One-Loop Two-Point Fermionic Diagrams in External Electromagnetic Fields in the Fock-Schwinger Formalism

Alexander Parkhomenko

P. G. Demidov Yaroslavl State University, Yaroslavl, Russia

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in collaboration with Alexandra Dobrynina, Ilya Karabanov, & Lubov Vassilevskaya

Outline

Strong Magnetic Field

- In vacuum, regime of quantizing magnetic field is achieved when field strength is comparable to fermion mass squared or larger. In QED, it is the electron critical value:

$$B \gtrsim B_e = m_e^2/e \simeq 4.41 \times 10^{13} \text{ G}$$

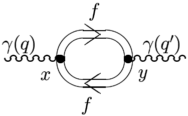
- In hot (dense) matter, besides the Landau energy scale, magnetic field should be comparable to or exceed the value of temperature squared (chemical potential squared)
- Such conditions are known despite being very rare:
  - agnetized neutron stars: up to $B \sim 10^{13}$ G in pulsurs and a few times 10^{14} G in magnetars
  - Non-central relativistic heavy ion collisions: strength could reach QCD energy scale, $B \sim B_\pi = m_\pi^2/e \simeq 3.1 \times 10^{18}$ G
  - Intense laser fields
- External magnetic field background is considered only

Photon Polarization Operator

- Photon polarization operator is typical example of two-point correlation function
- Lagrangian of spinor QED

$$\mathcal{L}_{\rm QED}(x) = eQ_f\left[\bar{f}(x)\gamma_\mu f(x)\right]A^\mu(x)$$

- Matrix element of $\gamma \to \gamma$ transition



$$\mathcal{M}_{\gamma\to\gamma} = -i\,\varepsilon'^{*}_{\mu}(q)\,\mathcal{P}^{\mu\nu}(q)\,\varepsilon_\nu$$

- $\mathcal{P}^{\mu\nu}(q)$ is two-point correlator of two vector currents
- Photon dispersion relations follow from the equations

$$q^2 - \Pi^{(\lambda)}(q) = 0 \quad (\lambda = 1, 2, 3)$$

- $\Pi^{(\lambda)}(q)$ are eigenvalues of the photon polarization operator
- In an external background field, corresponding modification of fermion propagator should be taken into account

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Propagator in Constant Homogenious Magnetic Field

- Dirac equation in an external electromagnetic field

$$\{[i\,\partial^\mu - e\,Q_f\,A^\mu(\mathrm{r},t)]\,\gamma_\mu - m_f\}\,\Psi(\mathrm{r},t) = 0$$

- Q_f and m_f are the relative charge and mass of the fermion
- Pure constant homogeneous magnetic field: $\mathrm{B} = (0, 0, B)$
- Equation for fermion propagator in the magnetic field

$$\{[i\,\partial^\mu - e\,Q_f\,A^\mu(x)]\,\gamma_\mu - m_f\}\,G_{\mathrm{F}}(x,y) = i\,\delta^{(4)}(x-y)$$

- Propagator can be constructed as an infinite sum of exact solutions of Dirac equation
- Alternatively, the Fock-Schwinger method can be used
- Lorentz-covariant four-potential: $A_\mu(x) = -F_{\mu\nu}x^\nu/2$
- $F_{\mu\nu}$ is the strength tensor of external electromagnetic field

Propagator in the Fock-Schwinger Representation

- General representation of the propagator in magnetic field [J. S. Schwinger, Phys. Rev. 82 (1951) 664]

$$G_{\mathrm{F}}(x,y) = \mathrm{e}^{i\Phi(x,y)}\, S_{\mathrm{F}}(x-y)$$

- Translationally and gauge non-invariant phase factor

$$\Phi(x,y) = -eQ_f \int_y^x d\xi^\mu \left[A_\mu(\xi) + \frac{1}{2} F_{\mu\nu}(\xi - y)^\nu\right]$$

- In two-point correlation function phase factors cancel each other

$$\Phi(x,y) + \Phi(y,x) = 0$$

- Gauge and translationally invariant part of a charged fermion propagator ($\beta = eB\,Q_f$)

$$\begin{aligned} S_{\mathrm{F}}(X) \quad = \quad & -\frac{i\beta}{2(4\pi)^2} \int\limits_0^\infty \frac{ds}{s^2} \Big\{ (X\widetilde{\Lambda}\gamma)\cot(\beta s) - i(X\widetilde{\varphi}\gamma)\gamma_5 - \\ & - \frac{\beta s}{\sin^2(\beta s)} (X\Lambda\gamma) + m_f s\, [2\cot(\beta s) + (\gamma\varphi\gamma)] \Big\} \times \\ & \times \quad \exp\left(-i\left[m_f^2 s + \frac{1}{4s}(X\widetilde{\Lambda}X) - \frac{\beta\cot(\beta s)}{4}(X\Lambda X)\right]\right) \end{aligned}$$

Basic Tensors in Presence of Magnetic Field

- Dimensionless tensor of the external magnetic field and its dual

$$\varphi_{\alpha\beta} = \frac{F_{\alpha\beta}}{B}\,, \qquad \tilde{\varphi}_{\alpha\beta} = \frac{1}{2}\,\varepsilon_{\alpha\beta\rho\sigma}\varphi^{\rho\sigma}$$

- Minkowski space is divided into two subspaces:
  - Euclidean with the metric tensor $\Lambda_{\mu\nu} = (\varphi\varphi)_{\mu\nu}$; plane orthogonal to the field strength vector
  - Pseudo-Euclidean with the metric tensor $\tilde{\Lambda}_{\mu\nu} = (\tilde{\varphi}\tilde{\varphi})_{\mu\nu}$
  - Metric tensor of Minkowski space $g_{\mu\nu} = \tilde{\Lambda}_{\mu\nu} - \Lambda_{\mu\nu}$
- Arbitrary four-vector $a^\mu = (a_0, a_1, a_2, a_3)$ can be decomposed into two orthogonal components

$$a_\mu = \tilde{\Lambda}_{\mu\nu}a^\nu - \Lambda_{\mu\nu}a^\nu = a_{\parallel\mu} - a_{\perp\mu}$$

- For the scalar product of two four-vectors one has

$$(ab) = (ab)_\parallel - (ab)_\perp$$

$$(ab)_\parallel = (a\tilde{\Lambda}b) = a^\mu\tilde{\Lambda}_{\mu\nu}b^\nu, \quad (ab)_\perp = (a\Lambda b) = a^\mu\Lambda_{\mu\nu}b^\nu$$

Orthogonal Basis Motivated by Magnetic Field

- Correlators having rank non-equal to zero, could be decomposed in some orthogonal set of vectors
- In magnetic field, such a basis naturally exists

$$b_\mu^{(1)} = (q\varphi)_\mu, \qquad b_\mu^{(2)} = (q\tilde{\varphi})_\mu$$
$$b_\mu^{(3)} = q^2\,(\Lambda q)_\mu - (q\Lambda q)\,q_\mu, \quad b_\mu^{(4)} = q_\mu$$

- Arbitrary vector a_μ can be presented as

$$a_\mu = \sum_{i=1}^{4} a_i\,\frac{b_\mu^{(i)}}{(b^{(i)}b^{(i)})}, \qquad a_i = a^\mu\,b_\mu^{(i)}$$

- Third-rank tensor $T_{\mu\nu\rho}$ can be decomposed similarly

$$T_{\mu\nu\rho} = \sum_{i,j,k=1}^{4} T_{ijk}\,\frac{b_\mu^{(i)}\,b_\nu^{(j)}\,b_\rho^{(k)}}{(b^{(i)}b^{(i)})\,(b^{(j)}b^{(j)})\,(b^{(k)}b^{(k)})},$$

$$T_{ijk} = T^{\mu\nu\rho}\,b_\mu^{(i)}\,b_\nu^{(j)}\,b_\rho^{(k)}.$$

Photon Polarization Operator in Magnetic Field

- $\Pi^{(\lambda)}(q)$ are eigenvalues of the photon polarization operator

$$\mathcal{P}_{\mu\nu}(q) = \sum_{\lambda=1}^{3} \frac{b_\mu^{(\lambda)} b_\nu^{(\lambda)}}{(b^{(\lambda)})^2}\, \Pi^{(\lambda)}(q)$$

- In vacuum, $\mathcal{P}_{\mu\nu}(q)$ has two physical eigenmodes
- In an external constant homogeneous magnetic field, the number of physical eigenmodes is the same
- Eigenvectors are determined by the field strength tensor

$$\varepsilon_\mu^{(1)} = b_\mu^{(1)} / \sqrt{q_\perp^2}, \quad \varepsilon_\mu^{(2)} = b_\mu^{(2)} / \sqrt{q_\parallel^2}$$

- In the magnetic field, $\Pi^{(\lambda)}(q)$ contains both vacuum and field-induced parts (for electron)

$$\Pi^{(\lambda)}(q) = -i\, \mathcal{P}(q^2) - \frac{\alpha}{\pi}\, Y_{VV}^{(\lambda)}$$

- Details on $Y_{VV}^{(\lambda)}$ can be found in A. Kuznetsov & N. Mikheev, Electroweak Processes in External Electromagnetic Fields (Springer, 2013)

AA-Correlator: Axion Self-Energy

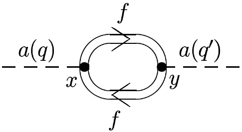
[Skobelev V. V., Phys. At. Nucl. 61 (1998); Borisov A. V. & Sizin P. E., JETP 86 (1999); Vassilevskaya L. A. et al., Phys. At. Nucl. 64 (2001)]

- Other example is the axion self-energy
- Lagrangian density of fermion-axion interaction

$$\mathcal{L}_{af}(x) = \frac{g_{af}}{2m_f}\left[\bar{f}(x)\gamma^{\mu}\gamma_5 f(x)\right]\partial_{\mu}\, a(x)$$

- $g_{af} = C_f m_f / f_a$ — dimensionless Yukawa constant
  C_f — dimensionless factor specifying the axion model
- Matrix element of $a \to a$ transition determines the electromagnetic correction to axion mass squared m_a^2

$$M_{a\to a} = -\delta m_a^2 = \frac{g_{af}^2}{4m_f^2}\,\Pi^{(AA)}_{\mu\nu}\, q^{\mu} q^{\nu}$$



- $\Pi^{(AA)}_{\mu\nu}$ is two-point correlator of two axial vectors

Axion Self-Energy in Magnetic Field

- Amplitude of axion self-energy

$$M_{a\to a}(q^2, q_\perp^2, \beta) = \sum_f \frac{g_{af}^2 \beta}{8\pi^2} \int_0^\infty \frac{dt}{\sin(\beta t)} \int_0^1 du \left[q_\parallel^2 \cos(\beta t) - q_\perp^2 \cos(\beta t u)\right] \times$$
$$\times \exp\left\{-i\left[m_f^2 t - \frac{q_\parallel^2}{4} t\,(1-u^2) + q_\perp^2 \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)}\right]\right\}$$

- Two proper times variables s_1 and s_2 replaced by $t = s_1 + s_2$ and $u = (s_1 - s_2)/t$
- Field-induced contribution to the $a \to a$ transition

$$\Delta M(q^2, q_\perp^2, \beta) = M_{a\to a}(q^2, q_\perp^2, \beta) - M_{a\to a}(q^2, 0, 0),$$

- This quantity is free from UV divergences

Generalized Local Fermionic Current

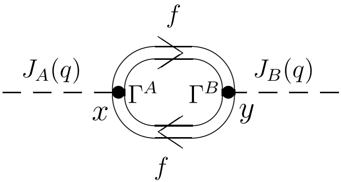
[M. Yu. Borovkov et al., Phys. At. Nucl. 62 (1999) 1601]

- Lagrangian density of local fermion interaction

$$\mathcal{L}_{\text{int}}(x) = \left[\bar{f}(x)\Gamma^A f(x)\right] J_A(x)$$

- J_A — generalized current (photon, neutrino current, etc.)
- Γ_A — any of γ-matrices from the set $\{1,\, \gamma_5,\, \gamma_\mu,\, \gamma_\mu\gamma_5, \sigma_{\mu\nu} = i\left[\gamma_\mu, \gamma_\nu\right]/2\}$
- Interaction constants are included into the current J_A

General Case of Two-Point Correlator



- Two-point correlation function of general form

$$\Pi_{AB} = \int d^4X \, \mathrm{e}^{-i(qX)} \, \mathrm{Sp}\left\{ S_{\mathrm{F}}(-X) \, \Gamma_A \, S_{\mathrm{F}}(X) \, \Gamma_B \right\}$$

- $S_{\mathrm{F}}(X)$ — gauge and translationally invariant part of the fermion propagator
- $X^\mu = x^\mu - y^\mu$ — integration variable
- Correlations of scalar, pseudoscalar, vector and axial-vector currents were studied by Borovkov et al. [Phys. At. Nucl. 62 (1999) 1601]
- Consider correlations of a tensor current with the other ones

Electromagnetic Dipole Interaction of Fermions

- Models beyond the SM can produce effective operators at current energies and Pauli Lagrangian density, in particular

$$\mathcal{L}_{\rm AMM}(x) = -\frac{\mu_f}{2}\left[\bar{f}(x)\sigma_{\mu\nu}f(x)\right]F^{\mu\nu}(x)$$

- For electron, the coupling can be written as $\mu_e = \mu_B a_e$, where $\mu_B = e/(2m_e)$ is Bohr magneton and a_e is electron AMM
- Total Lagrangian of interaction

$$\mathcal{L}_{\rm int}(x) = \mathcal{L}_{\rm QED}(x) + \mathcal{L}_{\rm AMM}(x)$$

- It gives additional contribution to the polarization operator
- Contribution linear in AMM is related with correlator of vector and tensor currents, $\Pi^{(VT)}_{\mu\nu\rho}$
- Contribution quadratic in AMM is determined by correlator of two tensor currents, $\Pi^{(TT)}_{\mu\nu\rho\sigma}$

Correlator of Vector and Tensor Currents

- Vector-tensor (VT) correlator, $\Pi^{(\mathrm{VT})}_{\mu\nu\rho}$, is rank-3 tensor
- Vector-current conservation and antisymmetry of the tensor current leave 8 non-trivial coefficients in the decomposition on basis vectors
- Of them, four coefficients only are independent
- Double-integral representation of coefficients is used

$$\Pi^{(\mathrm{VT})}_{ijk}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du\, e^{-i\Omega(t,u)}\, Y^{(\mathrm{VT})}_{ijk}(q^2, q_\perp^2, \beta; t, u)$$

- Phase definition

$$\Omega(t,u) = m_f^2 t - \frac{q_\parallel^2}{4}\, t\,(1-u^2) + q_\perp^2\, \frac{\cos(\beta t u) - \cos(\beta t)}{2\beta \sin(\beta t)}$$

- Integration variables and relation between momenta squared
  $t = s_1 + s_2,\ u = (s_1 - s_2)/(s_1 + s_2);\quad q_\parallel^2 = q^2 + q_\perp^2$

Integrands in Vector-Tensor Correlator

$$Y_{114}^{(\mathrm{VT})}(t,u) = -Y_{141}^{(\mathrm{VT})}(t,u) = -m_f\, q_\perp^2\, q^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}$$

$$Y_{223}^{(\mathrm{VT})}(t,u) = -Y_{232}^{(\mathrm{VT})}(t,u) = m_f\, q_\perp^2\, (q_\parallel^2)^2\, \frac{\beta t}{\sin(\beta t)} \left[\cos(\beta t) - \cos(\beta t u)\right]$$

$$Y_{224}^{(\mathrm{VT})}(t,u) = -Y_{242}^{(\mathrm{VT})}(t,u) = m_f\, q_\parallel^2\, \frac{\beta t}{\sin(\beta t)} \left[q_\perp^2\, \cos(\beta t) - q_\parallel^2\, \cos(\beta t u)\right]$$

$$Y_{334}^{(\mathrm{VT})}(t,u) = -Y_{343}^{(\mathrm{VT})}(t,u) = -m_f\, q_\perp^2\, q_\parallel^2\, (q^2)^2\, \frac{\beta t \cos(\beta t u)}{\sin(\beta t)}$$

- Choice of basis vectors is optimal because of vector current conservation $\quad q^\mu \Pi_{\mu\nu\rho}^{(\mathrm{VT})}$
- $Y_{4jk}^{(\mathrm{VT})}$ vanish naturally in this basis
- Antisymmetry in the last two indices is due to antisymmetric tensor current
- Parameters q^2, $q_\perp^2$, and β in $Y_{ijk}^{(\mathrm{VT})}$ are assumed implicitly

VT Contribution to $\gamma \to \gamma$ Amplitude

- Basis vectors are normalized, so $\gamma \to \gamma$ amplitude by itself is required to extract the photon polarization operator
- Vector and tensor currents in momentum space

$$j_V^{\mu} = -eQ_f \varepsilon'^{\mu}, \quad j_T^{\nu\rho} = -i\mu_f f^{*\nu\rho}/2 = -i\mu_f \left(q^{\nu}\varepsilon^{*\rho} - q^{\rho}\varepsilon^{*\nu}\right)/2$$

- Relation among the $\gamma \to \gamma$ amplitude and VT correlator

$$\mathcal{M}_{\mathrm{VT}} = ieQ_f \mu_f \varepsilon'^{\mu} \Pi^{(\mathrm{VT})}_{\mu\nu\rho} f^{*\nu\rho}/2$$

- The $\gamma \to \gamma$ amplitude in explicitly gauge invariant form

$$\mathcal{M}_{\mathrm{VT}} = -\frac{eQ_f \mu_f m_f \beta}{32\pi^2} \int_0^{\infty} \frac{dt}{\sin(\beta t)} \int_0^1 du\, e^{-i\Omega(t,u)} \times \left\{ \cos(\beta tu)\,(f' f^*) + \frac{q_\perp^2}{2q_\parallel^2} \left[\cos(\beta t) - \cos(\beta tu)\right] (\tilde{\varphi} f')\,(\tilde{\varphi} f^*) \right\}$$

- Used the notation for tensor contractions

$$\left(f' f^*\right) = f'^{\mu\nu} f^*_{\nu\mu}, \qquad \left(\tilde{\varphi} f^{(\prime)}\right) = \tilde{\varphi}^{\mu\nu} f^{(\prime)}_{\nu\mu}$$

Field Induced Part of the Amplitude

- The $\gamma \to \gamma$ amplitude in the fieldless limit

$$\mathcal{M}^{(0)}_{\mathrm{VT}} = -\frac{eQ_f \mu_f m_f}{32\pi^2} (f' f^*) \int_0^\infty \frac{dt}{t} \int_0^1 du \, e^{-it\left[m_f^2 - q^2 (1-u^2)/4\right]}$$

- Field-induced part is obtained after subtraction of $\mathcal{M}^{(0)}_{\mathrm{VT}}$

$$\Delta\mathcal{M}_{\mathrm{VT}} = \mathcal{M}_{\mathrm{VT}} - \mathcal{M}^{(0)}_{\mathrm{VT}}$$

- The strong field limit, i. e. lowest Landau level contribution

$$\mathcal{M}^{(\mathrm{smf})}_{\mathrm{VT}} = -\frac{eQ_f \mu_f m_f \beta q_\perp^2}{16\pi^2 (q_\parallel^2)^2} e^{-q_\perp^2/(2\beta)} (\tilde{\varphi} f') (\tilde{\varphi} f^*) F(z)$$

- Introduce $z = 4m_f^2/q_\parallel^2$ and used the function

$$F(z) = \begin{cases} \frac{1}{2\sqrt{1-z}} \left[\ln\left|\frac{\sqrt{1-z}-1}{\sqrt{1-z}+1}\right| - i\pi\Theta(z)\right], & z < 1 \\ \frac{1}{\sqrt{z-1}} \arctan \frac{1}{\sqrt{z-1}}, & z \geq 1 \end{cases}$$

Correlator of Two Tensor Currents

- Tensor-tensor (TT) correlator, $\Pi^{(\mathrm{TT})}_{\mu\nu\rho\sigma}$, is rank-4 tensor
- Antisymmetry of both tensor currents leaves 36 non-trivial coefficients in the basis decomposition
- Of them, eight coefficients only are independent
- Double-integral representation of coefficients is used

$$\Pi^{(\mathrm{TT})}_{ijkl}(q^2, q_\perp^2, \beta) = \frac{1}{4\pi^2} \int_0^\infty \frac{dt}{t} \int_0^1 du \, e^{-i\Omega(t,u)} Y^{(\mathrm{TT})}_{ijkl}(q^2, q_\perp^2, \beta; t, u)$$

Integrands in Tensor-Tensor Correlator

- Coefficients relevant for the photon polarization operator

$$Y^{(\mathrm{TT})}_{1414}(q_\parallel^2, q_\perp^2, \beta; t, u) = -q_\perp^2 \left\{ 2q_\perp^2 \left(q_\perp^2 + q_\parallel^2\right) \frac{\cos(\beta t) - \cos(\beta t u)}{\sin^2(\beta t)} \right.$$
$$+4q_\perp^2 q_\parallel^2 \left[\cos(\beta t u) - u \sin(\beta t u) \cot(\beta t)\right] - q_\parallel^2 \left[\left(1 - u^2\right) q_\parallel^2 + 4m_f^2\right] \cos(\beta t u)$$
$$\left. -q_\perp^2 \left[\left(1 - u^2\right) q_\parallel^2 - 4m_f^2\right] \cos(\beta t) + \frac{4i}{t} q_\parallel^2 \left[\cos(\beta t) - \frac{\beta t}{\sin(\beta t)}\right] \right\}$$

$$Y^{(\mathrm{TT})}_{2424}(q_\parallel^2, q_\perp^2, \beta; t, u) = \frac{q_\parallel^2}{q_\perp^2} Y^{(\mathrm{TT})}_{1414}(q_\parallel^2, q_\perp^2, \beta; t, u)$$

- Other six coefficients and TT part of the $\gamma \to \gamma$ amplitude will be presented in a forthcoming paper

AMM Contribution to Photon Polarization Operator

- For electron, tensor coupling can be written as $\mu_e = a_e \mu_B$
- Field-induced part of $\Pi^{(\lambda)}(q)$ is modified

$$\Pi^{(\lambda)}(q) = -i\,\mathcal{P}(q^2) - \frac{\alpha}{\pi}\, Y^{(\lambda)}_{VV} + 2\frac{\alpha}{\pi}\, a_e\, Y^{(\lambda)}_{VT} + \frac{\alpha}{\pi}\, a_e^2\, Y^{(\lambda)}_{TT}$$

- Last two terms can be presented in the form of double integral

$$Y^{(\lambda)}_{VT(TT)} = \int_0^\infty \frac{dt}{t} \int_0^1 du \left\{ \frac{\beta t}{\sin(\beta t)}\, y^{(\lambda)}_{VT(TT)}\, e^{-i\Omega} - q^2\, e^{-i\Omega_0} \right\}$$

- Notations are from the book by A. Kuznetsov and N. Mikheev
- Part independent on the field is subtracted
- Integrands entering the vector-tensor part

$$y^{(1)}_{VT} = y^{(3)}_{VT} = q^2 \cos(\beta t u)$$
$$y^{(2)}_{VT} = q_{\|}^2 \cos(\beta t u) - q_{\perp}^2 \cos(\beta t)$$

AMM Contribution to Photon Polarization Operator

- Integrands in the tensor-tensor part

$$y_{TT}^{(1)} = \frac{Y_{1414}^{(TT)}}{4m_e^2\, q_\perp^2}, \qquad y_{TT}^{(2)} = \frac{Y_{2424}^{(TT)}}{4m_e^2\, q_\parallel^2}$$

- For the electron, tensor-tensor term gives α-suppressed correction to vector-tensor one
- If neutrinos have local interaction with photon due to AMM, they contribute to TT part of photon polarization operator
- Taking into account the upper limit on neutrino AMM $\mu_\nu < 6.4 \times 10^{-12} \mu_B$ [PDG, 2022], this contribution, being $\sim \mu_\nu^2$, is negligible

Crossed-Field Limit

- Pure field invariant vanishes: $\beta \to 0$
- Dynamical parameter: $\chi_f^2 = e^2 Q_f^2 \, (qFFq) = \beta^2 q_\perp^2$
- The crossed-field limit is valid for an ultrarelativistic particle moving in the direction transverse to the field strength in a relatively weak magnetic field, $\chi_f^2 \gg \beta^3$
- Gauge and translationally invariant part of the fermion propagator [A. Kuznetsov & N. Mikheev, Electroweak Processes in External Electromagnetic Fields (Springer, 2013)]

$$
\begin{aligned}
S_F(X) = \frac{-i}{32\pi^2} \int_0^\infty \frac{ds}{s^3} \Big\{ & (X\gamma) - \frac{2\beta^2 s^2}{3} (X\Lambda\gamma) \\
& + i\beta s \, (X\tilde{\varphi}\gamma) \, \gamma_5 + m_f s \, [2 + \beta s \, (\gamma\varphi\gamma)] \Big\} \\
& \times \exp\left\{ -i \left[m_f^2 s + \frac{X^2}{4s} + \frac{\beta^2 s}{12} (X\Lambda X) \right] \right\}
\end{aligned}
$$

- $\varphi_{\mu\nu}$ and $\Lambda_{\mu\nu}$ have the same definitions as in the magnetic field

Crossed-Field Limit

- Pure field invariant vanishes $(\beta \to 0)$
- As basic vectors, accept the following orthonormalized set

$$b_\mu^{(1)} = \frac{eQ_f}{\chi_f}\,(qF)_\mu, \qquad b_\mu^{(2)} = \frac{eQ_f}{\chi_f}\,(q\tilde{F})_\mu$$

$$b_\mu^{(3)} = \frac{e^2 Q_f^2}{\chi_f^2\sqrt{q^2}}\left[q^2\,(qFF)_\mu - (qFFq)\,q_\mu\right], \quad b_\mu^{(4)} = \frac{q_\mu}{\sqrt{q^2}}$$

- Dynamical parameter: $\chi_f^2 = e^2 Q_f^2\,(qFFq) = \beta^2 q_\perp^2$
- Coefficients of the vector-tensor correlator in this basis:

$$\Pi_{ijk}^{(VT)}(q^2, \chi_f) = \frac{1}{4\pi^2}\int_0^\infty \frac{dt}{t}\int_0^1 du\, Y_{ijk}^{(VT)}(q^2, \chi_f; t, u)$$

$$\times \exp\left\{-i\left[\left(m_f^2 - \frac{q^2}{4}\,(1-u^2)\right)t + \frac{1}{48}\,\chi_f^2\,(1-u^2)^2 t^3\right]\right\}$$

Vector-Tensor Correlator Integrands in Crossed Fields

- Results for integrands in external electromagnetic crossed fields

$$
\begin{aligned}
Y_{114}^{(\mathrm{VT})} &= -Y_{141}^{(\mathrm{VT})} = -m_f\sqrt{q^2} \\
Y_{223}^{(\mathrm{VT})} &= -Y_{232}^{(\mathrm{VT})} = m_f\,\frac{\chi_f^2 t^2}{2\sqrt{q^2}}\left(1-u^2\right) \\
Y_{224}^{(\mathrm{VT})} &= -Y_{242}^{(\mathrm{VT})} = -m_f\sqrt{q^2}\left[1+\frac{\chi_f^2 t^2}{2q^2}\left(1-u^2\right)\right] \\
Y_{334}^{(\mathrm{VT})} &= -Y_{343}^{(\mathrm{VT})} = -m_f\sqrt{q^2}
\end{aligned}
$$

Integrands of Tensor-Tensor Correlator in Crossed Fields

- Double-integral representation of coefficients is used again

$$\Pi_{ijk\ell}^{(TT)}(q^2,\chi_f) = \frac{1}{4\pi^2}\int_0^\infty \frac{dt}{t}\int_0^1 du\, Y_{ijk\ell}^{(TT)}(q^2,\chi_f;t,u)$$

$$\times \exp\left\{-i\left[\left(m_f^2 - \frac{q^2}{4}\left(1-u^2\right)\right)t + \frac{1}{48}\chi_f^2\left(1-u^2\right)^2 t^3\right]\right\}$$

- Integrands of the tensor-tensor correlator contributing to the photon polarization tensor

$$Y_{1414}^{(\mathrm{TT})} = q^2\left(1-u^2\right) + 4m_f^2 - \frac{t^2\chi_f^2}{12}\left(1-u^2\right)\left(3+5u^2\right)$$

$$+\frac{2m_f^2 t^2\chi_f^2}{q^2}\left(1-u^2\right) + \frac{t^4\chi_f^4}{72q^2}\left(1-u^2\right)^2\left(9-u^2\right) + \frac{8it^2\chi_f}{3q^2}$$

$$Y_{2424}^{(\mathrm{TT})} = q^2\left(1-u^2\right) + 4m_f^2 - \frac{t^2\chi_f^2}{12}\left(1-u^2\right)\left(3+5u^2\right)$$

$$+\frac{2m_f^2 t^2\chi_f^2}{q^2}\left(1-u^2\right) + \frac{t^4\chi_f^4}{72q^2}\left(1-u^2\right)^2\left(9-u^2\right) + \frac{8it^2\chi_f}{3q^2}$$

- The other coefficients will be presented in a forthcoming paper

Conclusions

- Two-point fermionic correlators in presence of constant homogeneous external magnetic field are considered
- This analysis extends the previous one by inclusion the tensor current into consideration resulting the total set of two-point fermionic correlators
- Study of correlators of tensor fermionic current with the others allows to investigate effects of the fermion anomalous magnetic moment in the one-loop approximation
- Field-induced contribution to the photon polarization operator linear and quadratic in fermion anomalous magnetic moment are calculated
- Computer technique developed for two-point correlators is planned to be applied for three-point ones