easy-axis anisotropy and develops the collinear ground state. We apply an inhomogeneous magnetic field along the spin direction of the ground state. We assume that the magnetic field points to the direction of axis z and depends on the coordinate x, $B^a(x) = B(x)\,\delta^{a3}$, $B(x) > 0$. This field gives the collinear ground state with the Néel vector pointing to the direction of axis z as $\langle \mathbf{n} \rangle = (0, 0, 1)$. Then one can introduce magnon complex scalar fields $\Phi(X)$ and $\Phi^*(X)$ as fluctuations on the top of the ground state, which parametrize the vector $\mathbf{n}$ as

$$\mathbf{n} = \left(\frac{\Phi + \Phi^*}{\sqrt{2}}, \frac{\Phi - \Phi^*}{\sqrt{2}i}, \sqrt{1 - \Phi^*\Phi} \right)$$

EFT model

The effective Lagrangian of magnons at the quadratic order of fluctuation fields around the ground state in the following form:

$$\mathcal{L}^{(2)} = f_t^2 \left(D_0\Phi^* D_0\Phi - \Delta^2\Phi^*\Phi \right) - f_s^2\delta^{ij}\partial_i\Phi^*\partial_j\Phi,$$
$$D_0\Phi = (\partial_0 + iU)\,\Phi,\ D_0\Phi^* = (\partial_0 - iU)\,\Phi^*,$$

where $U = \mu B$ and $rC^{ab} = \frac{1}{2} f_t^2 \Delta^2 \delta^{a3}\delta^{b3}$. The *EFT* model can be identified with the scalar QED of a charged complex field $\Phi\,(X)$ (with μ playing the role of an electric charge) coupled to $A_0 = B$. Here the energy gap Δ plays the role of a mass term. The constant $v_s = f_s/f_t$ (plays the role of the speed of the light) is relatively small, e.g. $\Delta \sim 1\mathrm{meV}$ and $v_s \sim 60\mathrm{m/s}$ for antiferromagnetic MnF_2.

EFT model

The corresponding wave equation is a modification of the Klein-Gordon equation,

$$\left(D_0^2 - v_s^2\delta^{ij}\partial_i\partial_j + \Delta^2\right)\Phi(X) = 0,$$
$$D_0\Phi = (\partial_0 + iU)\Phi,\ \ U = \mu B(x)$$

The *EFT* model describing low-energy dynamics of magnons can technically be identified with the scalar QED. In this model, the magnetic field $B^a(x)$ plays the role of the electric field potential, $A_0^a(x)$ (an x-step). A complete set with $m = (p_0, \mathbf{p}_\perp)$:

$$\phi_m(X) = \exp(-i\varepsilon t + i\mathbf{p}_\perp\mathbf{r}_\perp)\,\varphi_m(x),\quad \mathbf{r}_\perp = (0, y, z),$$
$$\left\{v_s^2\partial_x^2 + [\varepsilon - U(x)]^2 - \pi_\perp^2\right\}\varphi_m(x) = 0,\ \ \pi_\perp = \sqrt{v_s^2\mathbf{p}_\perp^2 + \Delta^2},$$

x-case basics

We suppose that all the measurements are performed during a macroscopic time (say, the time T) when the external field can be considered as constant. $E(x) = -\partial_x A_0 = -\partial_x B$.

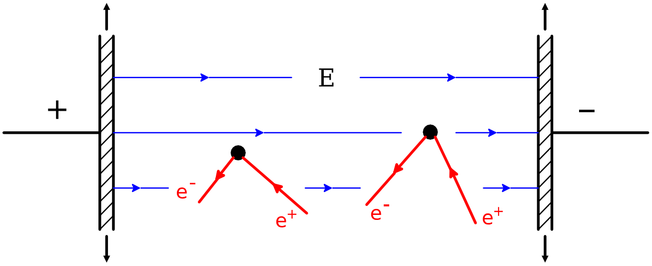


Figure: Capacitor

$$-\partial_x B = \text{const} > 0,\ x \in S_{\text{int}} = (x_{\text{L}}, x_{\text{R}});$$
$$-\partial_x B = 0,\ x \in S_{\text{L}} = (-\infty, x_{\text{L}}],\ x \in S_{\text{R}} = [x_{\text{R}}, \infty).$$

x-case basics

δU is the magnitude of the x-step,

$$\delta U = U_{\mathrm{L}} - U_{\mathrm{R}}, \quad U_{\mathrm{L}} = U(-\infty), \quad U_{\mathrm{R}} = U(+\infty).$$

If $\delta U < 2\Delta$ we deal with **noncritical steps: the range** Ω_3 (Klein zone) does not exist.
If $\delta U > 2\Delta$ we deal with **critical steps: the range** Ω_3, there exists the Klein zone.
$\pi_0(\mathrm{L}) = \varepsilon - U_{\mathrm{L}}$ asymptotic kinetic energy in the region $S_{\mathrm{L}} = (-\infty, x_{\mathrm{L}}]$,
$\pi_0(\mathrm{R}) = \varepsilon - U_{\mathrm{R}}$ asymptotic kinetic energy in the region $S_{\mathrm{R}} = [x_{\mathrm{R}}, \infty)$,

Solutions of KG equation with special left and right asymptotics

At left region S_{L} $(x < x_{\mathrm{L}})$ and right region S_{R} $(x > x_{\mathrm{R}})$, the KG equation has plane wave solutions which satisfy simple dispersion relations: $n = (\varepsilon, \mathbf{p}_\perp)$

$$
\begin{aligned}
{}_\zeta\varphi_m(x) &\sim \exp\left(ip^{\mathrm{L}}x\right) \ \text{as } x \in S_{\mathrm{L}}, \ \ \zeta = \mathrm{sgn}\left(p^{\mathrm{L}}\right) = \pm\,; \\
{}^\zeta\varphi_m(x) &\sim \exp\left(ip^{\mathrm{R}}x\right) \ \text{as } x \in S_{\mathrm{R}}, \ \ \zeta = \mathrm{sgn}\left(p^{\mathrm{R}}\right) = \pm, \\
p^{\mathrm{R/L}} &= \frac{\zeta}{v_s}\sqrt{\left[\pi_0(\mathrm{R/L})\right]^2 - \pi_\perp^2}, \ \ \pi_\perp = \sqrt{\mathbf{p}_\perp^2 + m^2}.
\end{aligned}
$$

The inner product on the hyperplane $x = \mathrm{const}$

$$
(\Phi, \Phi')_x = i\int\left[\Phi'(X)\,\partial_x\Phi^*(X) - \Phi^*(X)\,\partial_x\Phi'(X)\right]dt d\mathbf{r}_\perp
$$

is conserved. These solutions describe states with given conserved current along axis x.

Solutions of KG equation with special left and right asymptotics

If $(\Phi, \Phi')_x \neq 0$, solutions can be subjected to the following orthonormality conditions

$$\left({}_\zeta\phi_m, {}_{\zeta'}\phi_{m'} \right)_x = \zeta \delta_{\zeta,\zeta'} \delta_{m,m'}, \quad \left({}^\zeta\phi_m, {}^{\zeta'}\phi_{m'} \right)_x = \zeta \delta_{\zeta,\zeta'} \delta_{m,m'},$$
$${}_\zeta\mathcal{N} = \left| 2p^{\mathrm{L}} \right|^{-1/2} Y, \quad {}^\zeta\mathcal{N} = \left| 2p^{\mathrm{R}} \right|^{-1/2} Y, \quad Y = (V_\perp T)^{-1/2}$$

Nontrivial solutions ${}_\zeta\phi_m$ exist only for certain m,

$$[\pi_0(\mathrm{L})]^2 > \pi_\perp^2 \Longleftrightarrow \begin{cases} \pi_0(\mathrm{L}) > \pi_\perp \\ \pi_0(\mathrm{L}) < -\pi_\perp \end{cases}.$$

Nontrivial solutions ${}^\zeta\phi_m$ exist only for certain m,

$$[\pi_0(\mathrm{R})]^2 > \pi_\perp^2 \Longleftrightarrow \begin{cases} \pi_0(\mathrm{R}) > \pi_\perp \\ \pi_0(\mathrm{R}) < -\pi_\perp \end{cases}.$$

x-case basics: Ranges of quantum numbers

There exist five ranges Ω_k, $k = 1, \ldots, 5$ of quantum numbers m where the solutions have similar asymptotics,

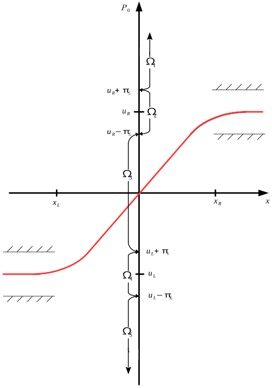


Figure: Potential energy $U(x)$ of a particle in an x-step and ranges of quantum numbers

Third range (Klein zone)

Ω_3

The Klein zone exists if $\delta U > 2m$. Here quantum numbers $\mathbf{p}_\perp$ are restricted by $2\pi_\perp \leq \delta U$,

$$\pi_0\left(\mathrm{L}\right) \leq -\pi_\perp,\ \pi_0\left(\mathrm{R}\right) \geq \pi_\perp \text{ if } m \in \Omega_3,$$

and there exist the following two complete sets of solutions

$$\left\{ {}_\zeta\phi_m\left(X\right)\right\},\ \left\{ {}^\zeta\phi_m\left(X\right)\right\},\ \ \zeta = \pm.$$

In contrast to the ranges Ω_1 and Ω_5, the naive one-particle interpretation of these solutions becomes erroneous. Approaches for treating quantum effects in the explicitly time-dependent external fields are not directly applicable to the critical potential steps. **A consistent nonperturbative formulation of quantum electrodynamics (QED) with such steps was given recently in**

[S.P. Gavrilov and D.M. Gitman, Phys. Rev. D 93, 045002 (2016); Eur. Phys. J. C **80**, 820 (2020)]

Decomposition

We assume that each pair ${}_{\zeta}\phi_{m}\left(X\right)$ and ${}^{\zeta}\phi_{m}\left(X\right)$, with given $m \in \Omega_1 \cup \Omega_3 \cup \Omega_5$ is complete in the space of solutions with each n. The corresponding mutual decompositions have the form

$$\begin{aligned} {}^{\zeta}\phi_{m}\left(X\right) &= {}_{+}\phi_{m}\left(X\right) g\left({}_{+}\middle|{}^{\zeta}\right) - {}_{-}\phi_{m}\left(X\right) g\left({}_{-}\middle|{}^{\zeta}\right), \\ {}_{\zeta}\phi_{m}\left(X\right) &= {}^{+}\phi_{m}\left(X\right) g\left({}^{+}\middle|{}_{\zeta}\right) - {}^{-}\phi_{m}\left(X\right) g\left({}^{-}\middle|{}_{\zeta}\right), \end{aligned}$$

where coefficients g:

$$\left({}_{\zeta}\phi_{m}, {}^{\zeta'}\phi_{m'}\right)_{x} = \delta_{m,m'} g\left({}_{\zeta}\middle|{}^{\zeta'}\right), \quad g\left({}^{\zeta'}\middle|{}_{\zeta}\right) = g\left({}_{\zeta}\middle|{}^{\zeta'}\right)^{*}$$

Unitary relations for the decomposition coefficients:

$$g\left({}^{\zeta'}\middle|{}_{+}\right) g\left({}_{+}\middle|{}^{\zeta}\right) - g\left({}^{\zeta'}\middle|{}_{-}\right) g\left({}_{-}\middle|{}^{\zeta}\right) = \zeta\delta_{\zeta,\zeta'} .$$

To extract results of the one-particle scattering theory, all the constituent quantities, such as reflection and transmission coefficients etc., have to be represented with the help of the g's, that is, the matrix elements of current in x-direction.

Orthogonality and normalization over space volume

However, it should be noted that **QFT deals with physical quantities that are presented by volume integrals on the hyperplane** $t = \mathrm{const}$.

$$\left(\Phi,\Phi'\right) \approx \int_{-K^{(\mathrm{L})}}^{x_{\mathrm{L}}} \Theta dx + \int_{x_{\mathrm{R}}}^{K^{(\mathrm{R})}} \Theta dx \ ,$$
$$\Theta = \frac{1}{v_s^2} \int \left\{ \Phi^* \left(i\partial_0 - U\right)\Phi + \left[\left(i\partial_0 - U\right)\Phi\right]^* \Phi \right\} d\mathbf{r}_\perp ,$$

where the improper integral is reduced to its special principal value,

$$K^{(\mathrm{L})}/v^{\mathrm{L}} - K^{(\mathrm{R})}/v^{\mathrm{R}} = O\left(1\right) , \quad v^{\mathrm{L/R}} = v_s^2 \left| p^{\mathrm{L/R}}/\pi_0\left(\mathrm{L/R}\right)\right|$$

and the limits $K^{(\mathrm{L/R})}/v^{\mathrm{L/R}} = T \ (T \to \infty)$ are assumed in final expressions. Note in the case under consideration, the potential step with different asymptotics at $x \longrightarrow \pm\infty$ cannot be subjected to any periodic boundary conditions in X-direction without changing its physical meaning.

Orthogonality and normalization over space volume

$$\left({}_{\zeta}\phi_m ,\ {}_{\zeta}\phi_m \right) = \left({}^{\zeta}\phi_m ,\ {}^{\zeta}\phi_m \right) = \mathrm{sgn}\ \pi_0\, (\mathrm{L/R})\, \mathcal{M}_m \ \mathrm{if}\ \ m \in \Omega_{1,5};$$

$$\left({}_{\zeta}\phi_m ,\ {}_{\zeta}\phi_m \right) = - \left({}^{\zeta}\phi_m ,\ {}^{\zeta}\phi_m \right) = \mathcal{M}_m ,\ \mathcal{M}_m = 2 \left| g\left({}_{+} \middle| {}^{-} \right) \right|^2 ,$$

$$\left({}_{\zeta}\phi_m ,\ {}^{\zeta}\phi_m \right) = 0 \ \mathrm{if}\ \ m \in \Omega_3 ,\ \ {}_{\zeta}\phi_m \ \mathrm{and}\ {}^{\zeta}\phi_m \ \ \mathrm{independent}.$$

Then we identify:

$$\mathrm{in-solutions}: {}_{+}\phi_{m_1},\ \ {}^{-}\phi_{m_1};\ {}_{-}\phi_{m_5},\ {}^{+}\phi_{m_5};\ {}_{-}\phi_{m_3},\ \ {}^{-}\phi_{m_3},$$

$$\mathrm{out-solutions}:\ {}_{-}\phi_{m_1},\ \ {}^{+}\phi_{m_1};\ {}_{+}\phi_{m_5},\ {}^{-}\phi_{m_5};\ {}_{+}\phi_{m_3},\ \ {}^{+}\phi_{m_3}$$

x-case basics: Quantization

$$\Psi(X) \Longrightarrow \hat{\Psi}(X), \quad \hat{\Psi}(X) = \begin{pmatrix} i\hat{\Pi}^{\dagger}(X) \\ \hat{\Phi}(X) \end{pmatrix}, \quad \hat{\Psi}(X) = \sum_{i=1}^{5} \hat{\Psi}_i(X)$$

$$\left[\hat{\Psi}(X), \hat{\Psi}^{\dagger}(X')\right]_{-}\Big|_{t=t'} = \delta(\mathbf{r} - \mathbf{r}') \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

$$\hat{\Psi}_3(X) = \sum_{m \in \Omega_3} \mathcal{M}_m^{-1/2} \left[{}_{-}a_m(\mathrm{in}) \, {}_{-}\phi_m(X) + {}^{-}b_m^{\dagger}(\mathrm{in}) \, {}^{-}\phi_m(X) \right]$$

$$= \sum_{m \in \Omega_3} \mathcal{M}_m^{-1/2} \left[{}_{+}a_m(\mathrm{out}) \, {}_{+}\phi_m(X) + {}^{+}b_m^{\dagger}(\mathrm{out}) \, {}^{+}\phi_m(X) \right].$$

Relations between in- and out-operators

In the range Ω_3 (the Klein zone)

$$\begin{aligned}
{}_{+}a_m(\mathrm{out}) &= g\left({}^{-}|{}_{+}\right)^{-1} g\left({}^{-}|{}_{-}\right)\ {}_{-}a_m(\mathrm{in}) - g\left({}^{-}|{}_{+}\right)^{-1}\ {}^{-}b_m^{\dagger}(\mathrm{in}),\\
{}^{+}b_m^{\dagger}(\mathrm{out}) &= -g\left({}_{-}|{}^{+}\right)^{-1}\ {}_{-}a_m(\mathrm{in}) + g\left({}_{-}|{}^{+}\right)^{-1} g\left({}_{-}|{}^{-}\right)\ {}^{-}b_m^{\dagger}(\mathrm{in}),\\
{}_{-}a_m(\mathrm{in}) &= g\left({}^{+}|{}_{-}\right)^{-1} g\left({}^{+}|{}_{+}\right)\ {}_{+}a_m(\mathrm{out}) + g\left({}^{+}|{}_{-}\right)^{-1}\ {}^{+}b_m^{\dagger}(\mathrm{out})\\
{}^{-}b_m^{\dagger}(\mathrm{in}) &= g\left({}_{+}|{}^{-}\right)^{-1}\ {}_{+}a_m(\mathrm{out}) + g\left({}_{+}|{}^{-}\right)^{-1} g\left({}_{+}|{}^{+}\right)\ {}^{+}b_m^{\dagger}(\mathrm{out})
\end{aligned}$$

show us that vacuum vectors $|0,\mathrm{in}\rangle$ and $|0,\mathrm{out}\rangle$,

$$a(\mathrm{in})\,|0,\mathrm{in}\rangle = b(\mathrm{in})\,|0,\mathrm{in}\rangle = 0,\ \ a(\mathrm{out})\,|0,\mathrm{out}\rangle = b(\mathrm{out})\,|0,\mathrm{out}\rangle = 0,$$

are different.

A differential mean number of particles created from vacuum can be expressed via these coefficients as

$$N_n^{\mathrm{cr}} = \left\langle 0,\mathrm{in}\left|\ {}_{+}a_m^{\dagger}(\mathrm{out})\ {}_{+}a_m(\mathrm{out})\right|0,\mathrm{in}\right\rangle = \left|g\left({}_{+}|{}^{-}\right)\right|^{-2}.$$

x-case basics

The total number of pairs N^{cr} and flux density $|j_x|$:

$$N^{\mathrm{cr}} = \sum_{n\in\Omega_3} N_n^{\mathrm{cr}} = V_\perp T\,|j_x|\,,$$
$$|j_x| = \frac{1}{(2\pi)^3}\int_{\Omega_3} d\varepsilon d\mathbf{p}_\perp N_n^{\mathrm{cr}},$$

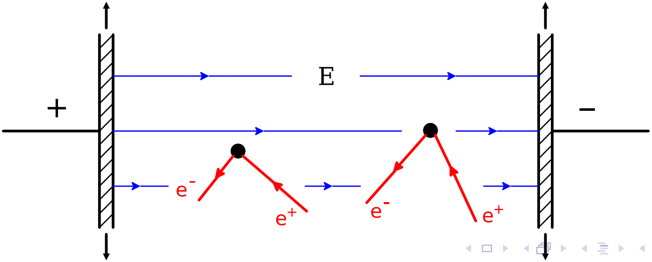
where $V_\perp T$ is transversal space-time volume.

L-constant gradient

$$N_n^{\rm cr} \approx N_n^0 = e^{-\pi\lambda}, \quad \lambda = \frac{\pi_\perp^2}{|\mu B'|\, v_s}, \quad \pi_\perp = \sqrt{v_s^2 \mathbf{p}_\perp^2 + \Delta^2}$$

is quasiconstant over the wide range of the energy ε for any given λ. Pair creation effects are proportional to the magnitude of potential step $\delta U = |\mu B'|\, L$ (the maximum increment of a particle energy) [S.P. Gavrilov and D.M. Gitman, Phys. Rev. D 93, 045033 (2016)]

(i) L-const. $E(x) = -B' = \text{const} > 0, \quad x \in S_{\rm int}$ within the spatial region L and is zero outside of it



Peak field

(ii) The Sauter-like magnetic step [S.P. Gavrilov and D.M. Gitman, Phys. Rev. D 93, 045002 (2016)]:

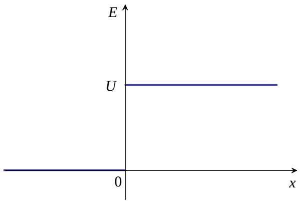
$$E(x) = -B' \cosh^{-2}(x/L_S), \; B(x) = L_S B' \tanh(x/L_S), \; L_S > 0;$$
$$\delta U = 2\left|\mu B'\right| L_S \gg 2\Delta, \; \delta U L_S / v_s \gg 1 - \text{smooth}$$
$$N_n^{cr} \approx N_n^{as} = e^{-\pi\tau}, \; \tau = \exp\left[-\pi L_S\left(2\left|\mu B'\right| L_S / v_s - \left|p^{\mathrm{R}}\right| - \left|p^{\mathrm{L}}\right|\right)\right]$$
$$\left|p^{\mathrm{R/L}}\right| \approx v_s^{-1}\sqrt{\left|\mu B' L_S\right|^2 - \pi_\perp^2}$$

Rectangular (the Klein) step

Sharp Sauter potential

$$\delta U = \left|\mu B'\right| L_S = \text{const} \gg 2\Delta,\ \delta U L_S / v_s \ll 1 - \text{sharp}\ \left(\textit{the Klein step}\right)$$

imitates a sufficiently high rectangular potential step (Klein step)



$$N_n^{cr} \approx \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\left(\frac{\delta U^2 L_S}{2v_s^2}\right)^2 + \left(\left|p^{\mathrm{L}}\right| - \left|p^{\mathrm{R}}\right|\right)^2} \to \frac{4\left|p^{\mathrm{L}}\right|\left|p^{\mathrm{R}}\right|}{\left(\left|p^{\mathrm{L}}\right| - \left|p^{\mathrm{R}}\right|\right)^2}$$

Observable physical quantities specifying the vacuum instability

Vacuum MM current and energy-momentum tensor of final pair created:
In weakly inhomogeneous fields:

$$J^1_{\rm cr} = \mu \left| j_x \right| , \quad J^0_{\rm cr}(x) = \begin{cases} -\mu \left| j_x \right| / v_s, & x \in S_{\rm L} \\ \mu \left| j_x \right| / v_s, & x \in S_{\rm R} \end{cases} ;$$

$$T^{10}_{\rm cr}\left(S_{\rm R}\right) = -T^{10}_{\rm cr}\left(S_{\rm L}\right) = \frac{1}{2}\delta U \left| j_x \right| ,$$

$$T^{00}_{\rm cr}(x) = \begin{cases} \left|\pi_0\left({\rm L}\right)\right| \left| j_x \right| / v_s, & x \in S_{\rm L} \\ \left|\pi_0\left({\rm R}\right)\right| \left| j_x \right| / v_s, & x \in S_{\rm R} \end{cases}$$

$$\left| j_x \right| = \frac{N^{\rm cr}}{V_\perp T}$$

Statistically-assisted Schwinger effect for bosons

An increment of the numbers of (anti)particles

$$\Delta N_m = N_m^{(\zeta)} - N_m^{(\zeta)}(\mathrm{in}),$$
$$\Delta N_m = N_m^{\mathrm{cr}}\left[1 + N_m^{(+)}(\mathrm{in}) + N_m^{(-)}(\mathrm{in})\right]$$

The number of created (anti)particles is growing in comparison with the one created from the vacuum.
E.g., **the flux of created antiparticles in the area S_{L} is growing proportionally to the flux of coming particles from the area S_{R}.**

The main new results obtained

- The *EFT* model describing low-energy dynamics of spin waves (magnons) can technically be identified with the scalar QED with a magnetic field plays the role of the electric field potential, $A_0^a(\mathbf{r})$ (an x-step). The magnetic field gradient acts on magnons in the same way as a constant electric field acts on charged scalar particles. We present a Fock space realization of the model.

The main new results obtained

- The *EFT* model describing low-energy dynamics of spin waves (magnons) can technically be identified with the scalar QED with a magnetic field plays the role of the electric field potential, $A_0^a(\mathbf{r})$ (an x-step). The magnetic field gradient acts on magnons in the same way as a constant electric field acts on charged scalar particles. We present a Fock space realization of the model.
- A magnon-antimagnon pair production on magnetic field inhomogeneities is studied.

The main new results obtained

- A method was proposed for the experimental implementation of the boson Klein effect (production of magnon-anti-magnon pairs) and its application to amplify magnon currents in magnetic nanostructures.

The main new results obtained

- A method was proposed for the experimental implementation of the boson Klein effect (production of magnon-anti-magnon pairs) and its application to amplify magnon currents in magnetic nanostructures.
- Statistically-assisted Schwinger effect for bosons is discovered

The end