Relativistic transformations for thermodynamic quantities

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Motivation: Ambiguities in the formulation of relativistic thermodynamics

- Planck, Einstein, von Laue and others
- No unique formulation for relativistic transformations of thermodynamic quantities exist

Thermodynamic system *A* is at rest in the moving reference frame K_0. Reference frame K_0 is moving with the speed **V** with respect to the laboratory reference frame *K*

Principle of entropy invariance: $\frac{\delta Q}{T} = dS = dS_0$ 4-invariant

D. Ter Haar, H. Wergeland, Phys. Rep. 1 (1971) 31
C. Farias et al., Scient. Rep. 7 (2017) 17657

1. *Planck's transformations:* $\delta Q = \delta Q_0 \sqrt{1-\mathbf{v}^2}, \quad T = T_0 \sqrt{1-\mathbf{v}^2}$

M. Planck, Sitzungsberichte der Königlich Preussischen, Akademie der Wissenschaften, 1907, pp. 542–570; Ann. Phys. 26 (1908) 1
A. Einstein, Jahrb.Radioakt.Elektron. 4 (1907) 411

2. *Ott's transformations:* $\delta Q = \frac{\delta Q_0}{\sqrt{1-\mathbf{v}^2}}, \quad T = \frac{T_0}{\sqrt{1-\mathbf{v}^2}}$

H. Ott, Z. Phys. 175 (1963) 70

3. *Landsberg transformations:* $T = T_0$ 4-invariant

P.T. Landsberg, Nature 212 (1966) 571

4. *Other transformations:* No general Lorentz transformation for temperature exist

Relativistically moving thermodynamic system

The reference frame K_0 moves relative to the laboratory frame K with constant velocity **v**.

Lorentz transformations for 4-momentum:

$$E_0 = \gamma(E - \mathbf{vP}), \quad \mathbf{P}_0 = \gamma\left(\mathbf{v}\frac{(\mathbf{vP})}{v^2} - \mathbf{v}E\right) + \mathbf{P} - \mathbf{v}\frac{(\mathbf{vP})}{v^2}$$

V.A. Fok (1961)

$$\gamma = (1 - \mathbf{v}^2)^{-1/2}, \quad v = |\mathbf{v}|, \quad \mathbf{v} = (v_x, v_y, v_z)$$

Particular case $\mathbf{v} = (v, 0, 0)$: $E_0 = \gamma\left(E - vP_x\right), \quad P_{0x} = \gamma\left(P_x - vE\right), \quad \mathbf{P}_{0\perp} = \mathbf{P}_\perp,$

Moving reference frame K0

- Thermodynamic system is at rest in K0 :

$$P_0^\mu = (E_0, \mathbf{P}_0), \quad E_0 = M, \quad \mathbf{P}_0 = 0$$

- E_0 is a fundamental thermodynamic potential:

$$E_0 = E_0(S_0, V_0, N_0),$$

$$dE_0 = T_0 dS_0 - p_0 dV_0 + \mu_0 dN_0,$$

$$T_0 \equiv \left(\frac{\partial E_0}{\partial S_0}\right)_{V_0 N_0}, \quad p_0 \equiv -\left(\frac{\partial E_0}{\partial V_0}\right)_{S_0 N_0}, \quad \mu_0 \equiv \left(\frac{\partial E_0}{\partial N_0}\right)_{S_0 V_0}$$

Laboratory reference frame K

- Energy, momentum, velocity:

$$E = \gamma E_0,$$

$$\mathbf{P} = \mathbf{v}E = \gamma E_0 \mathbf{v},$$

$$\mathbf{v} = \frac{\mathbf{P}}{E}$$

- Entropy, volume, number of particles:

$$S = S_0, \quad V = \frac{V_0}{\gamma}, \quad N = N_0$$

(based on A.S.P., Ann. Phys. 401 (2019) 130; A.S.P., arXiv:2406.12029 v1 [hep-ph])

Class I: Hamiltonian $E(\mathbf{P})$ as the fundamental thermodynamic potential

A. Physical independent state variables $(S_0, V_0, N_0, \mathbf{P})$.

Momentum $\mathbf{P}$ as an independent state variable.

Lorentz factor:

$$\gamma(S_0,V_0,N_0,\mathbf{P}) = \sqrt{1+\frac{\mathbf{P}^2}{E_0^2(S_0,V_0,N_0)}}$$

Energy - Fundamental thermodynamic potential:

$$E(S_0,V_0,N_0,\mathbf{P}) = \gamma E_0 = \sqrt{\mathbf{P}^2 + E_0^2(S_0,V_0,N_0)}$$

Velocity v-dependent variable, P-independent:

$$\mathbf{v}(S_0,V_0,N_0,\mathbf{P}) = \frac{\mathbf{P}}{E} = \frac{\mathbf{P}}{\sqrt{\mathbf{P}^2 + E_0^2(S_0,V_0,N_0)}}$$

Volume V-dependent variable, V_0 -independent:

$$V(S_0,V_0,N_0,\mathbf{P}) = \frac{V_0}{\gamma} = \frac{V_0}{\sqrt{1+\frac{\mathbf{P}^2}{E_0^2(S_0,V_0,N_0)}}}$$

- In the variables $(S_0,V_0,N_0,\mathbf{P})$, the relativistic transformations for physical variables of class I are consistent.
- We have obtained a new formula for pressure.

Differential of the Hamilton function:

$$dE(S_0,V_0,N_0,\mathbf{P}) = \frac{T_0}{\gamma}dS_0 - \frac{p_0}{\gamma}dV_0 + \frac{\mu_0}{\gamma}dN_0 + \mathbf{v}d\mathbf{P}$$
$$= T_I dS_0 - p_I dV_0 + \mu_I dN_0 + \mathbf{v}_I d\mathbf{P},$$

$$\left(\frac{\partial E}{\partial S_0}\right)_{V_0N_0\mathbf{P}} \equiv T_I, \qquad \left(\frac{\partial E}{\partial V_0}\right)_{S_0N_0\mathbf{P}} \equiv -p_I,$$

$$\left(\frac{\partial E}{\partial N_0}\right)_{S_0V_0\mathbf{P}} \equiv \mu_I, \qquad \left(\frac{\partial E}{\partial \mathbf{P}}\right)_{S_0V_0N_0} \equiv \mathbf{v}_I = \mathbf{v}$$

Lorentz transformation for differential of the Hamilton function:

$$dE_0 = \gamma\left(dE - \mathbf{v}d\mathbf{P}\right)$$

Relativistic transformations for physical thermodynamic quantities of class I (Planck's transformations for T and mu):

$$T_I = \frac{T_0}{\gamma}, \quad p_I = \frac{p_0}{\gamma}, \quad \mu_I = \frac{\mu_0}{\gamma}, \quad \mathbf{v}_I = \mathbf{v}$$

Class I: Hamiltonian $E(\mathbf{P})$ as the fundamental thermodynamic potential

B. Unphysical independent state variables $(S,V,N,\mathbf{P})$.

Momentum $\mathbf{P}$ as an independent state variable.

Volume V-independent variable, V_0 -dependent:

$$V=\frac{V_0}{\sqrt{1+\frac{\mathbf{P}^2}{E_0^2(S,V_0,N)}}},\quad \boxed{V_0=V_0(S,V,N,\mathbf{P})}\ \text{-solution}$$

Rest energy (unphysical - momentum dependent):

$$\boxed{E_0(S,V,N,\mathbf{P})=E_0(S,V_0(S,V,N,\mathbf{P}),N)}$$

Energy - Fundamental thermodynamic potential:

$$E(S,V,N,\mathbf{P})=\sqrt{\mathbf{P}^2+E_0^2(S,V_0(S,V,N,\mathbf{P}),N)}$$

Velocity v-dependent variable, P-independent:

$$\mathbf{v}(S,V,N,\mathbf{P})=\frac{\mathbf{P}}{\sqrt{\mathbf{P}^2+E_0^2(S,V_0(S,V,N,\mathbf{P}),N)}}$$

Lorentz factor:

$$\gamma(S,V,N,\mathbf{P})=\sqrt{1+\frac{\mathbf{P}^2}{E_0^2(S,V_0(S,V,N,\mathbf{P}),N)}}$$

- In the variables (S,V,N,P), the rest energy E_0 is not invariant.
- Thus, the relativistic transformations for unphysical variables of class I are inconsistent.

Differential of the Hamilton function:

$$dE(S,V,N,\mathbf{P})=\tilde{T}_I dS-\tilde{p}_I dV+\tilde{\mu}_I dN+\mathbf{v}_I d\mathbf{P},$$

$$\left(\frac{\partial E}{\partial S}\right)_{VN\mathbf{P}}\equiv\tilde{T}_I,\quad \left(\frac{\partial E}{\partial V}\right)_{SN\mathbf{P}}\equiv-\tilde{p}_I,$$

$$\left(\frac{\partial E}{\partial N}\right)_{SV\mathbf{P}}\equiv\tilde{\mu}_I,\quad \left(\frac{\partial E}{\partial \mathbf{P}}\right)_{SVN}\equiv\mathbf{v}_I$$

Differential of the rest volume:

$$dV_0(S,V,N,\mathbf{P})=-\alpha\frac{\mathbf{P}^2}{E^2}\frac{V_0T_0}{E_0}dS-\alpha\frac{\mathbf{P}^2}{E^2}\frac{V_0\mu_0}{E_0}dN+\alpha\gamma dV+\frac{\alpha}{\gamma}\frac{V_0}{E_0}\mathbf{v}d\mathbf{P}$$

$$\alpha\equiv\frac{1}{1-\frac{\mathbf{P}^2}{E^2}\frac{V_0p_0}{E_0}}=\frac{1}{1-\mathbf{v}^2\frac{V_0p_0}{E_0}}$$

Relativistic transformations for unphysical thermodynamic quantities of class I:

$$\boxed{\tilde{T}_I=\frac{\alpha T_0}{\gamma},\ \tilde{p}_I=\alpha p_0,\ \tilde{\mu}_I=\frac{\alpha\mu_0}{\gamma},\ \mathbf{v}_I=\alpha\left(1-\frac{V_0p_0}{E_0}\right)\mathbf{v}}$$

Class II: Lagrangian $L(\mathbf{P})$ as the fundamental thermodynamic potential

A. Physical independent state variables $(S_0, V_0, N_0, \mathbf{P})$.

Momentum $\mathbf{P}$ as an independent state variable

Lagrangian - Fundamental thermodynamic potential:

$$L(S_0,V_0,N_0,\mathbf{P}) = -\frac{E_0}{\gamma} = -\frac{E_0(S_0,V_0,N_0)}{\sqrt{1+\dfrac{\mathbf{P}^2}{E_0^2(S_0,V_0,N_0)}}}$$

Velocity v-dependent variable, P-independent:

$$\mathbf{v}(S_0,V_0,N_0,\mathbf{P}) = \frac{\mathbf{P}}{E} = \frac{\mathbf{P}}{\sqrt{\mathbf{P}^2 + E_0^2(S_0,V_0,N_0)}}$$

Volume V-dependent variable, V_0 -independent:

$$V(S_0,V_0,N_0,\mathbf{P}) = \frac{V_0}{\gamma} = \frac{V_0}{\sqrt{1+\dfrac{\mathbf{P}^2}{E_0^2(S_0,V_0,N_0)}}}$$

- The momentum P as an independent state variable in Lagrangian contradicts the requirements of the Lagrangian mechanics.
- Thus, the relativistic transformations for physical variables of class II are forbidden.

Differential of the Lagrange function:

$$dL(S_0,V_0,N_0,\mathbf{P}) = -\frac{\theta T_0}{\gamma}dS_0 + \frac{\theta p_0}{\gamma}dV_0 - \frac{\theta\mu_0}{\gamma}dN_0 + \frac{\mathbf{v}}{\gamma^2}d\mathbf{P}$$

$$= -T_{II}dS_0 + p_{II}dV_0 - \mu_{II}dN_0 + \mathbf{v}_{II}d\mathbf{P},$$

$$\theta \equiv 1 + \frac{\mathbf{P}^2}{E^2} = 1 + \mathbf{v}^2,$$

$$\left(\frac{\partial L}{\partial S_0}\right)_{V_0N_0\mathbf{P}} \equiv -T_{II}, \qquad \left(\frac{\partial L}{\partial V_0}\right)_{S_0N_0\mathbf{P}} \equiv p_{II},$$

$$\left(\frac{\partial L}{\partial N_0}\right)_{S_0V_0\mathbf{P}} \equiv -\mu_{II}, \qquad \left(\frac{\partial L}{\partial \mathbf{P}}\right)_{S_0V_0N_0} = \mathbf{v}_{II}$$

Relativistic transformations for physical thermodynamic quantities of class II:

$$T_{II} = \frac{\theta T_0}{\gamma}, \quad p_{II} = \frac{\theta p_0}{\gamma}, \quad \mu_{II} = \frac{\theta\mu_0}{\gamma}, \quad \mathbf{v}_{II} = \frac{\mathbf{v}}{\gamma^2}$$

$$T_{II} = \theta T_I, \qquad p_{II} = \theta p_I, \qquad \mu_{II} = \theta\mu_I$$

Class II: Lagrangian $L(\mathbf{P})$ as the fundamental thermodynamic potential

B. Unphysical independent state variables $(S,V,N,\mathbf{P})$.

Momentum $\mathbf{P}$ as an independent state variable

Volume V-independent variable, V_0 -dependent:

$$V = \frac{V_0}{\sqrt{1+\dfrac{\mathbf{P}^2}{E_0^2(S,V_0,N)}}}, \qquad V_0 = V_0(S,V,N,\mathbf{P}) \text{ -solution}$$

Rest energy (unphysical - momentum dependent):

$$E_0(S,V,N,\mathbf{P}) = E_0(S,V_0(S,V,N,\mathbf{P}),N)$$

Lagrangian - Fundamental thermodynamic potential:

$$L(S,V,N,\mathbf{P}) = -\frac{E_0(S,V_0(S,V,N,\mathbf{P}),N)}{\sqrt{1+\dfrac{\mathbf{P}^2}{E_0^2(S,V_0(S,V,N,\mathbf{P}),N)}}}$$

Velocity v-dependent variable, P-independent:

$$\mathbf{v}(S,V,N,\mathbf{P}) = \frac{\mathbf{P}}{\sqrt{\mathbf{P}^2 + E_0^2(S,V_0(S,V,N,\mathbf{P}),N)}}$$

- The momentum P as an independent state variable in the Lagrangian contradicts the requirements of the Lagrangian mechanics.
- In the variables (S,V,N,P), the rest energy E_0 is not invariant.
- Thus, the relativistic transformations for unphysical variables of class II are forbidden and inconsistent.

Differential of the Lagrange function:

$$dL(S,V,N,\mathbf{P}) = -\tilde{T}_{II}dS + \tilde{p}_{II}dV - \tilde{\mu}_{II}dN + \mathbf{v}_{II}d\mathbf{P},$$

$$\left(\frac{\partial L}{\partial S}\right)_{VN\mathbf{P}} \equiv -\tilde{T}_{II}, \quad \left(\frac{\partial L}{\partial V}\right)_{SN\mathbf{P}} \equiv \tilde{p}_{II},$$

$$\left(\frac{\partial L}{\partial N}\right)_{SV\mathbf{P}} \equiv -\tilde{\mu}_{II}, \quad \left(\frac{\partial L}{\partial \mathbf{P}}\right)_{SVN} \equiv \mathbf{v}_{II}$$

Differential of the rest volume:

$$dV_0(S,V,N,\mathbf{P}) = -\alpha\frac{\mathbf{P}^2}{E^2}\frac{V_0T_0}{E_0}dS - \alpha\frac{\mathbf{P}^2}{E^2}\frac{V_0\mu_0}{E_0}dN + \alpha\gamma dV + \frac{\alpha}{\gamma}\frac{V_0}{E_0}\mathbf{v}d\mathbf{P}$$

Relativistic transformations for unphysical thermodynamic quantities of class II:

$$\tilde{T}_{II} = \frac{\alpha\theta T_0}{\gamma}, \ \tilde{p}_{II} = \alpha\theta p_0, \ \tilde{\mu}_{II} = \frac{\alpha\theta\mu_0}{\gamma}, \ \mathbf{v}_{II} = (1+\frac{V_0p_0}{E_0})\frac{\alpha\mathbf{v}}{\gamma^2}$$

$$\tilde{T}_{II} = \theta\tilde{T}_I, \qquad \tilde{p}_{II} = \theta\tilde{p}_I, \qquad \tilde{\mu}_{II} = \theta\tilde{\mu}_I$$

Class III: Lagrangian $L(\mathbf{v})$ as the fundamental thermodynamic potential

A. Physical independent state variables $(S_0, V_0, N_0, \mathbf{v})$.

Velocity $\mathbf{v}$ as an independent state variable.

Lorentz factor:

$$\gamma(\mathbf{v}) = \frac{1}{\sqrt{1-\mathbf{v}^2}}$$

Lagrangian - Fundamental thermodynamic potential:

$$L(S_0,V_0,N_0,\mathbf{v}) = -\frac{E_0}{\gamma} = -E_0(S_0,V_0,N_0)\sqrt{1-\mathbf{v}^2}$$

Momentum P-dependent variable, v-independent:

$$\mathbf{P}(S_0,V_0,N_0,\mathbf{v}) = \mathbf{v}E = \mathbf{v}\frac{E_0(S_0,V_0,N_0)}{\sqrt{1-\mathbf{v}^2}}$$

Volume V-dependent variable, V_0 -independent:

$$V(V_0,\mathbf{v}) = \frac{V_0}{\gamma} = V_0\sqrt{1-\mathbf{v}^2}$$

- In the variables $(S_0,V_0,N_0,\mathbf{v})$, the relativistic transformations for physical variables of class III are consistent and equivalent to the relativistic transformations for physical variables of class I .
- We have obtained a new formula for pressure.

Differential of the Lagrangian function:

$$dL(S_0,V_0,N_0,\mathbf{v}) = -\frac{T_0}{\gamma}dS_0 + \frac{p_0}{\gamma}dV_0 - \frac{\mu_0}{\gamma}dN_0 + \mathbf{P}d\mathbf{v}$$
$$= -T_{III}dS_0 + p_{III}dV_0 - \mu_{III}dN_0 + \mathbf{P}_{III}d\mathbf{v},$$

$$\left(\frac{\partial L}{\partial S_0}\right)_{V_0N_0\mathbf{v}} \equiv -T_{III}, \qquad \left(\frac{\partial L}{\partial V_0}\right)_{S_0N_0\mathbf{v}} \equiv p_{III},$$

$$\left(\frac{\partial L}{\partial N_0}\right)_{S_0V_0\mathbf{v}} \equiv -\mu_{III}, \qquad \left(\frac{\partial L}{\partial \mathbf{v}}\right)_{S_0V_0N_0} \equiv \mathbf{P}_{III} = \mathbf{P}$$

Relativistic transformations for physical thermodynamic quantities of class III (Planck's transformations for T and mu):

$$T_{III} = \frac{T_0}{\gamma}, \quad p_{III} = \frac{p_0}{\gamma}, \quad \mu_{III} = \frac{\mu_0}{\gamma}, \quad \mathbf{P}_{III} = \mathbf{P}$$

$$T_I(S_0,V_0,N_0,\mathbf{P}) = T_{III}(S_0,V_0,N_0,\mathbf{v}(S_0,V_0,N_0,\mathbf{P})), \quad \text{etc.}$$

Class III: Lagrangian $L(\mathbf{v})$ as the fundamental thermodynamic potential

B. Unphysical independent state variables $(S,V,N,\mathbf{v})$.

Velocity $\mathbf{v}$ as an independent state variable.

Volume V-independent variable, V_0 -dependent:

$$V_0(V,\mathbf{v}) = \frac{V}{\sqrt{1-\mathbf{v}^2}}$$

Rest energy (unphysical - velocity dependent):

$$E_0(S,V,N,\mathbf{v}) = E_0(S,V_0(V,\mathbf{v}),N)$$

Lagrangian - Fundamental thermodynamic potential:

$$L(S,V,N,\mathbf{v}) = -E_0(S,V_0(V,\mathbf{v}),N)\sqrt{1-\mathbf{v}^2}$$

Velocity v-independent variable, P-dependent:

$$\mathbf{P}(S,V,N,\mathbf{v}) = \mathbf{v}\frac{E_0(S,V_0(V,\mathbf{v}),N)}{\sqrt{1-\mathbf{v}^2}}$$

Lorentz factor:

$$\gamma(\mathbf{v}) = \frac{1}{\sqrt{1-\mathbf{v}^2}}$$

- In the variables (S,V,N,v), the rest energy E_0 is not invariant.
- Thus, the relativistic transformations for unphysical variables of class III are inconsistent.

Differential of the Lagrangian function:

$$dL(S,V,N,\mathbf{v}) = -\tilde{T}_{III}dS + \tilde{p}_{III}dV - \tilde{\mu}_{III}dN + \mathbf{P}_{III}d\mathbf{v},$$

$$\left(\frac{\partial L}{\partial S}\right)_{VN\mathbf{v}} \equiv -\tilde{T}_{III}, \quad \left(\frac{\partial L}{\partial V}\right)_{SN\mathbf{v}} \equiv \tilde{p}_{III},$$

$$\left(\frac{\partial L}{\partial N}\right)_{SV\mathbf{v}} \equiv -\tilde{\mu}_{III}, \quad \left(\frac{\partial L}{\partial \mathbf{v}}\right)_{SVN} \equiv \mathbf{P}_{III}$$

Differential of the rest volume:

$$dV_0(V,\mathbf{v}) = \gamma\left(dV + \frac{V_0}{E_0}\mathbf{P}d\mathbf{v}\right)$$

Relativistic transformations for unphysical thermodynamic quantities of class III (Planck's transformations for T, p and mu):

$$\tilde{T}_{III} = \frac{T_0}{\gamma},\ \tilde{p}_{III} = p_0,\ \tilde{\mu}_{III} = \frac{\mu_0}{\gamma},\ \mathbf{P}_{III} = \left(1+\frac{V_0 p_0}{E_0}\right)\mathbf{P}$$

Class IV: Hamiltonian $E(\mathbf{v})$ as the fundamental thermodynamic potential

A. Physical independent state variables $(S_0, V_0, N_0, \mathbf{v})$.

Velocity $\mathbf{v}$ as an independent state variable

Lorentz factor:

$$\gamma(\mathbf{v}) = \frac{1}{\sqrt{1-\mathbf{v}^2}}$$

Hamiltonian - Fundamental thermodynamic potential:

$$E(S_0, V_0, N_0, \mathbf{v}) = \gamma E_0 = \frac{E_0(S_0, V_0, N_0)}{\sqrt{1-\mathbf{v}^2}}$$

Momentum P-dependent variable, v-independent:

$$\mathbf{P}(S_0, V_0, N_0, \mathbf{v}) = \mathbf{v}E = \mathbf{v}\frac{E_0(S_0, V_0, N_0)}{\sqrt{1-\mathbf{v}^2}}$$

Volume V-dependent variable, V_0 -independent:

$$V(V_0, \mathbf{v}) = \frac{V_0}{\gamma} = V_0\sqrt{1-\mathbf{v}^2}$$

- The velocity v as an independent state variable in the Hamiltonian contradicts the requirements of the Hamiltonian mechanics.
- Thus, the relativistic transformations for physical variables of class IV are forbidden.

Differential of the Hamiltonian function:

$$dE(S_0, V_0, N_0, \mathbf{v}) = \gamma T_0 dS_0 - \gamma p_0 dV_0 + \gamma \mu_0 dN_0 + \gamma^2 \mathbf{P} d\mathbf{v}$$
$$= T_{IV} dS_0 - p_{IV} dV_0 + \mu_{IV} dN_0 + \mathbf{P}_{IV} d\mathbf{v},$$

$$\left(\frac{\partial E}{\partial S_0}\right)_{V_0 N_0 \mathbf{v}} \equiv T_{IV}, \qquad \left(\frac{\partial E}{\partial V_0}\right)_{S_0 N_0 \mathbf{v}} \equiv -p_{IV},$$

$$\left(\frac{\partial E}{\partial N_0}\right)_{S_0 V_0 \mathbf{v}} \equiv \mu_{IV}, \qquad \left(\frac{\partial E}{\partial \mathbf{v}}\right)_{S_0 V_0 N_0} \equiv \mathbf{P}_{IV} = \gamma^2 \mathbf{P}$$

Relativistic transformations for physical thermodynamic quantities of class IV (Ott's transformations for T and mu):

$$T_{IV} = \gamma T_0, \quad p_{IV} = \gamma p_0, \quad \mu_{IV} = \gamma \mu_0, \quad \mathbf{P}_{IV} = \gamma^2 \mathbf{P}$$

$$T_{IV} = \gamma^2 T_{III}, \qquad p_{IV} = \gamma^2 p_{III}, \qquad \mu_{IV} = \gamma^2 \mu_{III}$$

Class IV: Hamiltonian $E(\mathbf{v})$ as the fundamental thermodynamic potential

B. Unphysical independent state variables $(S,V,N,\mathbf{v})$.

Velocity $\mathbf{v}$ as an independent state variable

Volume V-independent variable, V_0 -dependent:

$$V_0(V,\mathbf{v}) = \frac{V}{\sqrt{1-\mathbf{v}^2}}$$

Rest energy (unphysical - velocity dependent):

$$E_0(S,V,N,\mathbf{v}) = E_0(S,V_0(V,\mathbf{v}),N)$$

Hamiltonian - Fundamental thermodynamic potential:

$$E(S,V,N,\mathbf{v}) = \frac{E_0(S,V_0(V,\mathbf{v}),N)}{\sqrt{1-\mathbf{v}^2}}$$

Velocity v-independent variable, P-dependent:

$$\mathbf{P}(S,V,N,\mathbf{v}) = \mathbf{v}\frac{E_0(S,V_0(V,\mathbf{v}),N)}{\sqrt{1-\mathbf{v}^2}}$$

Lorentz factor:

$$\gamma(\mathbf{v}) = \frac{1}{\sqrt{1-\mathbf{v}^2}}$$

- The velocity v as an independent state variable in the Hamiltonian contradicts the requirements of the Hamiltonian mechanics.
- In the variables (S,V,N,v), the rest energy E_0 is not invariant.
- Thus, the relativistic transformations for unphysical variables of class IV are forbidden and inconsistent.

Differential of the Hamiltonian function:

$$dE(S,V,N,\mathbf{v}) = \tilde{T}_{IV}dS - \tilde{p}_{IV}dV + \tilde{\mu}_{IV}dN + \mathbf{P}_{IV}d\mathbf{v},$$

$$\left(\frac{\partial E}{\partial S}\right)_{VN\mathbf{v}} \equiv \tilde{T}_{IV}, \qquad \left(\frac{\partial E}{\partial V}\right)_{SN\mathbf{v}} \equiv -\tilde{p}_{IV},$$

$$\left(\frac{\partial E}{\partial N}\right)_{SV\mathbf{v}} \equiv \tilde{\mu}_{IV}, \qquad \left(\frac{\partial E}{\partial \mathbf{v}}\right)_{SVN} \equiv \mathbf{P}_{IV}$$

Differential of the rest volume:

$$dV_0(V,\mathbf{v}) = \gamma\left(dV + \frac{V_0}{E_0}\mathbf{P}d\mathbf{v}\right)$$

Relativistic transformations for unphysical thermodynamic quantities of class IV (Ott's transformations for T, p and mu):

$$\tilde{T}_{IV} = \gamma T_0,\ \tilde{p}_{IV} = \gamma^2 p_0,\ \tilde{\mu}_{IV} = \gamma\mu_0,\ \mathbf{P}_{IV} = \gamma^2\left(1-\frac{V_0 p_0}{E_0}\right)\mathbf{P}$$

$$\tilde{T}_{IV} = \gamma^2\tilde{T}_{III}, \qquad \tilde{p}_{IV} = \gamma^2\tilde{p}_{III}, \qquad \tilde{\mu}_{IV} = \gamma^2\tilde{\mu}_{III}$$

Incomplete formula for the differential of volume

Momentum $\mathbf{P}$ as an independent state variable

Unphysical independent state variables $(S, V, N, \mathbf{P})$.

Velocity $\mathbf{v}$ as an independent state variable

Unphysical independent state variables $(S, V, N, \mathbf{v})$

Lorentz volume contraction

$$V = \frac{V_0}{\gamma}$$

- Differential of the rest volume:

$$dV_0(S,V,N,\mathbf{P}) = -\alpha \frac{\mathbf{P}^2}{E^2}\frac{V_0 T_0}{E_0} dS - \alpha \frac{\mathbf{P}^2}{E^2}\frac{V_0 \mu_0}{E_0} dN + \alpha\gamma dV + \frac{\alpha}{\gamma}\frac{V_0}{E_0}\mathbf{v}d\mathbf{P},$$

$$\alpha \equiv \frac{1}{1 - \frac{\mathbf{P}^2}{E^2}\frac{V_0 p_0}{E_0}} = \frac{1}{1 - \mathbf{v}^2 \frac{V_0 p_0}{E_0}}$$

- Incomplete formula at $T_0 = p_0 = \mu_0 = 0, \quad \mathbf{v} = 0$:

$$\boxed{dV_0 = \gamma dV} \qquad \gamma = 1$$

- Planck's transformations:

$$dE(S,V,N,\mathbf{P}) = \tilde{T}_I dS - \tilde{p}_I dV + \tilde{\mu}_I dN + \mathbf{v}_I d\mathbf{P},$$

$$\boxed{\tilde{T}_I = \frac{T_0}{\gamma}, \quad \tilde{p}_I = p_0, \quad \tilde{\mu}_I = \frac{\mu_0}{\gamma}, \quad \mathbf{v}_I = \mathbf{v}}$$

- Differential of the rest volume:

$$dV_0(V,\mathbf{v}) = \gamma\left(dV + \frac{V_0}{E_0}\mathbf{P}d\mathbf{v}\right)$$

- Incomplete formula at $\mathbf{P} = 0$:

$$\boxed{dV_0 = \gamma dV} \qquad \gamma = 1$$

- Ott's transformations:

$$dE(S,V,N,\mathbf{v}) = \tilde{T}_{IV} dS - \tilde{p}_{IV} dV + \tilde{\mu}_{IV} dN + \mathbf{P}_{IV} d\mathbf{v},$$

$$\boxed{\tilde{T}_{IV} = \gamma T_0, \; \tilde{p}_{IV} = \gamma^2 p_0, \; \tilde{\mu}_{IV} = \gamma\mu_0, \; \mathbf{P}_{IV} = \gamma^2 \mathbf{P}}$$

Applications: The moving system of quark-gluon plasma

Ultrarelativistic ideal gas of quarks and gluons, $N_f = 2+1$ flavors (Stefan–Boltzmann limit)

Grand canonical ensemble (T_0, V_0, μ_0): $\mu_0 = 0$

$$\Omega_0 = -\frac{\pi^2 g V_0}{90} T_0^4, \quad E_0 = \frac{\pi^2 g V_0}{30} T_0^4 = -3\Omega_0, \quad S_0 = \frac{2\pi^2 g V_0}{45} T_0^3, \quad p_0 = \frac{\pi^2 g}{90} T_0^4 = -\frac{\Omega_0}{V_0} = \frac{E_0}{3V_0}$$

$$g = g_g + \frac{7}{8} g_q, \quad g_g = 2_{spin} \times (N_c^2 - 1), \quad g_q = 2_{spin} \times 2_{q\bar{q}} \times N_c \times N_f$$ - effective degeneracy factor

Fundamental ensemble (S_0, V_0, N_0):

$$E_0 = \Omega_0 + T_0 S_0 + \mu_0 N_0$$ -Legendre transform

$$E_0 = \frac{3S_0}{4}\left(\frac{aS_0}{V_0}\right)^{1/3} = \frac{3S_0 T_0}{4}, \quad p_0 = \frac{S_0}{4V_0}\left(\frac{aS_0}{V_0}\right)^{1/3} = \frac{S_0 T_0}{4V_0}, \quad T_0 = \left(\frac{aS_0}{V_0}\right)^{1/3}, \quad \mu_0 = 0 \qquad a = \frac{45}{2\pi^2 g}$$

Fundamental ensemble $(S, V, N, \mathbf{v})$:

$$S = S_0, \quad N = N_0$$ - 4-invariants

$$V_0 = \gamma V = \frac{V}{\sqrt{1-\mathbf{v}^2}}$$ - rest volume

$$E_0 = \frac{3S}{4}\left(\frac{aS}{V}\right)^{1/3}\left(1-\mathbf{v}^2\right)^{1/6}, \quad p_0 = \frac{S}{4V}\left(\frac{aS}{V}\right)^{1/3}\left(1-\mathbf{v}^2\right)^{2/3}, \quad T_0 = \left(\frac{aS}{V}\right)^{1/3}\left(1-\mathbf{v}^2\right)^{1/6}, \quad \mu_0 = 0$$

- The thermodynamic quantities of the system at rest in the moving reference frame K0 depend on the velocity v of the system in the laboratory reference frame K at fixed values of the independent variables of state (S,V,N) in the frame K.
- The rest energy E0 is not invariant. It depends on the velocity of the reference frame K0.
- Thus, the state variables $(S, V, N, \mathbf{v})$ are unphysical. If the rest energy E0 is a 4-invariant, then the state variables $(S, V, N, \mathbf{v})$ are not independent.

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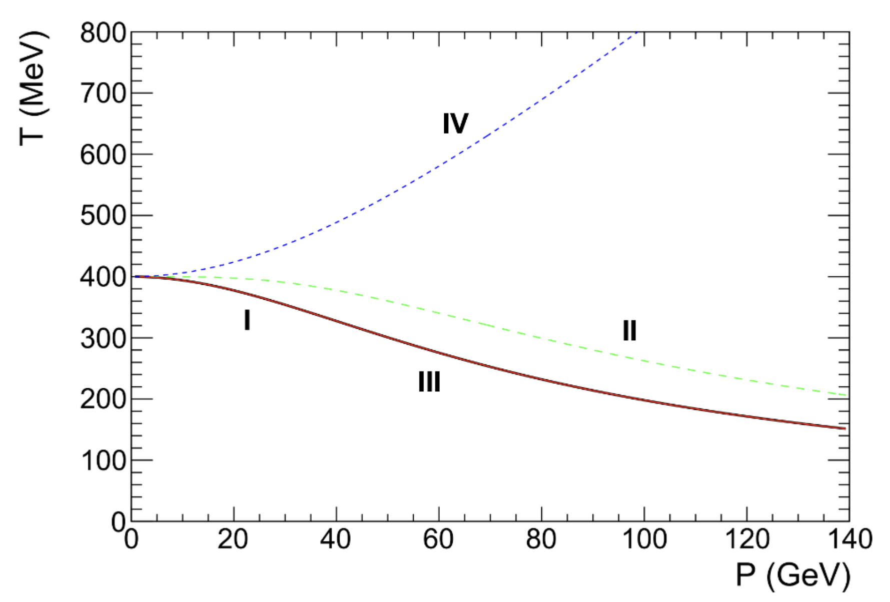
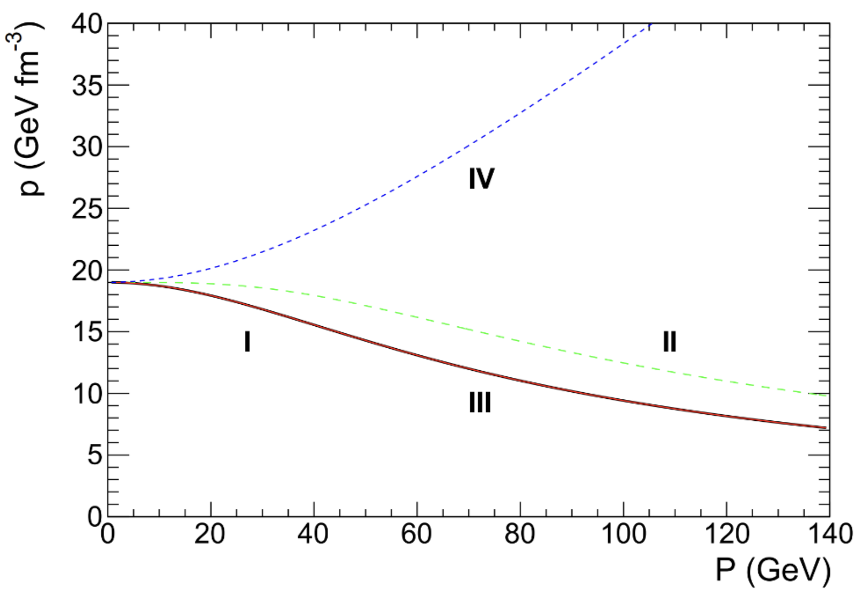
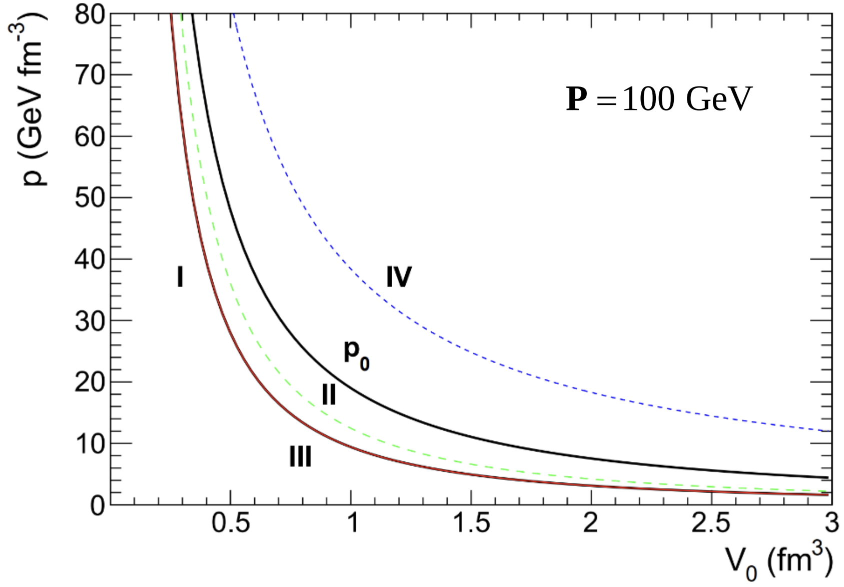




Ultrarelativistic ideal gas of quarks and gluons, $N_f = 2+1$ flavors

Physical state variables $(S_0, V_0, N_0, \mathbf{P})$ (Stefan–Boltzmann limit)



$V_0 = 1\ \text{fm}^3,\quad S_0 = 189.995,\quad \mu_0 = 0$

$E_0 = 56.998$ GeV,

$T_0 = 400$ MeV,

$p_0 = 18.999\ \text{GeV/fm}^3$

Conclusions

1. It was shown that eight different sets of relativistic transformations for thermodynamic quantities can be determined in the framework of the fundamental ensemble.

2. It was found that only two sets of relativistic transformations for the physical thermodynamic quantities of classes I and III are consistent with the requirements of the special theory of relativity and the Hamiltonian and Lagrangian mechanics. These relativistic transformations are equivalent and correspond exactly to the Planck transformations for physical temperature and physical chemical potential.

3. We have found a new formula for physical pressure of classes I and III.

4. Other six sets of relativistic transformations of thermodynamic quantities are either unphysical due to the dependence of rest energy of the thermodynamic system on 3-velocity and 3-momentum or forbidden by the Hamiltonian and Lagrangian mechanics.

5. It is found that the relativistic transformations for unphysical thermodynamic quantities of classes III and IV corresponding to Planck's and Ott's transformations are unphysical and contradict the requirements of the special theory of relativity since the rest energy of the thermodynamic system is not invariant.

6. The Ott transformation for physical thermodynamic quantities of class IV is forbidden by the Hamiltonian mechanics.

Thank you for your attention!