Parton models and limitations on the asymptotic behavior of cross-sections

O.V.Kancheli
Kurchatov.Institute ITEP

Efim Fradkin Centenial Conference 2024

Topics to be discussed

1. Partons as effective degrees of freedom of an ultra-relativistic particle
2. Partons in Regge theory and QCD
3. Limitations on parton distributions and cross-sections from the boost - invariance
4. Froissart behavior and the parton structure of the Froissart disc

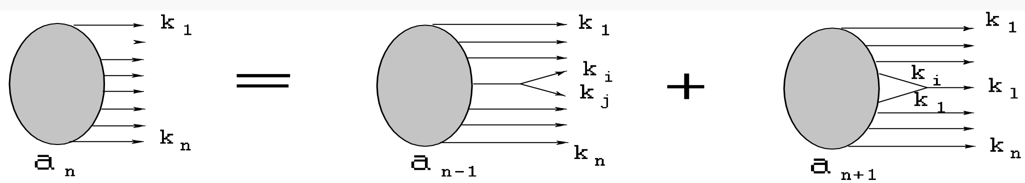
Fock equations for parton wave functions a_n

$$||P\rangle\rangle = a_1|\rightarrow\rangle + a_2|\rightrightarrows\rangle + a_3|\rightrightarrows\!\rightarrow\rangle + \cdots \Rightarrow$$

$$= \sum_{n=1}^{\infty}\int d\Omega_n(\vec{k}_i)\, a_n(P,\vec{k}_1,\ldots\vec{k}_n)\,|\vec{k}_1,\vec{k}_2,\ldots\vec{k}_n\rangle$$

$$(\hat{H}_0+\hat{H}_I)||P\rangle\rangle = E||P\rangle\rangle$$

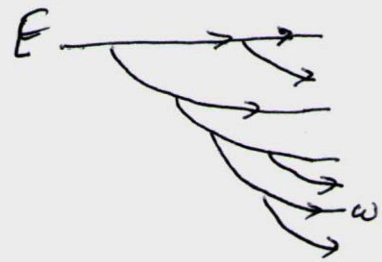
$$(E-\sum\omega_i)a_n(\vec{k}_i) = g\sum_{i\neq l}\frac{a_{n-1}(k_1,\ldots k_i+k_l,k_{n-1})}{\omega_i+\omega_l} + g\sum_{l=1}^{n}\int\frac{d^3q}{\omega(q)\,\omega(k_l-q)}\,a_{n+1}(q,k_i)$$



If we leave only parton fission in these equations --< , then we obtain the WF corresponding to the Regge pole : $a_n = x_n\,(-\frac{g}{m})^{n-1}\prod_i m^2/(m^2+k^2_{i\perp})$

QCD partons

QCD



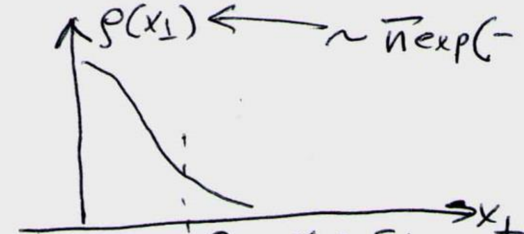
$\Delta \sim \alpha$

$$\to \quad dn \sim \alpha \left(\frac{E}{\omega}\right)^{\Delta} \frac{d\omega}{\omega} \frac{d^2k_\perp}{k_\perp^2+\mu^2}\, e^{-\frac{\ln^2(k_\perp^2+\mu^2)}{\ln E/\omega}}$$

$$\downarrow$$

$$\bar{n} \sim \left(\frac{E}{\mu}\right)^{\Delta}$$

$$\to \quad dn \sim \alpha \frac{d\omega}{\omega} \frac{d^2k_\perp}{k_\perp^2+\mu^2}$$



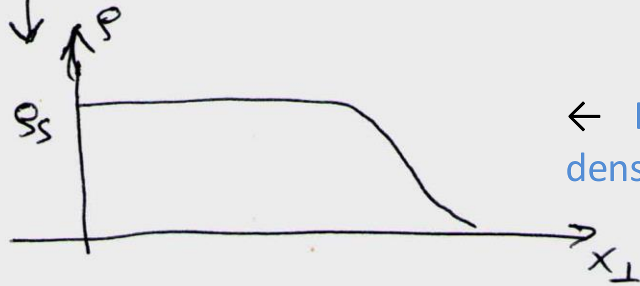
$$\rho(x_\perp) \leftarrow \sim \bar{n} \exp\left(-\frac{x_\perp^2}{R^2}\right)$$

$$R \sim \frac{\alpha}{\mu} \ln E/\mu$$

$$\frac{\partial\rho}{\partial y} = \Delta\rho - \lambda\rho^2 + r_0^2 \frac{\partial^2\rho}{\partial x_\perp^2} \quad \leftarrow \leftarrow$$

слияние партонов

$$\downarrow$$

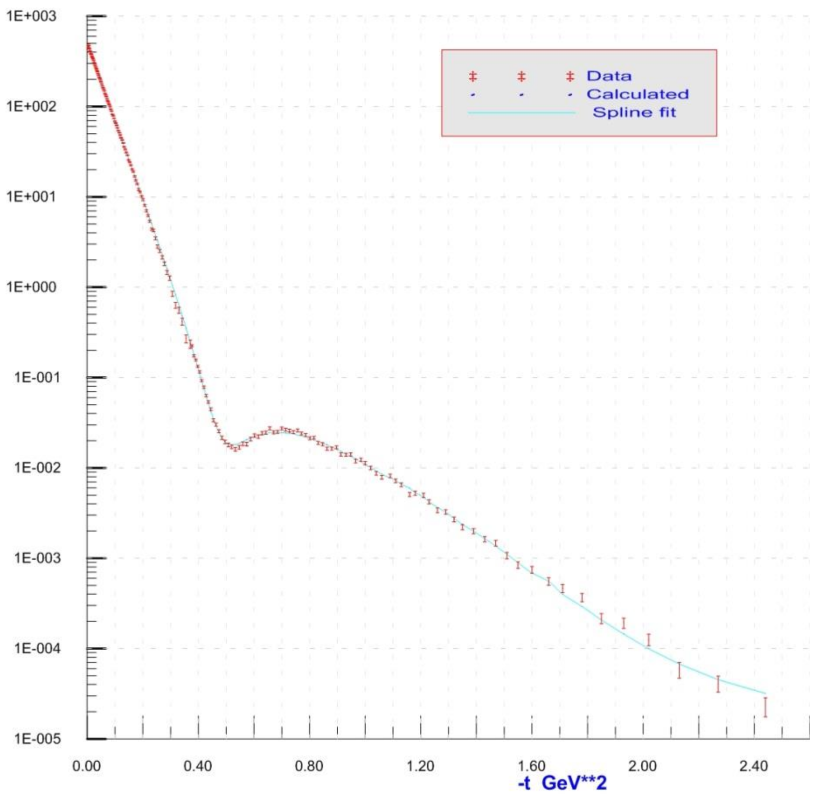


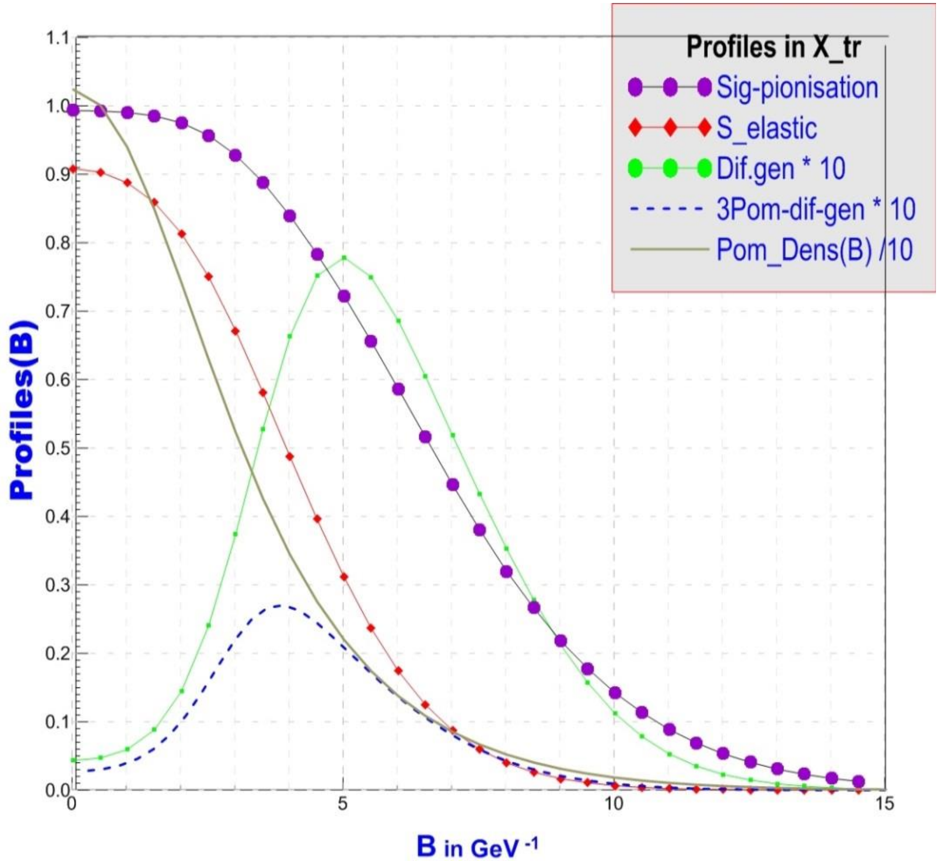
$$\rho_s \sim \frac{\Delta}{\lambda} \leftarrow \rho_s$$

← Distribution of soft parton density in the F disk and saturation

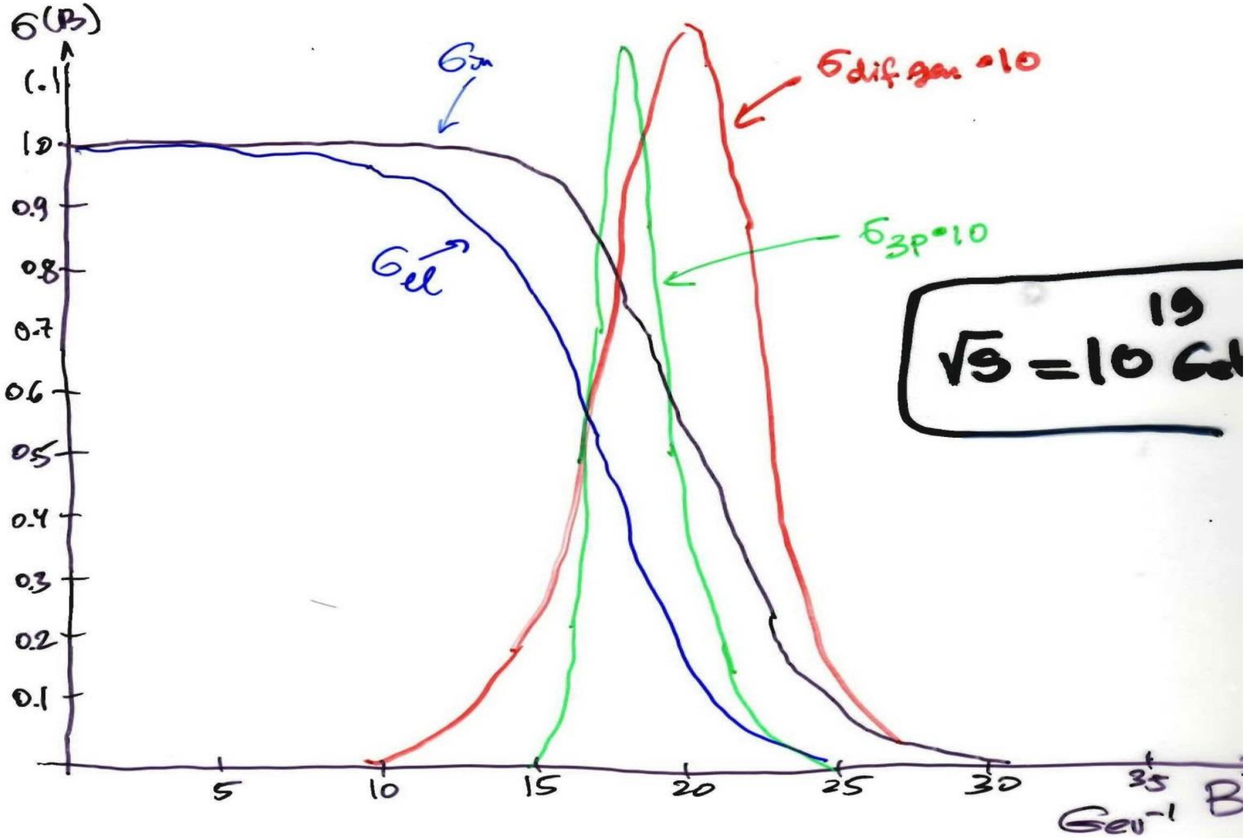
Elastic Scattering of p-p - 7 TeV (LHC)

Regge eikonal model





PP cross-sections in the same model at E $\sim$ m_{pl}



$\sigma_{tot} \simeq 820$ mb

$B \simeq 90$

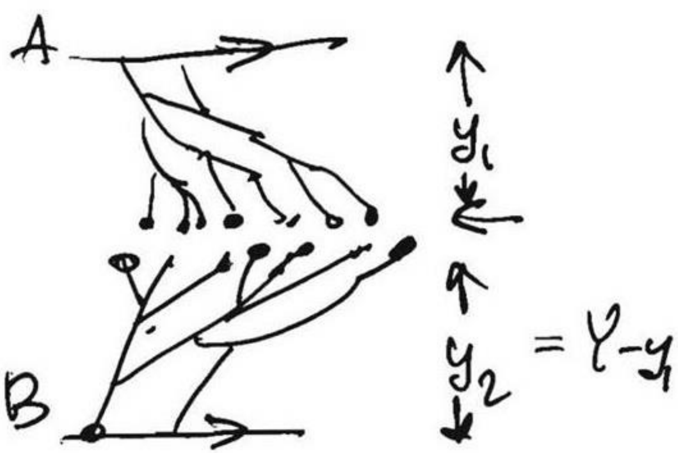
$\frac{Re}{Im} \sim 0.04$

$\sigma_{d.g} \simeq 36$ mb

$\sigma_{3P} \simeq 25$ mb

$\sigma_{elast} \simeq 360$ mb

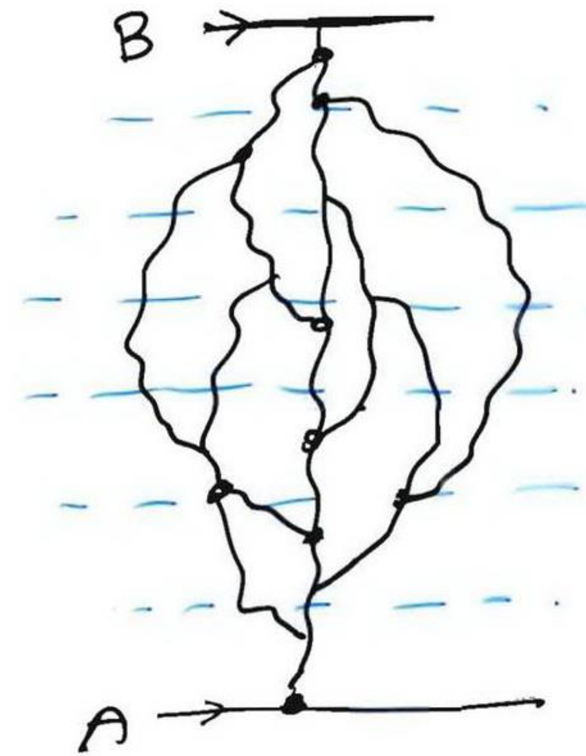
The boost invariance (BI) of cross sections in the parton model leads to the same restrictions as the t-unitarity of amplitudes ! ?



$Y = \text{const}$

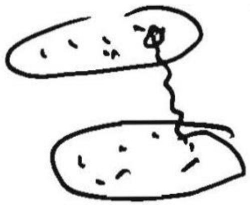
$y_1 \to y_1 + \xi$

$\sigma_n(y_1, y_2; \{k_i\}) \Longrightarrow \sigma_n(Y, \{k_i\})$



Limitation on cross-sections from **Boost Invariance**

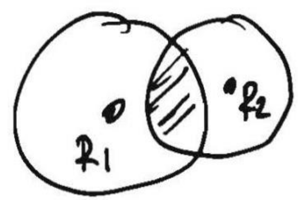
① Dilute clouds with parton gas ← Regge pole



$$\sigma_{in} \sim \sigma_0 \cdot n(y_1) \cdot n(y_2) \qquad y_2 = Y - y_1$$

$$\frac{\partial \sigma_{in}}{\partial y_1} = 0 \longrightarrow n(y) = n_0\, e^{\Delta y} \longleftrightarrow \frac{n_0}{y}\, e^{\Delta y - k_\perp^2/cy}$$

② Filled parton disks (Black disks)



$$\sigma_{in} \approx \pi \left(R(y_1) + R(y_2)\right)^2$$

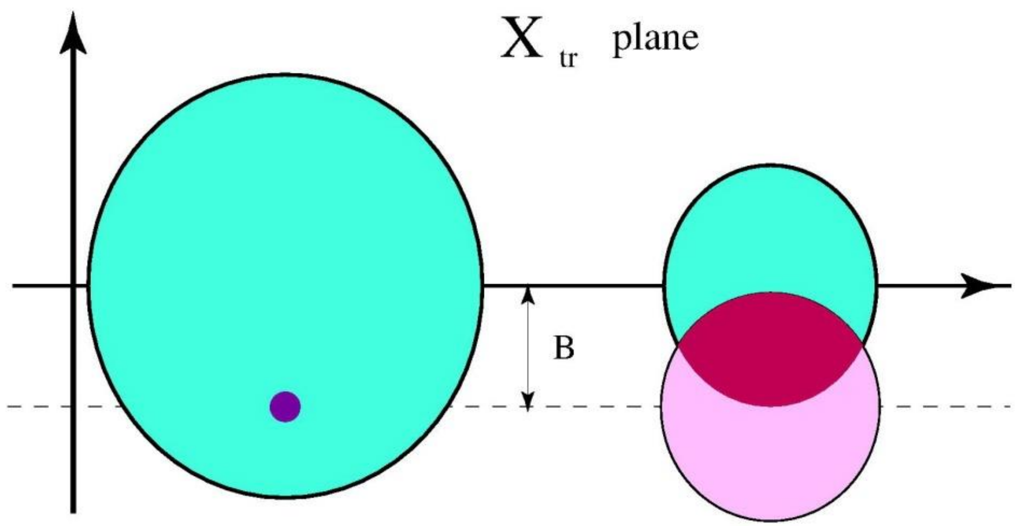
$$\frac{\partial \sigma_{in}}{\partial y} = 0 \longrightarrow R(y) = R_0 + R_1 y$$

← Froissat behavior

Parton saturation and BI
Calculating transparency T(Y) = | S(Y, $x_\perp$) $|^2$ = 1 - $\sigma_{in}(Y, x_\perp)$
in different systems

Lab. frame
↓

C.M. frame
↓



← Collision of gray F disks

T ~ 1

T ~ $e^{-c\,Y^2}$

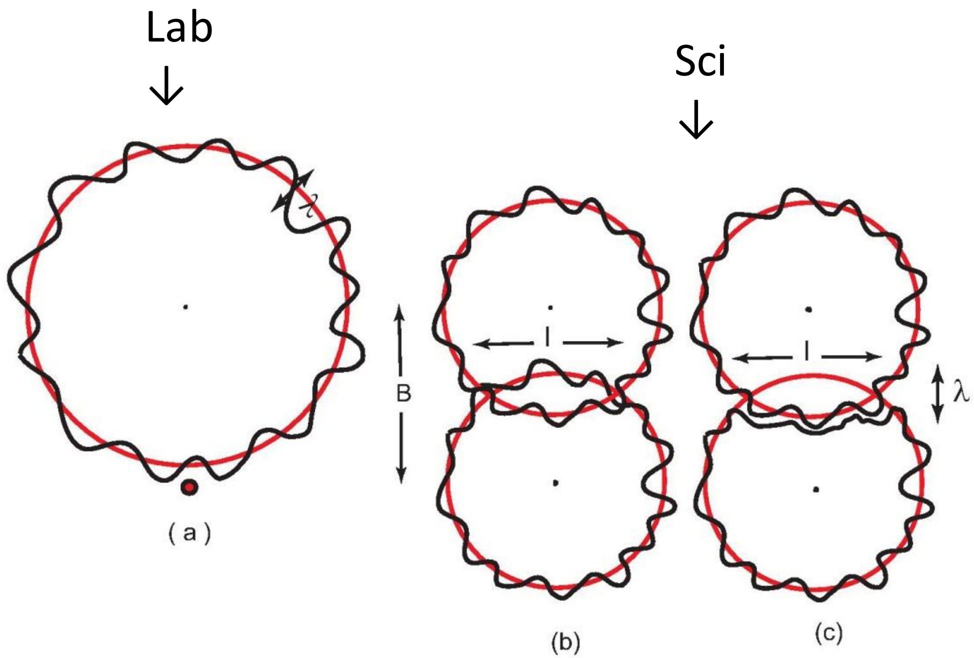
← Collision of a particle with a heavy nucleus

T ~ $e^{-c\,A^{1/3}}$

T ~ 1

Collisions near the Froissart Disk Boundary and BI

$$R(E) \sim \frac{1}{\mu} \ln E/\mu \quad ; \quad \delta R(E) \sim \sqrt{R/\mu} \sim \frac{1}{\mu}\sqrt{\ln E/\mu}$$



$$T(E,b) = |S(E,b)|^2$$

$$E \to \infty : \quad T_{lab}(E, b \simeq R(E)) \sim 1 \quad ; \quad T_{cm}(E, b \simeq R(E)) \sim e^{-(\ln E/\mu)^{1/8}}$$