B-mesons oscillations: applications of two-particle wave functions

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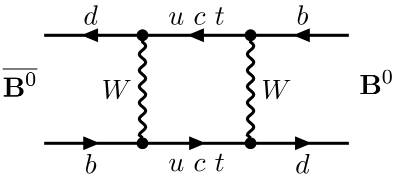
B-mesons oscillations overview

Feynman diagram of B^0-$\bar{B}^0$ oscillations



- Mixing:

$$B^0 = \frac{B_L + B_H}{2p},$$
$$\bar{B}^0 = \frac{B_L - B_H}{2q},$$

where p, q are responsible for CPV.

1. Oscillations occur if the following value is nonzero:

$$R := \frac{\Gamma(B^0 \to \bar{B}^0 \to \ell^- \bar{\nu}_\ell X)}{\Gamma(B^0 \to \ell^+ \nu_\ell \bar{X})} = \frac{x^2}{2 + x^2}.$$

2. The number $x = \frac{m_H - m_L}{\Gamma} = \frac{\Delta m}{\Gamma}$ determines the value of oscillations.
3. The oscillations can be measured experimentally in $\Upsilon \to B\bar{B} \to \ell^\pm \ell^\pm X$:

$$r = \frac{N_{++} + N_{--}}{N_{+-} + N_{-+}},$$

where $N_{\pm\pm}$ are the numbers of dileptons of the required sign.

One-particle and two-particles wave functions

- The value R can be derived by means of one-particle wave function:

$$\left|B^0(t)\right\rangle = e^{-iMt} e^{-(\Gamma/2)t} \left[\cos\left(\frac{\Delta m}{2} t\right) |B^0\rangle + i\frac{q}{p} \sin\left(\frac{\Delta m}{2} t\right) |\overline{B}^0\rangle \right].$$

- The value r describes the oscillations of C-odd $B^0 \bar{B}^0$ system $\rightarrow$ two-particles wave function needed.
- However $r = R \rightarrow$ tagging and no need for two-particle wave function.
- It doesn't work when B mesons are produced in C-even state or incoherently.
- Here we describe various problems where the expressions obtained by means of two-particles wave functions of oscillating B-mesons should be applied.

Two-particle wave function of the C-odd state

- C-odd state is produced in Υ decay: $\Upsilon \to B^0 \bar{B}^0$.
- Since quantum numbers of the resonance $1^{--} \to B^0 \bar{B}^0$ in P-wave with $C = -1$.
- Antisymmetric wave function:

$$\left|B^0 \bar{B}^0\right\rangle_{odd}(t_1, t_2) = \left|B^0(t_1)\right\rangle \left|\bar{B}^0(t_2)\right\rangle - \left|B^0(t_2)\right\rangle \left|\bar{B}^0(t_1)\right\rangle .$$

- Substitution $t = (t_1 + t_2)/2$ and $\Delta t = t_1 - t_2$ leads to:

$$\left|B^0 \bar{B}^0\right\rangle_{odd}(t, \Delta t) = e^{-2iMt} e^{-\Gamma t} \left[i \sin \frac{\Delta m}{2} \Delta t \left(\frac{q}{p} \bar{B}_1^0 \bar{B}_2^0 - \frac{p}{q} B_1^0 B_2^0 \right) + \right.$$
$$\left. + \cos \frac{\Delta m}{2} \Delta t \left(\bar{B}_1^0 B_2^0 - B_1^0 \bar{B}_2^0 \right) \right]$$

- Here indices $1, 2$ mesons distinguish B^0 states with different momenta.

Calculation of R_{odd}

- According to $N_{\pm\pm} = \frac{1}{2}\left| \left\langle B^0 B^0 \middle| B^0 \bar{B}^0(t_1, t_2) \right\rangle_{odd} \right|^2$, $N_{\pm\mp} = \frac{1}{2}\left| \left\langle B^0 \bar{B}^0 \middle| B^0 \bar{B}^0(t_1, t_2) \right\rangle_{odd} \right|^2$:

$$N_{++} = N_{--} = \frac{1}{2} \int\limits_{-\infty}^{+\infty} \mathrm{d}\Delta t \int\limits_{|\Delta t|/2}^{\infty} \mathrm{d}t e^{-2\Gamma t} \sin^2 \frac{\Delta m}{2} \Delta t = \frac{1}{4\Gamma^2} \frac{x^2}{1 + x^2},$$

$$N_{+-} = N_{-+} = \frac{1}{2} \int\limits_{-\infty}^{+\infty} \mathrm{d}\Delta t \int\limits_{|\Delta t|/2}^{\infty} \mathrm{d}t e^{-2\Gamma t} \cos^2 \frac{\Delta m}{2} \Delta t = \frac{1}{4\Gamma^2} \frac{2 + x^2}{1 + x^2}.$$

- Normalization is as follows: $\sum\limits_{i,j=+,-} N_{ij} = 1/\Gamma^2$.
- For the R_{odd} we reproduce the well-known expression obtained by means of one-particle wave function:

$$R_{odd} = \frac{N_{++} + N_{--}}{N_{+-} + N_{--}} = \frac{x^2}{2 + x^2}.$$

Two-particle wave function of the C-even state

- C-even state is produced in $\Upsilon(5S)$ decay with further e/m decay: $\Upsilon \to B^{0*}\bar{B}^0 \to B^0\bar{B}^0\gamma$.
- Symmetric wave function:

$$\left|B^0\bar{B}^0\right\rangle_{even}(t_1, t_2) = \left|B^0(t_1)\right\rangle\left|\bar{B}^0(t_2)\right\rangle + \left|B^0(t_2)\right\rangle\left|\bar{B}^0(t_1)\right\rangle;$$

- Analogously to the C-odd state we obtain:

$$N_{++} = N_{--} = \frac{1}{2}\int\limits_{-\infty}^{+\infty} \mathrm{d}\Delta t \int\limits_{|\Delta t|/2}^{\infty} \mathrm{d}t e^{-2\Gamma t}\sin^2\Delta m t = \frac{1}{4\Gamma^2}\frac{3x^2+x^4}{(1+x^2)^2};$$

$$N_{+-} = N_{-+} = \frac{1}{2}\int\limits_{-\infty}^{+\infty} \mathrm{d}\Delta t \int\limits_{|\Delta t|/2}^{\infty} \mathrm{d}t e^{-2\Gamma t}\cos^2\Delta m t = \frac{1}{4\Gamma^2}\frac{2+x^2+x^4}{(1+x^2)^2};$$

$$R_{even} = \frac{3x^2+x^4}{2+x^2+x^4}.$$

Incoherent B mesons production

- Incoherently B mesons are produced in hadron collisions at LHC.
- Wave function has no definite C-parity:

$$\left|B^0\bar{B}^0\right\rangle_{inc}(t_1,t_2)=\left|B^0(t_1)\right\rangle\left|\bar{B}^0(t_2)\right\rangle;$$

- Performing the analogous calculation as in the latter cases we obtain:

$$N_{++}=N_{--}=\frac{3x^2+x^4}{2+x^2+x^4};$$

$$N_{+-}=\frac{1}{4\Gamma^2}\frac{(2+x^2)^2}{(1+x^2)^2};\ N_{-+}=\frac{1}{4\Gamma^2}\frac{x^4}{(1+x^2)^2}.$$

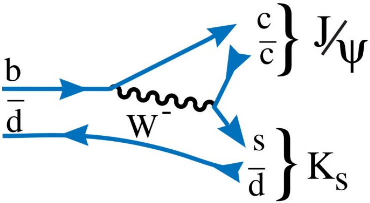
- It is instructive that $N_{+-}\neq N_{+-}$ unlike the C-odd/even case.
- Finally the R_{inc} value is:

$$R_{inc}=\frac{2x^2+x^4}{2+2x^2+x^4}.$$

- The obtained results are known in the literature but the derivation with two-particles wave function was not presented.

CPV and gold plated mode

Feynman diagram of $\bar{B}^0 \to J/\psi K$



$$a_{CP}(\Delta t) := \frac{N_{\ell^+ f} - N_{\ell^- f}}{N_{\ell^+ f} + N_{\ell^- f}} = \pm \sin 2\beta \sin \Delta m \Delta t$$

1. Direct CPV occurs when

$$A_{B^0 \to f} \neq A_{\bar{B}^0 \to f}.$$

2. Indirect CPV in $B^0 - \bar{B}^0$ mixing is small:

$$A_{CP} = \frac{N(\bar{B}^0 \to \ell^+ X) - N(B^0 \to \ell^- X)}{N(\bar{B}^0 \to \ell^+ X) + N(B^0 \to \ell^- X)} = O(10^{-4}).$$

3. Gold plated mode is $\Upsilon \to B^0 \bar{B}^0 \to (J/\psi K)(\ell^{\pm} X)$.
4. Leads to the best accuracy of β angle theoretical prediction and large enough branching to be detected:

$$Br(B \to J/\psi K) \approx 10^{-3}.$$

C-even state correction to a_{CP}

- Soft photon radiation in Υ decay leads to $B^0\bar{B}^0$ final state with $C = +1$.
- Such correction can be understood as:
  1. the bound on the accuracy of β angle measurement;
  2. bound on the probability of soft photon radiation.
- Modification of the formulas obtained above:
  1. keep CP-violating p/q parameter;
  2. perform the integration only over t since a_{CP} dependence on Δt is measured;
  3. introduce

$$A_{J/\psi K} = \left\langle J/\psi K \middle| H_{int} \middle| B^0 \right\rangle, \ \bar{A}_{J/\psi K} = \left\langle J/\psi K \middle| H_{int} \middle| \bar{B}^0 \right\rangle;$$

- Also let us define

$$\lambda = \frac{q}{p}\frac{\bar{A}_{J/\psi K}}{A_{J/\psi K}}, \ Im\lambda = \pm \sin 2\beta,$$

sign $\pm$ relates to K_S, K_L respectively and $|\lambda| \approx 1$.

Correction to a_{CP} calculation

- With the modifications mentioned we obtain for C-odd case:

$$N^{odd}_{\ell^+ f} = \frac{e^{-\Gamma\Delta t}}{2\Gamma}[1 + \sin 2\beta \sin \Delta m \Delta t].$$

$$N^{odd}_{\ell^- f} = \frac{e^{-\Gamma\Delta t}}{2\Gamma}[1 - \sin 2\beta \sin \Delta m \Delta t]$$

- For C-even case:

$$N^{even}_{\ell^+ f} = \frac{e^{-\Gamma\Delta t}}{2\Gamma} + \frac{\sin 2\beta}{2\Gamma} e^{-\Gamma\Delta t}\left[\frac{x}{1+x^2}\cos \Delta m \Delta t + \frac{1}{1+x^2}\sin \Delta m \Delta t\right],$$

$$N^{even}_{\ell^- f} = \frac{e^{-\Gamma\Delta t}}{2\Gamma} - \frac{\sin 2\beta}{2\Gamma} e^{-\Gamma\Delta t}\left[\frac{x}{1+x^2}\cos \Delta m \Delta t + \frac{1}{1+x^2}\sin \Delta m \Delta t\right].$$

Correction to a_{CP} calculation

- Let us suppose that there is an admixture η of C-even state to C-odd in Υ decay:

$$N_{\ell^\pm f} = N_{\ell^\pm f}^{odd} + \eta N_{\ell^\pm f}^{even}.$$

- For $a_{CP}(\Delta t)$ we obtain:

$$a_{CP}(\Delta t) = \sin 2\beta \left[\left(1 - \frac{x^2}{1+x^2}\eta \right) \sin \Delta m \Delta t + \frac{x}{1+x^2}\eta \cos \Delta m \Delta t \right]$$

- We can see that non-zero $\sim \cos \Delta m \Delta t$ term appeared.
- Using experimental data the bound on correction value η can be extracted:

$$C_{J/\psi K^0} = 0.004 \pm 0.010;$$

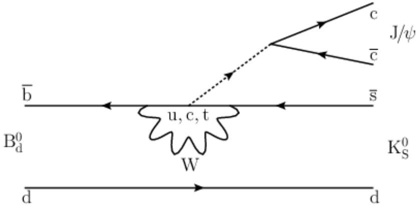
$$S_{J/\psi K^0} = 0.709 \pm 0.011;$$

$$\beta = \left(22.6^{+0.5}_{-0.4}\right)^\circ; \; x = 0.77;$$

$\eta \leq 0.05$ at the level of 2σ.

Penguin correction

$B^0 \to J/\psi K$ penguin diagram



- Our oscillation formulas also allow to take into account penguin diagram contribution.
- This contribution might be of the same order of the one from soft photon radiation.
- To perform the calculation one needs to introduce:

1. δ - difference of the tree and penguin strong phases;
2. $\gamma = arg \frac{V_{ub}^* V_{ud}}{-V_{cb}^* V_{cd}}$ is a unitarity triangle angle.
3. T and P are tree and penguin amplitudes without phases.

$$a_{CP} = \left\{ \sin 2\beta \left(1 - \eta \frac{x^2}{1+x^2} \right) - 2\frac{P}{T} \cos\delta \sin\gamma \cos 2\beta \right\} \sin \Delta m \Delta t +$$
$$+ \left\{ \eta \frac{x}{1+x^2} \sin 2\beta - 2\frac{P}{T} \sin\delta \sin\gamma \right\} \cos \Delta m \Delta t$$

$\Upsilon(4S) \to B^0\overline{B}^0 \to J/\psi K_S J/\psi K_S$

- It is remarkable that
  1. this decay holds **only** if CPV occurs;
  2. this decay happens **only** in presence of $B^0 - \bar{B}^0$ oscillations.
- With help of the C-odd two-particles wave function we obtain for the amplitude of the decay:

$$\langle J/\psi K_S, J/\psi K_S | B\bar{B}(t_1, t_2)\rangle_{\text{odd}} = -e^{-2iMt-\Gamma t}\left(i\frac{p}{q}A^2\right)\left[1-\lambda^2\right]\sin\left(\frac{\Delta m \Delta t}{2}\right).$$

- For the branching ratio of the considered decay one gets:

$$Br\left(\Upsilon(4S) \to B^0\overline{B}^0 \to J/\psi K_S J/\psi K_S\right) = 0.5 \cdot 2\sin^2 2\beta\left(\frac{x^2}{1+x^2}\right)Br^2(B^0 \to J/\psi K_S),$$

  where factor 0.5 takes into account $Br(\Upsilon \to B^0\bar{B}^0)$ and integration over t and Δt was performed.

$\Upsilon(4S) \to B^0\overline{B}^0 \to J/\psi K_S J/\psi K_S$

- The process under consideration is quite rare since $Br\big(\Upsilon(4S) \to B^0\overline{B}^0 \to J/\psi K_S J/\psi K_S\big) \approx 3.5 \times 10^{-8}$.
- The cross section of $\Upsilon(4S)$ production in e^+e^- collision is

$$\sigma = 2.1 \cdot 10^6 fb.$$

- The integrated luminosity of the SuperKEKB accelerator is supposed to be $L = 50$ ab^{-1}.
- The efficiency of Belle II to the considered mode is $\epsilon = 0.04$ (P.N. Pakhlov).
- Consequently for the number of events one obtains

$$N = \epsilon \cdot N(\Upsilon(4S)) \cdot Br(\Upsilon(4S) \to J/\psi K_S J/\psi K_S) \approx 150.$$

- If we take into account similar modes: $\psi' K_S$, $\chi_{c1} K_S$ it will double the number of events.

The way of $Br(B_s \to \mu^+\mu^-)$ precise measurement

Normalization channel:

$$Br(B_q \to X) = Br(B_{q'} \to X')\frac{f_{q'}}{f_q}\frac{\epsilon_{X'}}{\epsilon_X}\frac{N_X}{N_{X'}}.$$

- f_q is a fragmentation function $b \to B_q$;
- ϵ_X is an efficiency to X final state;
- N_X is a total number of X events.

1. Rare $B_s \to \mu^+\mu^-$ decay $\to$ possible New Physics:

$$Br = (3.63^{+0.15}_{-0.10}) \times 10^{-9} - \text{ SM;}$$
$$Br = (3.01 \pm 0.35) \times 10^{-9} - \text{ LHC.}$$

2. $f_s = (22.0^{+2.0}_{-2.1})\% \to$ it is important to decrease the error.
3. Our approach based on analysis of time-dependent numbers of dileptonic events allows to extract $Br\left(\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}\right)$ with 1% accuracy.
4. Determine $Br(B_s \to D_S\pi)/Br(B^0 \to D\pi)$ at Belle II $\to B_s \to D_S\pi$ as normalization channel.

Extraction of ϵ_{ss}

- Two-particles wave functions allow to calculate the time-dependant numbers of dileptons $dN_{\pm\pm}/d\Delta t$ and $dN_{\pm\mp}/d\Delta t$ in $\Upsilon(5S) \to B\bar{B}X \to \ell^{\pm}\ell^{\pm}X'$:

$$\frac{d(N_{+-} + N_{-+} - N_{--} - N_{++})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t},$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t}.$$

- The introduced ratios allow to decrease the uncertainty.

C-parity of $B\bar{B}$ pair	Decay modes				Branching notation
$B^0\bar{B}^0$ in the final state					
C-odd state	$B^0\bar{B}^0$ $B^{0*}\bar{B}^{0*}\pi^0$	$B^{0*}\bar{B}^{0*}$	$B^0\bar{B}^0\pi^0$		$(\epsilon_{00})^{\text{odd}}$
C-even state	$B^0\bar{B}^{0*}$ $B^0\bar{B}^{0*}\pi^0$	$B^{0*}\bar{B}^0$	$B^{0*}\bar{B}^0\pi^0$		$(\epsilon_{00})^{\text{even}}$
B^+B^- in the final state					
C-odd and C-even states	B^+B^- $B^{+*}B^{-*}\pi^0$	$B^{+*}B^{-*}$	$B^+B^-\pi^0$		ϵ_{+-}
	B^+B^{-*} $B^+B^{-*}\pi^0$	$B^{+*}B^-$	$B^{+*}B^-\pi^0$		
$B_S\bar{B}_S$ in the final state					
C-odd state	$B_S\bar{B}_S$	$B_S^*\bar{B}_S^*$			$(\epsilon_{SS})^{\text{odd}}$
C-even state	$B_S^*\bar{B}_S$	$B_S\bar{B}_S^*$			$(\epsilon_{SS})^{\text{even}}$
B^+B^0 in the final state					
No definite C-parity	$B^+\bar{B}^0\pi^-$	$B^{+*}\bar{B}^0\pi^-$	$B^+\bar{B}^{0*}\pi^-$	$B^{+*}\bar{B}^{0*}\pi^-$	ϵ_+
	$B^-B^0\pi^+$	$B^{-*}B^0\pi^+$	$B^-B^{0*}\pi^+$	$B^{-*}B^{0*}\pi^+$	ϵ_-

Extraction of ϵ_{ss}

- Experimental fit of the ratios determines the relative probability of $\Upsilon(5S) \to B_s^{(*)} \bar{B}_s^{(*)}$:

$$\frac{d(N_{+-} + N_{-+} - N_{--} - N_{++})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C + A \sin(\Delta m \Delta t + \varphi),$$

$$\frac{d(N_{++} + N_{--})/d\Delta t}{d(N_{+-} + N_{-+} + N_{--} + N_{++})/d\Delta t} = C' - \frac{A}{2} \sin(\Delta m \Delta t + \varphi),$$

where

$$C = \epsilon_{+-} + \left(\epsilon_{+} + \epsilon_{-}\right) \frac{2}{4 + x^2}, \tag{1}$$

$$C' = \frac{(\epsilon_{00})^{\text{odd}} + (\epsilon_{00})^{\text{even}} + (\epsilon_{SS})^{\text{odd}} + (\epsilon_{SS})^{\text{even}}}{2} + \left(\epsilon_{+} + \epsilon_{-}\right) \frac{1 + x^2/2}{4 + x^2}. \tag{2}$$

Extraction of ϵ_{ss}

- Using isotopic invariance we finally obtain:

$$2C' - C = \epsilon_{SS} + \left(\epsilon_{+} + \epsilon_{-}\right)\frac{x^2}{4 + x^2}.$$

- The relative uncertainty from the value of the last term is less than 1%.
- It allows to determine branching ratio of the normalization channel:

$$Br(B_s \to D_s^- \pi^+) = \frac{N(B_s \to D_s^- \pi^+)}{N(B^0 \to D^- \pi^+)} \frac{N(B^0)}{N(B_s)} Br(B^0 \to D^- \pi^+).$$

Conclusions

- The approach based on two-particles wave functions describing oscillating $B\bar{B}$ system was presented.
- The upper bound on the probability of soft photon radiation in $\Upsilon(4S) \to B^0\bar{B}^0\gamma$ decay was determined.
- The branching ratio of $\Upsilon(4S) \to B^0\overline{B}^0 \to J/\psi K_S J/\psi K_S$ and the possible number of such events which can be detected at Belle II were calculated.
- The model-independent method allowing to measure $Br(B_s \to \mu^+\mu^-)$ with 1% accuracy by means of determining $Br\left(\Upsilon(5S) \to B_s^{(*)}\bar{B}_s^{(*)}\right)$ was suggested.