Adler model for spin-3/2 field: some applications

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Outline of the talk

Talk is based on dicussions and papers
made in collaboration with
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1. Adler model and results for
gauge and gravitational anomalies (briefly).

2. Perturbative calculations in hydrodynamics and
gauge anomaly

3. Perturbative calculations in hydrodynamics and
gravitational anomaly.

I Adler model

Spin 3/2 left-handed Rarita-Schwinger field ψ_μ
interacting with spin 1/2 left-handed field λ

$$S = \int d^4x \Big(-\epsilon^{\lambda\rho\mu\nu} \bar{\psi}_\lambda \gamma_5 \gamma_\mu \partial_\nu \psi_\rho + i\bar{\lambda}\gamma^\mu \partial_\mu \lambda - im\bar{\lambda}\gamma^\mu \psi_\mu + im\bar{\psi}_\mu \gamma^\mu \lambda \Big)$$

the central point is the non-diagonal Majorana mass which tends to infinity, $m \to \infty$ and removes spin -1/2 fields $(\gamma^\mu \psi_\mu, \lambda)$ from the physical spectrum

Adler model. Basic idea

Bremsstrahlung:
spin-1/2, Left-Left transition

$$\sim \frac{\theta^2 d\theta^2}{(\theta^2+m^2/E^2)^2} \sim \ln(E/m)$$

spin-1/2 Left-Right transition

$$\sim \frac{(m^2/E^2)d\theta^2}{(\theta^2+(m^2/E^2))^2} \sim const$$

Rarita-Schwinger field

Left-Left transition :

$$\sim \frac{d\theta^2}{(\theta^2+(m^2/E^2))^2} \sim IR \quad \text{quadratic divergence}$$

Removing chirality-1/2 part removes this divergence

Anomalies in Adler model

$$\partial_\mu J^\mu_{A,Adler} = -\frac{5}{16\pi^2}\epsilon^{mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}$$

$$5/2 = 3/2 + 2(1/2)$$

Gauge anomaly is ptoportional to spin of constituents

$$\nabla_\mu J^\mu_{A,Adler} = \frac{-19}{384\pi^2\sqrt{-g}}\epsilon^{\alpha\beta\gamma\delta}R_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\gamma\delta}$$

$$19/4 = (3/2) - 2(3/2)^3 - +2(1/2) - 4(1/2)^3$$

Gravitational anomaly is proportional to
$S - 2S^3$, where S is spin of constituents (Duff, 1988)

Summary I

Calculation of anomalies for the Adler model is self-consistent, and we used it in applcations, along with spin-$1/2$.

II Quantum hydrodynamics

Son+Surowka paper (2009) on anomalous transport
Input: conservation laws, including anomalous current, $\Delta(entropy) \geq 0$

$$\partial_\alpha J^\alpha = e^2 C_{anomaly} F\tilde{F} \qquad (1)$$

Output: a few chiral effects including chiral vortical effect:

$$J^\alpha_{CVE} = \mu^2 C_{anomaly} \Omega^\alpha \qquad (2)$$

where $\Omega^\alpha = (1/2)\epsilon^{\alpha\beta\gamma\delta} u_\beta \partial_\gamma u_\delta$, u_β is fluid's velocity

Note that CVE survives in absence of external e-m fields, i.e. in absence of anomaly

Density Operator

In quantum statistics matrix elements are averaged with density operator $\hat{\rho}$

$$\hat{\rho} \; = \; \exp\left(-\hat{H}_{eff}/T\right)$$

where $\hat{H}_{eff}$ is built on conserved quantities: charges $\hat{Q}_i$, angular momentum $\hat{\vec{J}}$ (Landau-Lifshitz) + boost $\hat{\vec{K}}$ (F. Becattini (2017))

$$\hat{H}_{eff} \; = \; \hat{H}_0 \; - \; \Sigma_i \mu_i \hat{Q}_i - \vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\vec{\Omega}$ angular velocity, $\vec{a}$ is acceleration
$\hat{H}_{eff}$ picks up maximum entropy state
while in case of $\hat{H}_0$ we look for for minimum of energy

Adaptation to hydrodynamics

In the standard form:

$$\delta\hat{H} = -\mu_V \hat{Q}_V \ - \mu_A \hat{Q}_A$$

In localized form (Sadofyev, Shevchenko,VZ (2011))

$$\delta\hat{H} \quad \rightarrow \quad \delta L(x) \ = \text{`} \ \mu_V u_\alpha j_V^\alpha(x) + \mu_A j_A^\alpha(x)$$

$$eA_\alpha^V \ \rightarrow \ eA_\alpha^V + \mu_V u_\alpha; \qquad g_A A_\alpha^A \ \rightarrow \ g_A A_\alpha + \ \mu_A u_\alpha$$

where 4-velocity u_α is treated as an external field.

All chiral effects are immediately generated through this substitution, without considering fluid dynamics further

Summary II

Finding solutions to hydrodynamic equations is greatly simplified by using substitution $eA_\alpha \to \mu u_\alpha$. But in gravitational case there is no such a simple recipe.

III Gravitational anomaly. Motivations

Gravity effects certainly unobservable. Then Why ?
Quark-gluon plasma, produced in heavy-ion collisions is strongly de-accelerated immediately after the collision.
Estimates (D.Kharzeev (2005)) tell us that temperature of the plasma is around Unruh temperature

$$T_{Unruh} \sim \frac{a}{2\pi}$$

Can we learn smth about matter in gravitational field?
Analogy: Chiral vortical effect $\vec{J}_A = C_{anomaly}\mu^2\vec{\Omega}$ does not depend on strength of e-m interaction but has e-m analogy

Thus, observing effect of acceleration could tell us about gravitational interactions. "kinematical gravity"

Duality of statistical and gravitational approaches

Properties of fluids in equilibrium are evaluated statistically in terms of density operator, or effective interaction:

$$\delta \hat{H}_{eff} = -\vec{\Omega}\hat{\vec{J}} - \vec{a}\hat{\vec{K}}$$

where $\hat{\vec{K}}, \hat{\vec{J}}$ are operators of boost and angular momentum and $\vec{a}$, $\vec{\Omega}$ are acceleration and angular velocity.

Cont'd

In field theory, gravitational interaction is described by fundamental interaction Lagrangian:

$$\delta L = -\frac{1}{2}\theta^{\alpha\beta} h_{\alpha\beta}$$

where $\theta^{\alpha\beta}$ is the energy-momentum tensor of matter, $h_{\alpha\beta}$ is the gravitational potential, also accommodating $\vec{\Omega}_{grav}, \vec{a}_{grav}$.

Cont'd

Furthermore, one evaluates "external probes", $< \theta^{\alpha\beta} >, < J_5^\alpha >$ within both approaches, statistical and gravitational. Results compared for the same values of a, Ω. Expect the same results

The duality is confirmed on a number of examples. Our first example ((2019) Dubna) is for energy density of gas of massless spin-1/2 particles

$$(T_0^0)_{s=1/2} = \frac{7\pi^2 T^4}{60} + \frac{T^2 a^2}{24} - \frac{17a^4}{960\pi^2}$$

both statistically, in non-inertial frame, and, geometrically, on Rindler space with a conical singularity.

Duality, cont'd

The duality holds in the limit:

$$G_{Newton} \to 0, \; G_N \cdot M_{source} \to const \; or \; (M_{source} \to infty)$$

that is, in the quasiclassical limit for gravity.

There are well known bulk-boundary dualities in this limit. Our example is a kind of a simplest case in this series.

Duality for gravitational anomaly

For non-inerial frame: $\vec{J}_{A,KVE} = \left(\lambda_1 \Omega^2 + \lambda a^2\right)\vec{\Omega}$

In external gravitational field: $\nabla_\mu J^\mu_A = NR\tilde{R}$

Independent calculations of 1-loop graphs in two theories

For spin-1/2:

$$\lambda_1 - \lambda_2 = -\frac{1}{24\pi^2} + \frac{1}{8\pi^2} = \frac{32}{384\pi^2}$$

For Adler spin-3/2:

$$\lambda_1 - \lambda_2 = -\frac{53}{24\pi^2} + \frac{5}{8\pi^2} = -\frac{32 \cdot 19}{384\pi^2}$$

i.e. full agreement

Summary III

So far, no inconsistencies. Interpretation of the effects might be difficult.