Kinetic theory of electron-positron pair production in rotating electric fields

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Plan of the talk

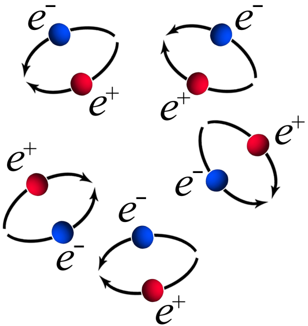
1. Introduction. Vacuum pair production in strong fields
2. Furry-picture quantization of the electron-positron field
3. One-particle correlation functions and quantum kinetic equations (QKE)
4. Dirac-Heisenberg-Wigner (DHW) formalism
5. Application. Helicity effects in rotating electric fields

Introduction. Vacuum e^+e^- pair production

Spatial and temporal scales
of the vacuum fluctuations in QED:

$$\ell = \frac{\hbar}{mc} \sim 10^{-13}\ \text{m},$$

$$\tau = \frac{\hbar}{mc^2} \sim 10^{-21}\ \text{s}.$$



Critical (Schwinger) field strength:

$$|e|E\ell \sim mc^2 \quad \Longrightarrow \quad E \sim E_\text{c} \equiv \frac{m^2c^3}{|e|\hbar} \sim 10^{16}\ \text{V/cm}$$

Introduction. Vacuum e^+e^- pair production

- Vacuum decay rate in a constant electric field is $\sim \mathrm{e}^{-\pi E_{\mathrm{c}}/E_0}$.
- Actual particle yield contains the volume prefactor V and depends on the temporal profile $E(t)$.
- Pair production threshold is $I \sim 10^{26}$ W/cm^2 ($I_{\mathrm{c}} \sim 10^{29}$ W/cm^2).

  [N. B. Narozhny *et al.*, JETP Lett. **80**, 382 (2004); A. G. Tkachev, I. A. Aleksandrov, V. M. Shabaev, arXiv:2408.04084]
- Maximal reported intensity is $I = 1.1 \times 10^{23}$ W/cm^2.

  [J. W. Yoon *et al.*, Optica **8**, 630 (2021)]
- Theoretical description requires nonperturbative numerical techniques.

I. Furry-picture quantization

Define two sets of the *in* and *out* solutions $\{_\zeta\varphi_n\}$ and $\{^\zeta\varphi_n\}$, respectively ($\zeta = \pm$).

The field operator in the Heisenberg representation reads:

$$\psi(x) = \sum_n \left[a_n(\text{in})\, {}_+\varphi_n(x) + b_n^\dagger(\text{in})\, {}_-\varphi_n(x) \right],$$

$$\psi(x) = \sum_n \left[a_n(\text{out})\, {}^+\varphi_n(x) + b_n^\dagger(\text{out})\, {}^-\varphi_n(x) \right].$$

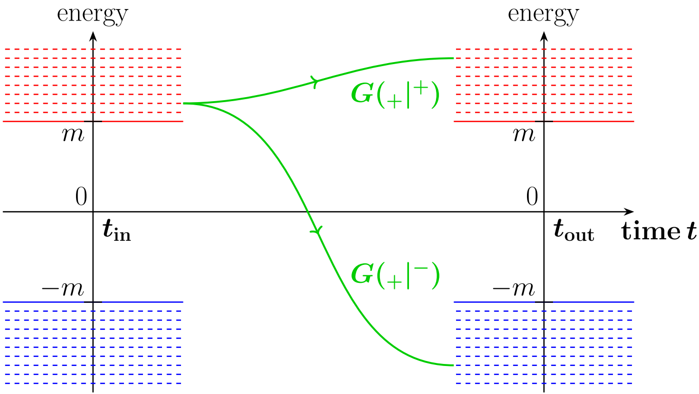
The matrices $G({}_\zeta|{}^\kappa)_{nk} = ({}_\zeta\varphi_n, {}^\kappa\varphi_k)$ and $G({}^\zeta|{}_\kappa)_{nk} = ({}^\zeta\varphi_n, {}_\kappa\varphi_k)$ contain all the information on the particle spectra:

$$n_m^{(e^-)} = \sum_n G({}^+|{}_-)_{mn} G({}_-|{}^+)_{nm} = \{G({}^+|{}_-)G({}_-|{}^+)\}_{mm},$$

$$n_m^{(e^+)} = \sum_n G({}^-|{}_+)_{mn} G({}_+|{}^-)_{nm} = \{G({}^-|{}_+)G({}_+|{}^-)\}_{mm}.$$

E. S. Fradkin, D. M. Gitman, S. M. Shvartsman, *Quantum Electrodynamics with Unstable Vacuum* (Springer-Verlag, Berlin, 1991)

I. Furry-picture quantization



$$n_m^{(e^-)} = \{G(^+|_-)G(_-|^+)\}_{mm} = \sum_k \left|G(_-|^+)_{km}\right|^2$$

I. Furry-picture quantization

The approach (in-out formalism) allows one to treat multidimensional inhomogeneities of the external field!

M. Ruf, G. R. Mocken, C. Müller, K. Z. Hatsagortsyan, C. H. Keitel, Phys. Rev. Lett. **102**, 080402 (2009)

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Let us consider a spatially homogeneous rotating field $\mathbf{E}(t)$ and decompose the field operator in terms of the adiabatic Hamiltonian eigenfunctions:

$$\psi(x) = \sum_s \int d\mathbf{p} \left[a_{\mathbf{p},s}(t) \varphi^{(+)}_{\mathbf{p},s}(\mathbf{x}; t) + b^\dagger_{\mathbf{p},s}(t) \varphi^{(-)}_{\mathbf{p},s}(\mathbf{x}; t) \right].$$

The electron number density reads

$$n^{(e^-)}_{\mathbf{p},s} \equiv \frac{dN_{\mathbf{p},s}}{d\mathbf{p}} = \lim_{t \to t_{\text{out}}} \langle 0, \text{in} | a^\dagger_{\mathbf{p},s}(t) a_{\mathbf{p},s}(t) | 0, \text{in} \rangle.$$

One can obtain a closed-form ODE system governing the one-particle correlation functions!

In terms of 10 yet-unknown functions f, $\mathbf{f}$, $\mathbf{u}$, and $\mathbf{v}$ defined via

$$\langle 0, \text{in}| a^\dagger_{\mathbf{p},s}(t) a_{\mathbf{p}',s'}(t) |0, \text{in}\rangle = \delta(\mathbf{p}-\mathbf{p}')\big[f(\mathbf{p},t)\delta_{s's} + \mathbf{f}(\mathbf{p},t)\boldsymbol{\sigma}_{s's}\big]\,,$$
$$\langle 0, \text{in}| b^\dagger_{\mathbf{p},s}(t) b_{\mathbf{p}',s'}(t) |0, \text{in}\rangle = \delta(\mathbf{p}-\mathbf{p}')\big[f(-\mathbf{p},t)\delta_{ss'} - \mathbf{f}(-\mathbf{p},t)\boldsymbol{\sigma}_{ss'}\big]\,,$$
$$\langle 0, \text{in}| a^\dagger_{\mathbf{p},s}(t) b^\dagger_{\mathbf{p}',s'}(t) |0, \text{in}\rangle = \delta(\mathbf{p}+\mathbf{p}')\big\{ [\mathbf{u}(\mathbf{p},t) - \text{i}\mathbf{v}(\mathbf{p},t)]\boldsymbol{\sigma}_{s's}\big\}\,,$$
$$\langle 0, \text{in}| b_{\mathbf{p},s}(t) a_{\mathbf{p}',s'}(t) |0, \text{in}\rangle = \delta(\mathbf{p}+\mathbf{p}')\big\{ [\mathbf{u}(-\mathbf{p},t) + \text{i}\mathbf{v}(-\mathbf{p},t)]\boldsymbol{\sigma}_{s's}\big\}$$

we find

$$\dot{f} = -2\boldsymbol{\mu}_2\mathbf{u},$$
$$\dot{\mathbf{f}} = 2(\boldsymbol{\mu}_1 \times \mathbf{f}) - 2(\boldsymbol{\mu}_2 \times \mathbf{v}),$$
$$\dot{\mathbf{u}} = 2(\boldsymbol{\mu}_1 \times \mathbf{u}) + \boldsymbol{\mu}_2(2f-1) + 2\omega\mathbf{v},$$
$$\dot{\mathbf{v}} = 2(\boldsymbol{\mu}_1 \times \mathbf{v}) - 2(\boldsymbol{\mu}_2 \times \mathbf{f}) - 2\omega\mathbf{u}.$$

The coefficients $\boldsymbol{\mu}_1$, $\boldsymbol{\mu}_2$, and ω contain the external field $\mathbf{E}(t)$.

I. A. Aleksandrov, A. Kudlis, A. I. Klochai, arXiv:2403.17204

II. Quantum kinetic equations (QKE)

The system [I. A. Aleksandrov, A. Kudlis, A. I. Klochai, arXiv:2403.17204]

$$\dot{f} = -2\boldsymbol{\mu}_2 \mathbf{u},$$
$$\dot{\mathbf{f}} = 2(\boldsymbol{\mu}_1 \times \mathbf{f}) - 2(\boldsymbol{\mu}_2 \times \mathbf{v}),$$
$$\dot{\mathbf{u}} = 2(\boldsymbol{\mu}_1 \times \mathbf{u}) + \boldsymbol{\mu}_2(2f - 1) + 2\omega \mathbf{v},$$
$$\dot{\mathbf{v}} = 2(\boldsymbol{\mu}_1 \times \mathbf{v}) - 2(\boldsymbol{\mu}_2 \times \mathbf{f}) - 2\omega \mathbf{u}$$

differs from its previous version! [V. N. Pervushin, V. V. Skokov, Acta Phys. Polon. B **37**, 2587 (2006)]

At $t = t_{\text{out}}$ the QKE components are equivalent to the G matrices.

$$f_s^{(e^-)}(\mathbf{p}) = \frac{(2\pi)^3}{V}\, n_{\mathbf{p},s}^{(e^-)} = f(\mathbf{p}, t_{\text{out}}) - (\text{sign}\, s)\, f_z(\mathbf{p}, t_{\text{out}})$$

In the case of linear polarization, the two systems coincide ($\mathbf{f} = 0$)!

In the temporal gauge $A_0 = 0$, let us define the following operator:

$$\hat{C}(\mathbf{x}, \mathbf{s}, t) = \exp\left[-\mathrm{i}e \int_{-1/2}^{1/2} \mathbf{A}(t, \mathbf{x} + \lambda \mathbf{s})\mathbf{s}\, d\lambda \right] [\psi(t, \mathbf{x} + \mathbf{s}/2), \overline{\psi}(t, \mathbf{x} - \mathbf{s}/2)].$$

The Wigner operator and Wigner function read

$$\hat{W}(\mathbf{x}, \mathbf{p}, t) = -\frac{1}{2} \int \hat{C}(\mathbf{x}, \mathbf{s}, t)\, \mathrm{e}^{-\mathrm{i}\mathbf{p}\mathbf{s}} d\mathbf{s},$$

$$W(\mathbf{x}, \mathbf{p}, t) = \langle 0, \text{in} | \hat{W}(\mathbf{x}, \mathbf{p}, t) | 0, \text{in} \rangle.$$

Let us decompose the Wigner function in terms of the basis of the Clifford algebra:

$$W = \frac{1}{4}\left[\mathfrak{s}\mathrm{I} + \mathrm{i}\mathfrak{p}\gamma^5 + \mathfrak{v}_\mu \gamma^\mu + \mathfrak{a}_\mu \gamma^\mu \gamma^5 + \mathfrak{t}_i^{(1)} \sigma^{0i} + \frac{1}{2} \varepsilon^{ijk} \mathfrak{t}_k^{(2)} \sigma_{ij} \right].$$

In the case of a spatially uniform field, in terms of $\tilde{\mathfrak{s}}(\mathbf{p},t) = \mathfrak{s}(\mathbf{p} - e\mathbf{A}(t), t)$, one obtains

$$\begin{aligned}
\dot{\tilde{\mathfrak{s}}} &= 2\mathbf{q}\tilde{\mathfrak{t}}, \\
\dot{\tilde{\mathfrak{v}}} &= -2\mathbf{q} \times \tilde{\mathfrak{a}} - 2m\tilde{\mathfrak{t}}, \\
\dot{\tilde{\mathfrak{a}}} &= -2\mathbf{q} \times \tilde{\mathfrak{v}}, \\
\dot{\tilde{\mathfrak{t}}} &= -2\mathbf{q}\tilde{\mathfrak{s}} + 2m\tilde{\mathfrak{v}},
\end{aligned}$$

where $\mathbf{q} = \mathbf{p} - e\mathbf{A}(t)$.

The 10 DHW functions can be connected with the G matrices and also with the 10 QKE components! [I. A. Aleksandrov, A. Kudlis, A. I. Klochai, arXiv:2403.17204]

The explicit relations are given by

$$
\begin{aligned}
\tilde{\mathfrak{s}} - \tilde{\mathfrak{s}}_{\mathbf{A}=\mathbf{0}} &= \frac{4}{q^0}\big[mf - (\mathbf{q}\mathbf{u})\big], \\
\tilde{\mathfrak{v}} - \tilde{\mathfrak{v}}_{\mathbf{A}=\mathbf{0}} &= \frac{4}{q^0}\left[q^0\mathbf{u} + \mathbf{q}f - \frac{\mathbf{q}(\mathbf{q}\mathbf{u})}{q^0+m}\right], \\
\tilde{\mathfrak{a}} &= -\frac{4}{q^0}\left[m\mathbf{f} - (\mathbf{q}\times\mathbf{v}) + \frac{\mathbf{q}(\mathbf{q}\mathbf{f})}{q^0+m}\right], \\
\tilde{\mathfrak{t}} &= -\frac{4}{q^0}\left[m\mathbf{v} - (\mathbf{q}\times\mathbf{f}) + \frac{\mathbf{q}(\mathbf{q}\mathbf{v})}{q^0+m}\right].
\end{aligned}
$$

I. A. Aleksandrov, A. Kudlis, A. I. Klochai, arXiv:2403.17204

Observable quantities can be directly obtained by properly projecting the Wigner function: [I. A. Aleksandrov, A. Kudlis, A. I. Klochai, arXiv:2403.17204]

$$f_s^{(e^-)}(\mathbf{p}) \equiv \frac{(2\pi)^3}{V} n_{\mathbf{p},s}^{(e^-)} = \mathrm{Tr}\left\{\gamma^0 u_{\mathbf{q},s} u_{\mathbf{q},s}^\dagger \left[\tilde{W}(\mathbf{p}, t_{\text{out}}) - \tilde{W}_{\mathbf{A}=\mathbf{0}}(\mathbf{p}, t_{\text{out}})\right]\right\}.$$

cf. [A. Blinne, E. Strobel, Phys. Rev. D **93**, 025014 (2016); L. N. Hu *et al.*, arXiv:2402.16476]

This yields

$$f_s^{(e^-)}(\mathbf{p}) = f(\mathbf{p}, t_{\text{out}}) - (\mathrm{sign}\, s)\, f_z(\mathbf{p}, t_{\text{out}})$$

and coincides with the QKE result.

What is s? It is a mere number of the basis bispinor in the above derivations!

Helicity-resolved spectra

We suggest that the final states possess well-defined helicity:

$$\frac{(\boldsymbol{\Sigma}\mathbf{p})}{\mathbf{p}}\, u_{\mathbf{p}}^{(\mathsf{L/R})} = \mp u_{\mathbf{p}}^{(\mathsf{L/R})}.$$

The unitary transformation $u_{\mathbf{p},s} = \alpha_{\mathbf{p},s}^{(\mathsf{L})}\, u_{\mathbf{p}}^{(\mathsf{L})} + \alpha_{\mathbf{p},s}^{(\mathsf{R})}\, u_{\mathbf{p}}^{(\mathsf{R})}$ leads to

$$f^{(e^-\mathsf{L})}(\mathbf{p}) \equiv \frac{(2\pi)^3}{V}\langle 0, \mathsf{in}|\left[a_{\mathbf{p}}^{(\mathsf{L})}(t_{\mathsf{out}})\right]^\dagger a_{\mathbf{p}}^{(\mathsf{L})}(t_{\mathsf{out}})|0, \mathsf{in}\rangle = f(\mathbf{p}, t_{\mathsf{out}}) - \frac{\mathbf{q}\mathbf{f}(\mathbf{p}, t_{\mathsf{out}})}{|\mathbf{q}|},$$

$$f^{(e^-\mathsf{R})}(\mathbf{p}) \equiv \frac{(2\pi)^3}{V}\langle 0, \mathsf{in}|\left[a_{\mathbf{p}}^{(\mathsf{R})}(t_{\mathsf{out}})\right]^\dagger a_{\mathbf{p}}^{(\mathsf{R})}(t_{\mathsf{out}})|0, \mathsf{in}\rangle = f(\mathbf{p}, t_{\mathsf{out}}) + \frac{\mathbf{q}\mathbf{f}(\mathbf{p}, t_{\mathsf{out}})}{|\mathbf{q}|}.$$

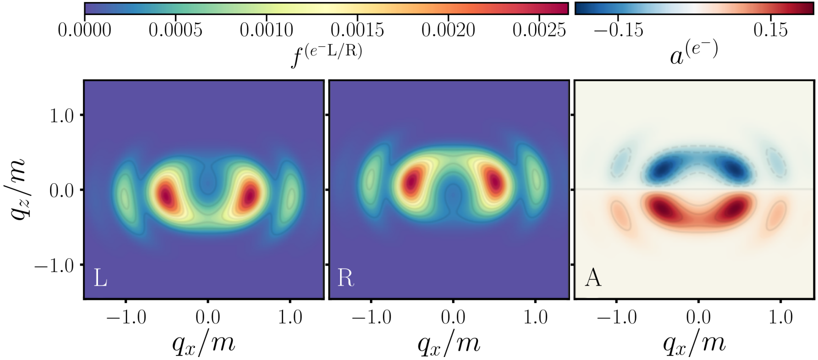
The same can be retrieved via the Wigner-function (basis-independent) approach:

$$f^{(e^-\mathsf{L/R})}(\mathbf{p}) = \mathrm{Tr}\left\{\gamma^0 u_{\mathbf{q}}^{(\mathsf{L/R})}\left[u_{\mathbf{q}}^{(\mathsf{L/R})}\right]^\dagger \left[\tilde{W}(\mathbf{p}, t_{\mathsf{out}}) - \tilde{W}_{\mathbf{A}=\mathbf{0}}(\mathbf{p}, t_{\mathsf{out}})\right]\right\}.$$

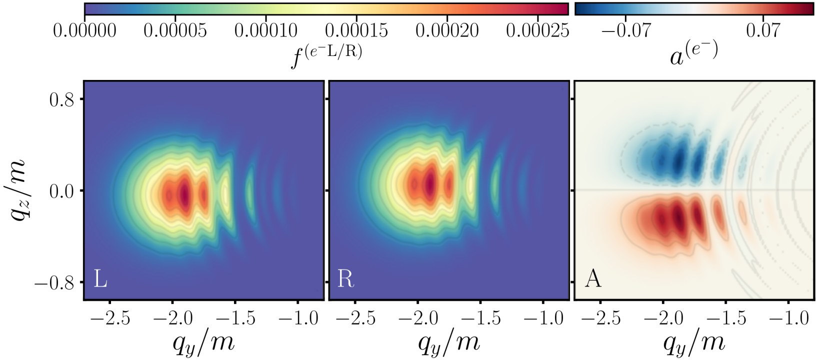
Helicity-resolved spectra

Let us consider a circularly polarized electric pulse:

$$\mathbf{E}(t) = \frac{E_0}{\sqrt{2}} F(\Omega t)\left[\cos(\Omega t)\,\mathbf{e}_x + \sin(\Omega t)\,\mathbf{e}_y\right].$$



Helicity-resolved spectra



I. A. Aleksandrov, A. Kudlis, Phys. Rev. D **110**, L011901 (2024)

Conclusion

What we found:

- an explicit connection between the QKE and DHW techniques for arbitrary polarization of the external field (+ canonical Furry-picture quantization),
- the correct version of the QKEs,
- direct prescriptions for computing spin-resolved particle densities via the QKE and DHW approaches,
- helicity asymmetry in circularly polarized fields.

Other avenues for our research:

- first-order (radiation) processes and second-order effects (e.g., vacuum birefringence),
- analysis of other observables and setups.

Acknowledgments

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