Fine structure of the masless pereturbation theory series in QCD

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Topics to be considered :
Asymptotic structure of the analytical PT series in gauge theories with fermions (say QCD or QED)
Gauge theories with N_f-number of fermions flavours
AVV - anomalous triangle diagram with both conformal and axial anomalies
Analytical results obtained with participation of the members of INR (Moscow, Troitsk) and JINR (Dubna)
Generalized Crewther relation studies postponed

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Plan

- Large N_f or $O(1/N_f)$ expansion for RG-invariants : Adler e^+e^--annihilation and Bjorken polarized DIS sum rule coefficient functions
- $\{\beta\}$- decomposed or RG β-function related expanded represntations for coefficients of these PT -series
- Relation of $O(1/N_f)$ and β-expansions and theory ambiguities
- Addition of N=1 SUSY QCD possible limitations (INR-TH-2024-010 with K.V. Stepanyantz, in progress)
- Adler and Bjorken polarized sum rule coefficient functions PMC/BLM considerations (staus of warnings on scale and conformal symmetry related applications in phenomenology)
- Comments on analogy with Adler (1972) clarification on status of Finite *quenched* QED Program by Johnson, Baker , Willey et al (63 up to 70s)

Basis for e^+e^- to hadrons Adler function

$$D(a_s(Q^2)) = -\frac{d\Pi(a_s)}{d\ln Q^2} = Q^2\int\limits_0^\infty ds \frac{R^{th}_{e^+e^-}(a_s(s))}{(s+Q^2)^2} \to Q^2\int\limits_0^\infty ds \frac{R^{exp+th}_{e^+e^-}(s)}{(s+Q^2)^2},$$

$$R^{th}_{e^+e^-}(a_s) = \sigma_{tot}^{e^+e^-\to hadrons}(a_s(s))/\sigma_0(e^+e^-\to\mu^+\mu^-)$$

$$\left(\frac{\partial}{\partial\ln\mu^2} + \beta(a_s)\frac{\partial}{\partial a_s}\right) D(a_s(\mu^2)) = 0,$$

$$\frac{\partial a_s}{\partial\ln\mu^2} = \beta(a_s) = -\sum_{n\geq 0}\beta_n a_s^{n+2}.$$

$$D\left(a_s(Q^2)\right) = \left(\sum_i q_i^2\right) D^{ns}\left(a_s(Q^2)\right) + \left(\sum_i q_i\right)^2 D^{si}\left(a_s(Q^2)\right)$$

The a_s^4 Baikov,Chetyrkinand Kuhn (2010+...) BChK group ;

Bjorken sum rule for polarized $l^- N$ DIS

$$S_{Bjp}(Q^2) = \int_0^1 dx[g_1^{(lp)}(x,Q^2) - g_1^{(ln)}(x,Q^2)] = \frac{1}{6}|\frac{g_A}{g_V}|C_{Bjp}(a_s(Q^2))$$

$$C_{Bjp}(a_s(Q^2)) = C_{Bjp}^{NS}(a_s(Q^2)) + d_R\left(\sum_f Q_f\right)C_{Bjp}^{SI}(a_s(Q^2))$$

$$C_{Bjp}^{NS}(a_s)D^{NS}(a_s) = 1 + \beta(a_s)(K_0 + K_1 a_s + K_2 a_s^2 + O(a_s^3))(CBK$$

$$generalized\ Crewther) = (Mikhailov, K(10-12)) =$$

$$1 + \sum_{n\geq 0}\left(\frac{\beta(a_s)}{a_s}\right)^n P_n(a_s)$$

Singlet not considered here in GLS-related JETP Lett 94 (2011) (AK); Considered in Bjp -related Phys.Lett. B238 (2013) (Larin S.A.) and Reconsidered by BChK NPPP (2015) and Dubna talk by KGCh (work in (re)progress (?))

The $\overline{MS}$-scheme large N_f BLM approach (1983) generalization

In the $\overline{MS}$-scheme BLM prescribes to absorb into the SCALE the N_f dependence

$$D^{ns}(a_s) = 1 + d_{10}a_s + (d_{20} + d_{21}N_f)a_s^2 + (d_{30} + d_{31}N_f + d_{32}N_f^2)a_s^3 \\ +(d_{40} + d_{41}N_f + d_{42}N_f^2 + d_{43}N_f^3)a_s^4$$

Generalized Grunberg,Kataev (91-92); AK (92) ; G (92); Beneke, Braun (95) ; Neubert (95) Brodsky,Wu (2012)
d_{n0}- scale-invariant contributions ; absorbing all N_f dependence into the BLM related scales (Grunberg-Kataev generalization of BLM)
$d_{10} = +1$; $d_{20} = \frac{1}{12} \approx 0.085$**;** (BLM) $d_{30} \approx= -23.227$**;** (GK-92)
$d_{40} = +82.344$ (Brodsky-Wu (2012) (Sign ! ; Not small !)
As shown by Goriachuk, K., Molokoedov (22) agree with β-expanded model (see next page) and Brodsky, Wu et al (12)
R_δ procedure $a_s(\mu^2) = a_s(\mu_\delta) + \sum_{n\geq 1} \frac{1}{n!}\frac{d^n a_s(\mu_\delta^2)}{dln(\mu_\delta^2)}(-\delta)^n$ with
$\delta = ln(\mu_\delta^2/\mu^2)$

The $\{\beta\}$-expansion PT approach for the RG-invariant quantities . Adler function as example

Consider the PT expansion

$$D^{ns}(a_s) = 1 + d_1 a_s + d_2 a_s^2 + d_3 a_s^3 + d_4 a_s^4 + O(a_s^5)$$

In the MS-like schemes β-expansion prescription is:

$$
\begin{gathered}
d_1 = d_1[0] \\
d_2 = \beta_0 d_2[1] + \mathbf{d_2[0]} - \textbf{ the Basis of BLM procedure} \\
d_3 = \beta_0^2 d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1] + \mathbf{d_3[0]}, \\
d_4 = \beta_0^3 d_4[3] + \beta_2 d_4[0,0,1] + \beta_1\beta_0 d_4[1,1] + \beta_0^2 d_4[2] + \beta_1 d_4[0,1] \\
+ \beta_0 d_4[1] + \mathbf{d_4[0]}; \ldots
\end{gathered}
$$

Suggested by Mikhailov (Quarks2004, JHEP(07)) Further on Bakulev,Mikhailov, Stefanis(10) ; Kataev, Mikhalov M(12-16); Brodsky,Wu, Mojaza et al(12-23); Cvetic,Kataev(16); Kataev,Molokoedov (22,23) ; Baikov, Mikhailov (22-23) ; Mikhailov (24)

Theory ambiguity in terms of the $\{\beta\}$-expansion. Why ? Where ?

The problem appears starting from N^2LO QCD:

$$d_3 = d_{32}n_f^2 + d_{31}n_f + d_{30} \to \beta_0^2\ d_3[2] + \beta_1 d_3[0,1] + \beta_0 d_3[1,0] + d_3[0],$$

where $\beta_0 = \beta_{00} + \beta_{10}n_f$, $\beta_1 = \beta_{10} + \beta_{11}n_f$. How to get from single n_f- term two terms $\beta_1 d_3[0,1] + \beta_0 d_3[1]$. Mikhailov(07): Add to QCD **additional degree of freedom**, i.e. $n_{\tilde{g}}$ flavour number of multiplet of *MSSM gluino* . **Broken SUSY case model . Regularization** $\overline{MS}$ **not** $\overline{DR}$
There $\beta_0 = \beta_0(n_f, n_{\tilde{g}})$, $\beta_1 = \beta_0(n_f, n_{\tilde{g}})$ (Clavelli,Surguladze(97) and $d_3(n_f, n_{\tilde{g}})$ (Chetyrkin (97)) are known analytically. In extended QCD (eQCD) D- and β-function are evaluated analytically by Chetyrkin(22); Zoller (2016) and β-expansion has solution; though model dependence exist Cvetic, K (16); K Molokoedov (23) Bednyakov (24, in private) and Mikhailov (22,24) who gives $d_{20} = \frac{1}{12} \approx 0.085$; $d_{30} \approx= -35.87(model)$; $d_{40} \approx -98$ (sign !);

Representations for the D^{ns} in not only QCD

Whether expansion in powers of $\beta(a_s)/a_s$, where $\beta(a_s) = -\sum_{j\geq 0}\beta_j a_s^{j+2}$ (not in all RENORMALIZATION SCHEMES conformal anomaly) is valid for the D^{ns}? Cvetic, Kataev (16); K,Mikhailov (09-12) motivated; Valid say for static potential as well K, Molokoedo

$$D^{ns}(a_s) = 1 + \sum_{n\geq 0}\left(\frac{\beta(a_s)}{a_s}\right)^n D_n(a_s)$$

$$D_n(a_s) = \sum_{r=1}^{4-n} a_s^r \sum_{k=1}^{r} D_n^{(r)}[k, r-k] C_F^k C_A^{r-k} + a_s^4 \delta_{n0} \times$$
$$\left(D_0^{(4)}[F,A]\frac{d_F^{abcd}d_A^{abcd}}{d_R} + D_0^{(4)}[F,F]\frac{d_F^{abcd}d_F^{abcd}}{d_R} n_f\right) + O(a_s^5)$$

with $D_0^{(4)}[F,A]$ and $D_1^{(4)}[F,F]$; $D_n^{(r)}[k, r-k]$ analytical In comment by Shen,Wu, Ma,Brodsky (16) β-expansion of $\gamma^{photon}(a_s)$ in RG equation for Π is not applied (from Brodsky, Mojaza, Wu (14) despite Mikhailov, K (14-16) and AK (14))

The $\{\beta\}$ expanded QCD terms for D^{ns} in $SU(N_c)$ non-diagrammatic and diagarammatic (!) differences

Using the $\overline{MS}$-scheme factorized representation, Cvetic,Kataev(16). The results differs from QCD+gluino theory (Mikhailov (07))

$$d_1[0] = \frac{3}{4}C_F \ d_2[0] = \left(-\frac{3}{32}C_F^2 + \frac{1}{16}C_F C_A \right) \ d_2[1] = \left(\frac{33}{8} - 3\zeta_3\right)C_F$$

$$d_3[0] = -\frac{69}{128}C_F^3 - \left(\frac{101}{256} - \frac{33}{16}\zeta_3\right) \neq +\mathbf{\frac{71}{64}} \ \ C_F^2 C_A$$

$$-\left(\frac{53}{192} + \frac{33}{16}\zeta_3\right) \neq +\left(\mathbf{\frac{523}{768}} - \mathbf{\frac{27}{8}}\boldsymbol{\zeta_3}\right) \ \ C_F C_A^2$$

As the result one has $d_3[0] = -23.227 \neq \mathbf{-35.87}$, $d_4[0] = +83.344 \neq \mathbf{-98}$ (Cvetic,Kataev (16) $\neq$ K, Mikhailov (15) and Baikov,Mikhailov (22,23) gluino QCD and extanded QCD related results KGCh (97; 22) and Zoller (16)

The $\{\beta\}$ expansion QCD expression for d_4 and c_4 was also obtained

We present model dependent one from Cvetic, K (2016)

$$
\begin{aligned}
d_4[0] = \left(\frac{3}{16} - \frac{1}{4}\zeta_3 - \frac{5}{4}\zeta_5\right)\frac{d_F^{abcd}d_A^{abcd}}{d_R} - \left(\frac{13}{16} + \zeta_3 - \frac{5}{2}\zeta_5\right)\frac{d_F^{abcd}d_F^{abcd}}{d_R}n_f \\
+\left(\frac{4157}{2048} + \frac{3}{8}\zeta_3\right)C_F^4 \\
-\left(\frac{3509}{1536} + \frac{73}{128}\zeta_3 + \frac{165}{32}\zeta_5\right) \neq -\left(\mathbf{\frac{2409}{512}} + \mathbf{\frac{27}{16}}\boldsymbol{\zeta_3}\right) \quad C_F^3 C_A \\
+\left(\frac{9181}{4608} + \frac{299}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq -\left(\mathbf{\frac{3105}{1024}} + \mathbf{\frac{81}{32}}\boldsymbol{\zeta_3}\right) \quad C_F^2 C_A^2 \\
\left(-\frac{30863}{36864} - \frac{147}{128}\zeta_3 + \frac{165}{64}\zeta_5\right) \neq \left(\mathbf{\frac{68047}{12288}} + \mathbf{\frac{8113}{512}}\boldsymbol{\zeta_3} - \mathbf{\frac{3555}{128}}\boldsymbol{\zeta_5}\right) C_F C_A^3
\end{aligned}
$$

The difference is from diagrammatic related expression of Mikhailov (22-24) which is closer to Ball, Beneke, Braun (95) . Not clear whether is it possible to get theory relation between the results in general

PMC/BLM vs massless $\overline{MS}$: K,Molokoedov PRD(23): in Adler function $\gamma_{ph}(a_s)$ corrctly β expanded as K, Mikhailov (15) ; Salinas-Arzimendi ,Schmidt (2210.01851)

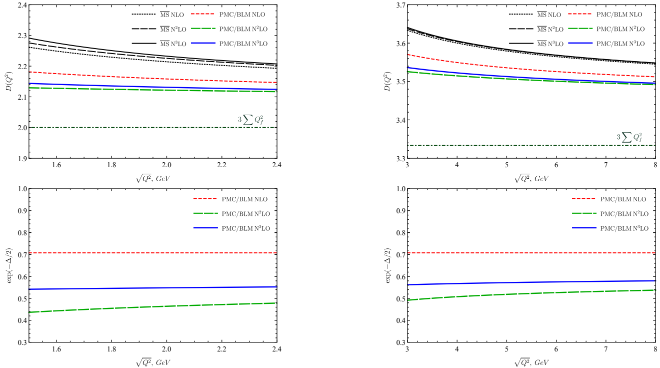
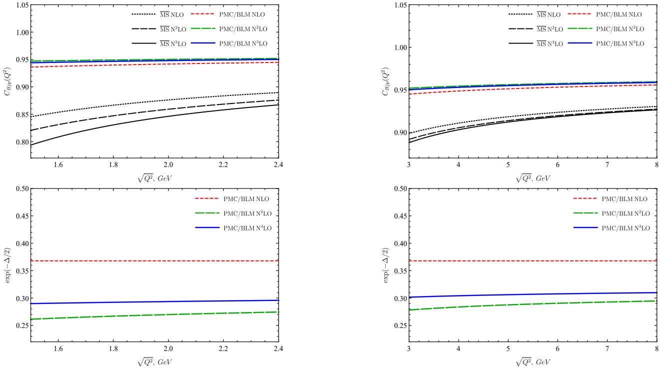


Figure: (1a) Adler function $D(Q^2)$ on $\sqrt{Q^2}$ at $n_f = 3, 4$ in the massless limit. (1b) PMC Factor $\exp(-\Delta/2)$ on $\sqrt{Q^2}$. Experimental related data higher (!) $\overline{MS}$ Eidelman, Jegerlehner, K, Veretin (98); Davier et al (23). Bad for PMC/BLM and in cases of SUSY QCD related effective model and eQCD as well .

PMC/BLM vs massless $\overline{MS}$: Bjorken polarized SR at n_f=3,4 $S_{Bjp}(Q^2) = \frac{1}{6}(g_A/g_V)C_{Bjp}(Q^2)$ by AK and Molokoedov drawn @ 23



Experimental data lower (!) PMC/BLM and a higher than $\overline{MS}$
Deur et al (23) and Shirkov et al (08) (and Kotikov 24 talks)
Effects of conformal symmetry violation by both PT and non-PT effects ARE NOT SEEN in PMC but ARE SEEN in NATURE (!) . Considerations see also
D.Kortlorz,Mikhailiov,Teryaev,A.Kotlorz (19);

Conclusions

- PMC/BLM do not feel running of QCD coupling constant and is useful tool for study of CS limit results in theory BUT not in phenomenology
- Analogy with Finite QED Program treatment by Adler.
- Is it possible to understand better the existing model dependence in coefficients of β-expanded terms of PT series ? (Leading renormalon chains and subleading renormalon chains)
- Leading renormalon chains desribe nicely effects of growth of PT coefficvients of Eucledian PT series
- Claim of α_s CERN Working group gided with participation of Michelangelo Mangano (2024). We should take into account in α_s extraction "scale systematics" or "missing higher order systematics" or "procedure dependence systermatics" .

Remained theory questions

- Why R_δ agree with multiple β-expasnion ? . Whether it agrees with useful Cvetic-Valenzuela (08) study cosniderede e.g. in Kotikov (24) talks ?
- Possible study of applying variants of β-expansion for Adler and Bjorken polarized sum rule PT coefficient function phenomenology related study .
- Whether mutiple β-expansion and thus is R_δ are distingushed in $N = 1$ SUSY QCD NSVZ-related D-function considerations ? At next-to-leading order level yes (Aleshin,Kataev,Stepanyantz (19)) .
- Why β-function is factorized in the CSB PT QCD expression for of $\pi^0 \to \gamma\gamma$ formfactor in the gauge-invariant and definite MOM schemes in Landau fauge ??? (Crewther -type relation).

Status of started from HSFI-2014 (09.07.24 Gatchina) and Protvono-2024 Workshop (24.07.24) Round Table Discussions

- PMC/BLM topic :
  Brodsky, Stanley J. пт, 30 авг., 03:42 (4 дня назад)
  кому: kataev@ms2.inr.ac.ru, leonardo, leonardo.digiustino@uninsubria.it, Alexandre Deur <deurpam@jlab.org>
  Dear Leonardo and Alexandre,
  I don't believe that Andrei Kataev and I are very far apart on BLM/PMC issues, but I would greatly appreciate your insight and comments.
  Thanks Stan

Possible further theory consideretion

A.L.Kataev (INR RAS) and K.V. Stepanyantz (MSU) INR-TH-2024-010
Exact relations between running of α_s and α in N=1 SQCD+SQED

THANK YOU FOR YOUR ATTENTION