Superfluidity in a neutrino cluster

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Plan of the talk

- Neutrino interaction with scalar bosons
- Neutrino superfluidity: formation of neutrino pairs, gap equation and its solution
- Formation of a neutrino cluster
- Results
- Parameters of a scalar boson
- Cluster cooling
- Summary

References

M. Dvornikov, Superfluidity in neutrino clusters,
J. Phys. G **51**, 075201 (2024), arXiv:2310.04806

Scalar fields in elementary particle physics

- Nowadays the Higgs boson is the only known scalar field
- The interaction of other particles with the Higgs field generates their masses
- Higgs boson was experimentally discovered in 2012
- Various extentions of the standard model alllow the presence of additional scalar particles, which can have rather small masses
- The constraints on the characteristics of these new scalat particles are present in J.M. Berryman, et al.[arXiv:1802.00009]

Neutrino interaction with a scalar field

Lagrangian of a system is

$$\mathcal{L} = \bar{\nu}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m)\nu + \frac{1}{2}(\partial_{\mu}\phi\partial^{\mu}\phi - M^{2}\phi^{2}) + g\phi\bar{\nu}\nu$$

Using the standard technique, we exclude the scalar field

$$\mathcal{L} = \bar{\nu}(\mathrm{i}\gamma^{\mu}\partial_{\mu} - m)\nu + \frac{g^{2}}{2M^{2}}(\bar{\nu}\nu)(\bar{\nu}\nu)$$

The interaction of fermions, e.g., neutrinos, mediated by a scalar field is attractive (Peskin & Schroeder, 1995). Thus, neutrinos can form bound states.

Neutrino superfluidity

Kapusta (2004) and some succeeding authors previously studied the neutrino superfluidity owing to the scalar boson exchange

We present the neutrino bispinor as $\nu^{\mathrm{T}} = (\varphi, \chi)$

The definition of the neutrino condensate: $\langle \varphi_{-a}(\mathbf{p}) \varphi_{+b}(\mathbf{p}) \rangle = \epsilon_{ab} D$

The neutrino interaction Hamiltonian

$$H_{\text{int}} = -\frac{4g^2}{M^2} \sum_{\mathbf{p}} \frac{m^2}{(\varepsilon + m)^2} \Big\{ D^* e^{-2\mathrm{i}\varepsilon t} \left[b_+(\mathbf{p}) b_-(-\mathbf{p}) - b_-(\mathbf{p}) b_+(-\mathbf{p}) \right] + D e^{2\mathrm{i}\varepsilon t} \left[b_-^\dagger(\mathbf{p}) b_+^\dagger(-\mathbf{p}) - b_+^\dagger(\mathbf{p}) b_-^\dagger(-\mathbf{p}) \right] \Big\}$$

We also take into account the Hamiltonian of noninteracting neutrinos

$$H_0 = \sum_{\mathbf{p}} (\varepsilon - \mu) \left[b_+^\dagger(\mathbf{p}) b_+(\mathbf{p}) + b_-^\dagger(\mathbf{p}) b_-(\mathbf{p}) \right]$$

Diagonalization of Hamiltonian and the energy gap equation

After the Bogoliubov transformation

$$c_+(\mathbf{p}) = e^{-\mathrm{i}\varepsilon t - \mathrm{i}\alpha} \cos\frac{\theta}{2} b_+(\mathbf{p}) + e^{\mathrm{i}\varepsilon t + \mathrm{i}\alpha} \sin\frac{\theta}{2} b_-^\dagger(-\mathbf{p}), \quad c_-(\mathbf{p}) = e^{-\mathrm{i}\varepsilon t - \mathrm{i}\alpha} \cos\frac{\theta}{2} b_-(\mathbf{p}) - e^{\mathrm{i}\varepsilon t + \mathrm{i}\alpha} \sin\frac{\theta}{2} b_+^\dagger(-\mathbf{p})$$

the Hamiltonian takes the form,

$$H = \sum_{\mathbf{p}} E \left[c_+^\dagger(\mathbf{p}) c_+(\mathbf{p}) + c_-^\dagger(\mathbf{p}) c_-(\mathbf{p}) \right], \quad E = \sqrt{(\varepsilon - \mu)^2 + \Delta^2}, \quad \Delta = \frac{8 g^2 m^2 |D|}{M^2 (\varepsilon + m)^2}$$

The gap equation for Δ for the degenerate neutrino gas,

$$\frac{2g^2}{M^2} \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \frac{m^2}{(\varepsilon + m)^2} \frac{1}{\sqrt{(\varepsilon - \mu)^2 + \Delta^2}} = 1$$

Solution of the gap equation

The integral is divergent at great momenta. One should use the cut-off parameter Λ. While considering the Yukawa interaction, it is unclear what the magnitude of Λ is. Following Lifshits & Pitaevskii (2002) we take that $\Lambda \sim \mu$

We look for the solution near the Fermi sphere, $\varepsilon \approx \mu + v_{\mathrm{F}}(p - p_{\mathrm{F}})$

The solution is

$$\Delta = \frac{v_{\mathrm{F}}}{\sinh y} \sqrt{(\Lambda - p_{\mathrm{F}})^2 + p_{\mathrm{F}}^2 + 2p_{\mathrm{F}}(\Lambda - p_{\mathrm{F}}) \cosh y}, \quad y = \frac{\pi^2 M^2 (\mu + m)^2}{g^2 m^2 \mu p_{\mathrm{F}}}$$

The phase transition temperature is $T_c \approx 0.57\Delta$

Neutrino cluster

Smirnov & Xu (2022) described how a neutrino cluster is formed

The system of wave equations for a scalar boson and a neutrino

$$(\partial_\mu \partial^\mu + M^2)\phi = g\bar{\nu}\nu, \quad [\mathrm{i}\gamma^\mu \partial_\mu - (m - g\phi)]\nu = 0$$

Neutrino acquires the effective mass $m_{\text{eff}} = m - g\langle\phi\rangle$

The value of the condensate for degenerate neutrinos

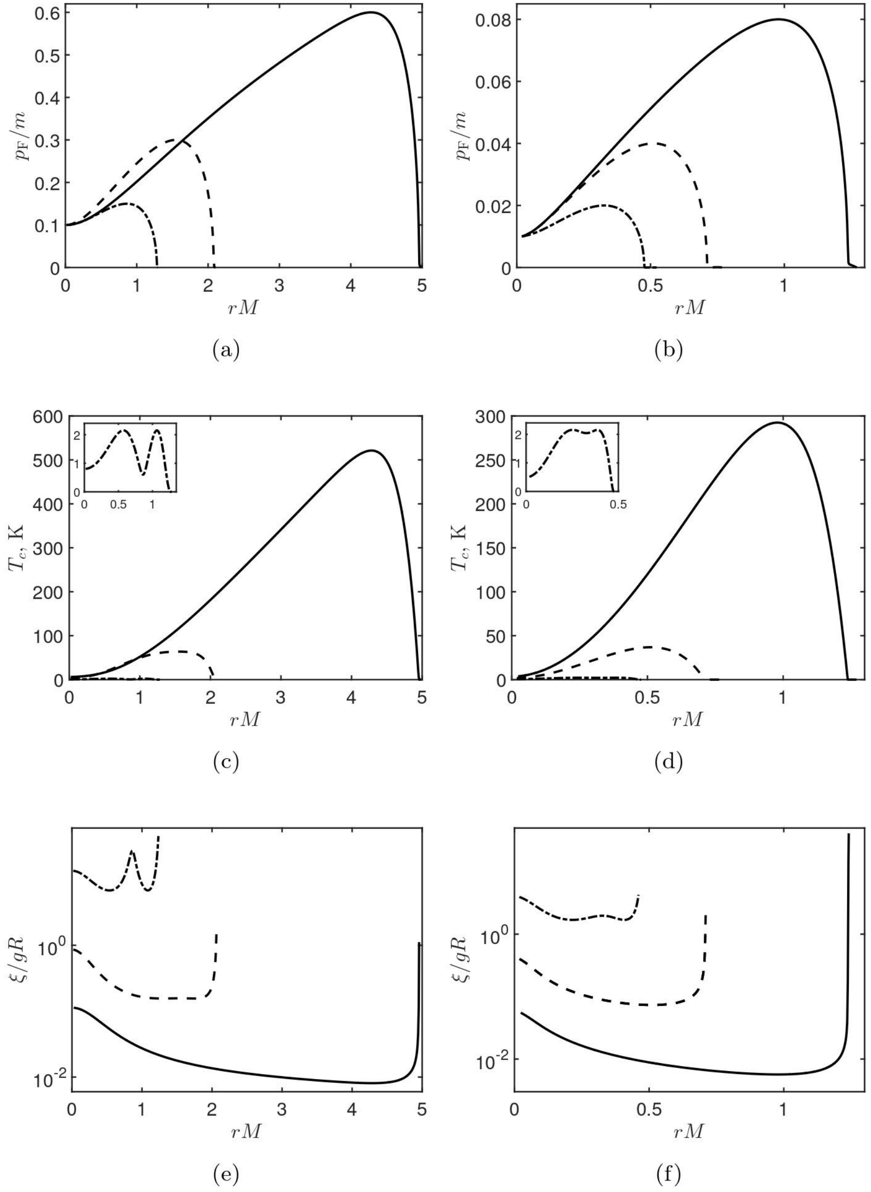
$$\langle\bar{\nu}\nu\rangle = \frac{1}{\pi^2}\int_0^{p_\mathrm{F}} p^2 \mathrm{d}p \frac{m_{\text{eff}}}{\varepsilon_{\text{eff}}} = \frac{m_{\text{eff}}}{2\pi^2}\left[p_\mathrm{F}\sqrt{p_\mathrm{F}^2 + m_{\text{eff}}^2} - m_{\text{eff}}^2 \ln\left(\frac{p_\mathrm{F} + \sqrt{p_\mathrm{F}^2 + m_{\text{eff}}^2}}{m_{\text{eff}}}\right)\right]$$

The wave equation for boson which are distributed in a spherically symmetrical way

$$\left(M^2 - \frac{\mathrm{d}^2}{\mathrm{d}r^2} - \frac{2}{r}\frac{\mathrm{d}}{\mathrm{d}r}\right)\langle\phi\rangle = \frac{g m_{\text{eff}}}{2\pi^2}\left[p_\mathrm{F}\sqrt{p_\mathrm{F}^2 + m_{\text{eff}}^2} - m_{\text{eff}}^2 \ln\left(\frac{p_\mathrm{F} + \sqrt{p_\mathrm{F}^2 + m_{\text{eff}}^2}}{m_{\text{eff}}}\right)\right]$$

The value of the scalar boson condensate of the Fermi momentum are not independent since the chemical potential is constant inside a cluster: $\mu = \sqrt{p_\mathrm{F}^2 + m_{\text{eff}}^2} = \text{const}$

Results: Fermi momenta, phase transition temperature, and coherence length



(a), (b) - Fermi momenta

(c), (d) - phase transition temperature

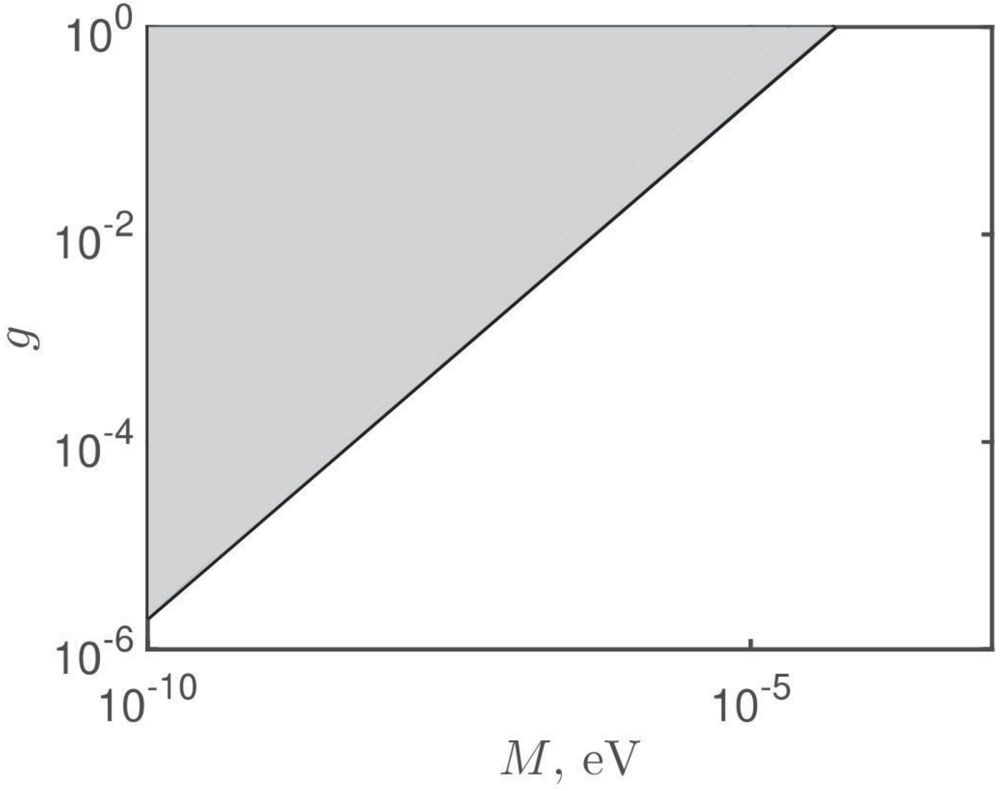
(e), (f) - coherence length $\xi = v_F/\pi\Delta$

(a), (c), (e): $\chi = M/gm = 0.1$, $p_F(0)/m = 0.1$. Solid line - $\mu/m = 0.6$, dashed line - $\mu/m = 0.3$, dash-dotted line - $\mu/m = 0.15$.

(b), (d), (f): $\chi = 0.01$, $p_F(0)/m = 0.01$. Solid line - $\mu/m = 0.08$, dashed line - $\mu/m = 0.04$, dash-dotted line - $\mu/m = 0.02$.

One can see that the superfluidity takes place if a cluster consists of relic neutrinos having T = 1.95 K

Limits on the scalar boson properties to have a neutrino superfluidity in a cluster



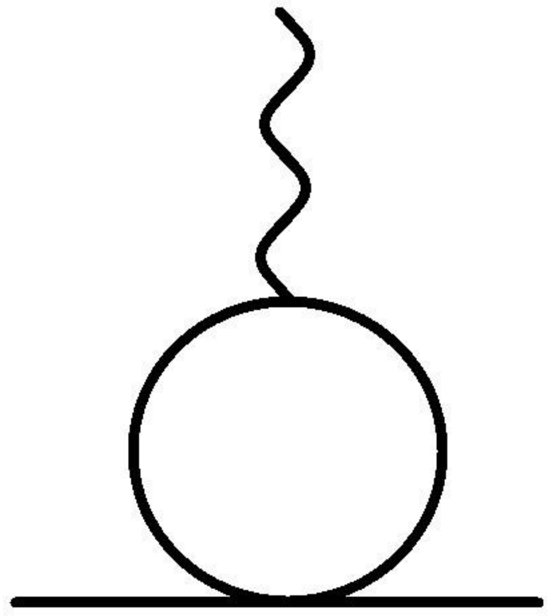
- Excluded areas are grey
- The upper border corresponds to a solution for the equation for a cluster where neutrino gas is still superfluid
- The extablished constrains on g and M do not contradict the experimental bounds on the scalar boson properties (Berryman et al., 2018)

Cluster cooling

- We suggest that a cluster is formed by relic neutrinos. If a neutrino density rises, the temperature of the neutrino gas also increases locally
- The enhanced temperature, first, can destroy the cluster. (We recall that the neutrino gas is supposed to be highly degenerate. The temperature effects on the cluster characteristics are studied by Dvornikov (2024).)
- Second, if the cluster temperature is above the phase transition temperature, the superfluidity disappers
- Smirnov & Xu (2022) suggested the mechanism for the cluster cooling based on bremsstrahlung ($\nu\nu \rightarrow \nu\nu\phi$) and annihilation ($\nu\bar{\nu} \rightarrow \phi\phi$). However, the cooling time is greater than the universe age. Thus, these mechanisms are unsatisfactory
- We take that a neutrino cluster appears in the early universe after the electroweak phase transition. In this case neutrinos are massive particles
- We assumme that it cools cools down by the neutrino Cherenkov radiation in hot primordial plasma

Neutrino Cherenkov radiation

- A charged particle moving in matter can emit Cherenkov photons with the refration index $n = k/\omega > 1$
- A neutrino is electrically neutral particle. However, in matter with nonzero temperature and density, it can acquire the induced electric charge, see, e.g., Oraevsky & Semikoz (1987)
- The Cherenkov radiation of neutrinos is possible even for massless neutrinos in frames of the standard model



Cluster cooling by Cherenkov radiation

- Dvornikov (2024) calculated the Cherenkov plasmon luminosity $\dot{E}$ in a hot primordial plasma with zero chemical potential
- We take into account only longitudinal plasmons
- In this calculation, we suggest that (hot neutrino gas)->(hot neutrino gas)+plasmon, rather than consider a single neutrino propagating with a certain velocity
- Cherenkov plasmons are unstable, i.e. they decay after propagating cetrain length L. In happens since the longitudinal form factor acquires the imaginary part at n>1. This process is analogous to the Landau damping
- We assume that a cluster cools down layer-by-layer: when the outer layer cools down, the second one starts to emit plasmons, etc. Thus, we should replace $\dot{E} \to E(L/R)$, where R is the cluster radius
- If a cluster appears in the early universe and we require that $T_{\mathrm{clust}}/\dot{E} < H^{-1}$, its cooling rate is greater than the universe expansion
- We get that, if a cluster appears in the epoch with 26 keV < T < 100 GeV, it successfully cools down
- The temperature of the neutrino decopling is (2-3) MeV. Thus, if a cluster is formed at 26 keV < T < (2-3) MeV, it is destroyed by the termal fluctuations of relic neutrinos gas

Summary

- We showed that the neutrino interaction by a scalar boson is attractive
- This interaction leads to the formation of a neutrino cluster
- Neutrinos can form pairs with opposite spins. These pairs constitute the superfluid condensate
- Superfluid condensate can consist of relic neutrinos
- The constains on the Yukawa interaction necessary for the existance of superfluidity are not ruled out by the experimental bounds on the scalar boson properties
- Cherenkov radiation is a possible channel for the cluster cooling if it appears in the early universe