Equivalent photons approximation for pp colliders: `libepa` and its applications

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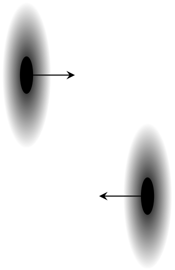
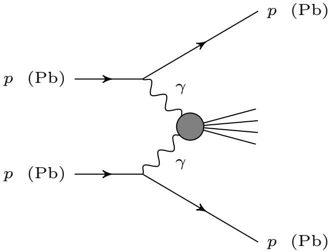
Ultraperipheral collisions (UPC) at the LHC





- It is possible to detect protons in forward detectors to reconstruct full kinematics.
- Accessible analytically with equivalent photons approximation (EPA).
- Formulae can be easily adopted for new particles (γ couples to electric charge).

Particle production in UPC

`libepa` approaches and code were developed while the authors were working on papers

- Phys. Usp. **62**, no.9, 910-919 (2019)
- JHEP **01**, 143 (2020)
- Phys. Rev. D **103**, no.3, 035016 (2021)
- JHEP **10**, 234 (2021)

Many of these results are included in the library documentation as examples.

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It was applied to semi-inclusive processes (where only one of the colliding particles remains intact, and the other disintegrates) in papers

- Eur. Phys. J. C **82**, no.11, 1055 (2022)
- Phys. Rev. D **108**, no.9, 093006 (2023)
- JETP Lett. **119**, no.1, 5-9 (2024)

Required features

- Proton form factors (including magnetic contribution)

- Fiducial cross section
  For example, typical cuts for particle pair production are
  - $p_T > \hat{p}_T$ — transverse momentum of each particle.
  - $|\eta| < \hat{\eta}$ — pseudorapidity of each particle.
  - $\sqrt{s_{\min}} < \sqrt{s} < \sqrt{s_{\max}}$— invariant mass of produced pair.
  - $\hat{\omega}_{1,\min} < \omega_1 < \hat{\omega}_{1,\max}$, $\hat{\omega}_{2,\min} < \omega_2 < \hat{\omega}_{2,\max}$ — bounds on photons energies due to forward detectors.

- Survival factor — distribution in the impact parameter space is needed

Notations!

The following notation is popular in the literature:

- $\sqrt{s}$ for the invariant mass of the colliding particles ($\Rightarrow 2E$ in what follows)
- W for the invariant mass of the produced particles, i.e. invariant mass of the colliding *photons* ($\Rightarrow \sqrt{s}$ in what follows).

UPC cross section with EPA

Many references are not provided in this talk, see arXiv:2311.01353 for details.
See the review on two photon physics: Budnev *et al*, Phys. Rep. 15, 181 (1975).

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$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \, \sigma(\gamma\gamma \to X) \, n_A(\omega_1) \, n_B(\omega_2),$$

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It is convenient to change the integration variables from the photons energies ω_1, ω_2 to the invariant mass of the produced system $\sqrt{s} = \sqrt{4\omega_1\omega_2}$ and its rapidity $y = \frac{1}{2}\ln\frac{\omega_1}{\omega_2}$:

$$\frac{\mathrm{d}\sigma(AB \to ABX)}{\mathrm{d}\sqrt{s}} = \sigma(\gamma\gamma \to X) \cdot \frac{\mathrm{d}L_{AB}}{\mathrm{d}\sqrt{s}},$$

where L_{AB} is the photon-photon luminosity in the collision of particles A and B,

$$\frac{\mathrm{d}L_{AB}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int_{-\infty}^{\infty} n_A\left(\frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(\frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right) \mathrm{d}y.$$

EPA spectra

$$\mathcal{J}_\mu = Ze \cdot \bar{\psi} \left[F_1(Q^2)\gamma_\mu - \frac{\sigma_{\mu\nu} q^\nu}{2m_\psi} F_2(Q^2) \right] \psi, \quad Q^2 \equiv -q^2, \quad \sigma_{\mu\nu} = \frac{\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu}{2},$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2), \qquad F_1(Q^2) = \frac{G_E(Q^2) + \frac{Q^2}{4m_\psi^2} G_M(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}},$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_\psi^2} F_2(Q^2), \quad F_2(Q^2) = \frac{G_M(Q^2) - G_E(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}}.$$

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$$n(\omega) = \frac{2Z^2\alpha}{\pi\omega} \int\limits_0^\infty \frac{D(q_\perp^2 + (\omega/\gamma)^2)}{(q_\perp^2 + (\omega/\gamma)^2)^2} \, q_\perp^3 \, \mathrm{d}q_\perp, \quad D(Q^2) = \frac{G_E^2(Q^2) + \frac{Q^2}{4m_\psi^2} G_M^2(Q^2)}{1 + \frac{Q^2}{4m_\psi^2}}$$

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We need distribution in impact parameter space:

$$n(\omega) = \int n(b,\omega) \, \mathrm{d}^2 b = 2\pi \int\limits_0^\infty n(b,\omega) \, b \, \mathrm{d}b, \quad n(b,\omega) = ?$$

$$G_E(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2},$$
$$G_M(Q^2) = \frac{\mu_\psi}{\left(1 + \frac{Q^2}{\Lambda^2}\right)^2},$$

where μ_ψ is the fermion magnetic moment expressed in units of $e/2m_\psi$, and Λ is a parameter of the approximation related to the fermion charge radius R through

$$R^2 = -6 \lim_{Q^2 \to 0} \frac{\mathrm{d}G_E(Q^2)}{\mathrm{d}Q^2} \quad \Rightarrow \quad \Lambda^2 = \frac{12}{R^2}.$$

Using the modern value of 0.8414 fm for the proton charge radius we get that for proton $\Lambda^2 = 0.66$ GeV^2.

$F_2(Q^2) = 0 \; (F_1(Q^2) = G_E(Q^2) = G_M(Q^2))$

$$n_2(\omega) = \frac{Z^2\alpha}{\pi\omega}\left[(4a+1)\ln\left(1+\frac{1}{a}\right) - \frac{24a^2+42a+17}{6(a+1)^2}\right], \; a = \left(\frac{\omega}{\Lambda\gamma}\right)^2,$$

$$n_2(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right) - \sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}K_1\left(b\sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}\right) - \frac{b\Lambda^2}{2}K_0\left(b\sqrt{\Lambda^2+\left(\frac{\omega}{\gamma}\right)^2}\right)\right]^2.$$

n_{2D}

If the Pauli form factor is neglected, i.e. $\mathcal{J}_\mu = ZeF_1(Q^2)\bar{\psi}\gamma_\mu\psi$, but the electric and magnetic form factors are not assumed to be equal ($G_E(Q^2) \neq G_M(Q^2)$), then

$$n_{\text{2D}}(\omega) = \frac{Z^2\alpha}{\pi\omega}\left\{\left(1+4u-2(\mu_\psi-1)\frac{u}{v}\right)\ln\left(1+\frac{1}{u}\right)\right.$$
$$+\frac{\mu_\psi-1}{(v-1)^4}\left[\frac{\mu_\psi-1}{v-1}(1+4u+3v)-2\left(1+\frac{u}{v}\right)\right]\ln\frac{u+v}{u+1}-\frac{24u^2+42u+17}{6(u+1)^2}$$
$$+(\mu_\psi-1)\,\frac{6u^2(v^2-3v+3)+3u(3v^2-9v+10)+2v^2-7v+11}{3(u+1)^2(v-1)^3}$$
$$\left.-(\mu_\psi-1)^2\,\frac{24u^2+6u(v+7)-v^2+8v+17}{6(u+1)^2(v-1)^4}\right\},\quad u=\left(\frac{\omega}{\Lambda\gamma}\right)^2,\ v=\left(\frac{2m_\psi}{\Lambda}\right)^2,$$

$$n_{\text{2D}}(b,\omega) = \frac{Z^2\alpha}{\pi^2\omega}\left[\frac{\omega}{\gamma}K_1\left(\frac{b\omega}{\gamma}\right)-\left(1+\frac{(\mu_\psi-1)\frac{\Lambda^4}{16m_\psi^4}}{\left(1-\frac{\Lambda^2}{4m_\psi^2}\right)^2}\right)\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\,K_1\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right.$$
$$+\frac{(\mu_\psi-1)\frac{\Lambda^4}{16m_\psi^4}}{\left(1-\frac{\Lambda^2}{4m_\psi^2}\right)^2}\sqrt{4m_\psi^2+\frac{\omega^2}{\gamma^2}}\,K_1\left(b\sqrt{4m_\psi^2+\frac{\omega^2}{\gamma^2}}\right)$$
$$\left.-\frac{1-\frac{\mu_\psi\Lambda^2}{4m_\psi^2}}{1-\frac{\Lambda^2}{4m_\psi^2}}\cdot\frac{b\Lambda^2}{2}\,K_0\left(b\sqrt{\Lambda^2+\frac{\omega^2}{\gamma^2}}\right)\right]^2.$$

EPA spectrum with all form factors

$$n_p(\omega) = \frac{Z^2\alpha}{\pi\omega}\left\{\left(1+4u-(\mu_\psi^2-1)\frac{u}{v}\right)\ln\left(1+\frac{1}{u}\right)-\frac{24u^2+42u+17}{6(u+1)^2}\right.$$
$$\left.-\frac{\mu_\psi^2-1}{(v-1)^3}\left[\frac{1+u/v}{v-1}\ln\frac{u+v}{u+1}-\frac{6u^2(v^2-3v+3)+3u(3v^2-9v+10)+2v^2-7v+11}{6(u+1)^2}\right]\right\},$$
$$u=\left(\frac{\omega}{\Lambda\gamma}\right)^2,\ v=\left(\frac{2m_\psi}{\Lambda}\right)^2.$$

This is the correct spectrum for proton, however its spatial counterpart has not been derived yet.

Fiducial cross section

$$\frac{\mathrm{d}\sigma_{\text{fid.}}(AB \to AB\chi^+\chi^-)}{\mathrm{d}\sqrt{s}} = \int\limits_{\max(\hat{p}_T, \tilde{p}_T)}^{\frac{\sqrt{s}}{2}\sqrt{1-\frac{4m_\chi^2}{s}}} \mathrm{d}p_T \, \frac{\mathrm{d}\sigma(\gamma\gamma \to \chi^+\chi^-)}{\mathrm{d}p_T} \, \frac{\mathrm{d}L_{AB}^{\text{fid.}}}{\mathrm{d}\sqrt{s}},$$

$$\frac{\mathrm{d}L_{AB}^{\text{fid.}}}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int\limits_{\max(-\hat{y}, \tilde{y})}^{\min(\hat{y}, \tilde{Y})} \mathrm{d}y \, n_A\left(\frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(\frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right),$$

$$\hat{y} = \ln\left(\frac{2p_T}{\sqrt{s}} \cdot \frac{\sinh\hat{\eta} + \sqrt{\cosh^2\hat{\eta} + \frac{m_\chi^2}{p_T^2}}}{1 \mp \sqrt{1 - \frac{p_T^2 + m_\chi^2}{s/4}}}\right).$$

and $\tilde{y}$ and $\tilde{Y}$ are the constraints on rapidity coming from the constraints on photon energies,

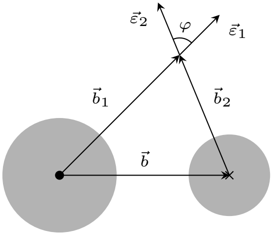
$$\tilde{y} = \max\left(\ln\frac{\hat{\omega}_{1,\min}}{\sqrt{s}/2}, \ln\frac{\sqrt{s}/2}{\hat{\omega}_{2,\max}}\right),$$

$$\tilde{Y} = \min\left(\ln\frac{\hat{\omega}_{1,\max}}{\sqrt{s}/2}, \ln\frac{\sqrt{s}/2}{\hat{\omega}_{2,\min}}\right),$$

and $\tilde{p}_T$ is an extra constraint on p_T that ensures that integrations are performed over physically meaningful domains:

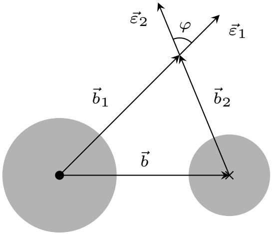
$$\hat{y} > 0, \;\; -\hat{y} < \tilde{Y}, \;\; \hat{y} > \tilde{y}.$$

Survival factor



$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X)\, n_A(b_1, \omega_1)\, n_B(b_2, \omega_2)\, P_{AB}(b),$$

Survival factor



$$\sigma(AB \to ABX) = \int_0^\infty \mathrm{d}\omega_1 \int_0^\infty \mathrm{d}\omega_2 \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \, \sigma(\gamma\gamma \to X)\, n_A(b_1, \omega_1)\, n_B(b_2, \omega_2)\, P_{AB}(b),$$

$$\frac{\mathrm{d}\sigma(AB \to ABX)}{\mathrm{d}\sqrt{s}} = \sigma_\parallel(\gamma\gamma \to X) \frac{\mathrm{d}L_{AB}^\parallel}{\mathrm{d}\sqrt{s}} + \sigma_\perp(\gamma\gamma \to X) \frac{\mathrm{d}L_{AB}^\perp}{\mathrm{d}\sqrt{s}},$$

where

$$\frac{\mathrm{d}L_{AB}^\parallel}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \int_{-\infty}^{\infty} \mathrm{d}y \, n_A\left(b_1, \frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(b_2, \frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right) P_{AB}(b) \cos^2\varphi,$$

$$\frac{\mathrm{d}L_{AB}^\perp}{\mathrm{d}\sqrt{s}} = \frac{\sqrt{s}}{2} \int \mathrm{d}^2 b_1 \int \mathrm{d}^2 b_2 \int_{-\infty}^{\infty} \mathrm{d}y \, n_A\left(b_1, \frac{\sqrt{s}}{2}\mathrm{e}^{y}\right) n_B\left(b_2, \frac{\sqrt{s}}{2}\mathrm{e}^{-y}\right) P_{AB}(b) \sin^2\varphi.$$

$$P_{pp}(b) = \left(1 - \mathrm{e}^{-\frac{b^2}{2B}}\right)^2,$$

where B is an empirical parameter depending on the collision energy E.

$$\frac{\mathrm{d}L_{pp}^{\parallel}}{\mathrm{d}\sqrt{s}} = \pi^2\sqrt{s}\int\limits_0^\infty b_1\,\mathrm{d}b_1\int\limits_0^\infty b_2\,\mathrm{d}b_2\int\limits_{-\infty}^\infty \mathrm{d}y\, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right)\, n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$

$$\times\left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}}\left[I_0\left(\frac{b_1b_2}{B}\right) + I_2\left(\frac{b_1b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}}\left[I_0\left(\frac{2b_1b_2}{B}\right) + I_2\left(\frac{2b_1b_2}{B}\right)\right]\right\},$$

$$\frac{\mathrm{d}L_{pp}^{\perp}}{\mathrm{d}\sqrt{s}} = \pi^2\sqrt{s}\int\limits_0^\infty b_1\,\mathrm{d}b_1\int\limits_0^\infty b_2\,\mathrm{d}b_2\int\limits_{-\infty}^\infty \mathrm{d}y\, n_p\left(b_1, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{y}\right)\, n_p\left(b_2, \tfrac{\sqrt{s}}{2}\,\mathrm{e}^{-y}\right)$$

$$\times\left\{1 - 2\mathrm{e}^{-\frac{b_1^2+b_2^2}{2B}}\left[I_0\left(\frac{b_1b_2}{B}\right) - I_2\left(\frac{b_1b_2}{B}\right)\right] + \mathrm{e}^{-\frac{b_1^2+b_2^2}{B}}\left[I_0\left(\frac{2b_1b_2}{B}\right) - I_2\left(\frac{2b_1b_2}{B}\right)\right]\right\}.$$

- *Developer's repository link:* https://github.com/jini-zh/libepa.
- *Licensing provisions:* GNU General Public License 3 (GPL3).
- *Programming Language:* C++, Python.
- *Solution method:* Cross sections are expressed in terms of multiple integrals over the phase space parameters and numerically calculated through recurrent application of algorithms for one-dimensional integration. Functional programming approach is used to simplify the interface and optimize the calculations.
- *Physics description:* CPC 305 (2024) 109347
- *Programmer reference:* included in the repository, see also https://jini-zh.org/libepa/libepa.html

Simple cases

The differential cross section for the production of a pair of muons with the invariant mass 100 GeV in collisions of protons with the energy 13 TeV (C++):

```
#include <epa/proton.hpp>

int main(void)
{
  const double muon_mass = 105.6583745e-3; // GeV
  const double collision_energy = 13e3; // GeV
  const double invariant_mass = 100; // GeV

  auto luminosity = epa::pp_luminosity(collision_energy);
  auto photons_to_muons = epa::photons_to_fermions(muon_mass);
  auto cross_section = epa::xsection(photons_to_muons, luminosity);

  double result = cross_section(invariant_mass); // barn/GeV
  printf("%e\n", result);

  return 0;
}
```

Cross section for the production of a pair of fermions in pp collisions with the energy $E = 13$ TeV for the fermion mass range from 90 to 250 GeV (Python interface):

```
import epa

integrate = epa.default_integrator(0)
luminosity = epa.pp_luminosity(13e3, integrator = epa.default_integrator(1))
def xsection(mass):
    return integrate(
            epa.xsection(epa.photons_to_fermions(mass), luminosity),
            2 * mass,
            6.5e3
    )
for mass in range(90, 251, 5):
    print(f'{mass:3d} {xsection(mass):19.12e}')
```

ATLAS, PLB 777, 303 (2018)

The measured value is the fiducial cross section for the $pp \to pp\mu^+\mu^-$ reaction with the following constraints:

- for 12 GeV $< \sqrt{s} <$ 30 GeV, $p_T >$ 6 GeV,
- for 30 GeV $< \sqrt{s} <$ 70 GeV, $p_T >$ 10 GeV,
- $|\eta| < 2.4$.

Experimental value:

$$\sigma_{\rm exp} = 3.12 \pm 0.07\ (\text{stat.}) \pm 0.10\ (\text{syst.})\ \text{pb}.$$

Available luminosities

Notation	$\tilde{L}$	$L_{2\mathrm{D}}$	$\tilde{L}_{2\mathrm{D}}$	L_2	$\tilde{L}_2$
Non-electromagnetic interactions	no	yes	no	yes	no
Pauli form factor	yes	no	no	no	no
Electric and magnetic form factors	distinct	distinct	distinct	equal	equal
Survival factor		$S_{2\mathrm{D}} = \frac{\mathrm{d}L_{2\mathrm{D}}/\mathrm{d}\sqrt{s}}{\mathrm{d}\tilde{L}_{2\mathrm{D}}/\mathrm{d}\sqrt{s}}$		$S_2 = \frac{\mathrm{d}L_2/\mathrm{d}\sqrt{s}}{\mathrm{d}\tilde{L}_2/\mathrm{d}\sqrt{s}}$	

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Non-electromagnetic interactions	no	yes	no	yes	no
Pauli form factor	yes	no	no	no	no
Electric and magnetic form factors	distinct	distinct	distinct	equal	equal
Survival factor		$S_{2D} = \frac{dL_{2D}/d\sqrt{s}}{d\tilde{L}_{2D}/d\sqrt{s}}$		$S_2 = \frac{dL_2/d\sqrt{s}}{d\tilde{L}_2/d\sqrt{s}}$	

Since the spatial equivalent photon spectrum for proton $n_p(b, \omega)$ is unavailable, we calculate three cross sections $\tilde{\sigma}$, $\tilde{\sigma}_{2D}$, σ_{2D} corresponding to the luminosities $\tilde{L}$, $\tilde{L}_{2D}$, L_{2D} and then obtain an estimation for the cross section taking into account non-electromagnetic interactions *and* the Pauli form factor as $\sigma = \tilde{\sigma} \cdot (\sigma_{2D}/\tilde{\sigma}_{2D})$.

Results

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`libepa` (with required accuracy for differential cross sections):

$$\begin{aligned} \tilde{\sigma}_{\text{2D}} &= 3.46 \text{ pb} \quad (10^{-3}), \\ \sigma_{\text{2D}} &= 3.31 \text{ pb} \quad (10^{-2}), \quad \Rightarrow \quad \sigma = \tilde{\sigma} \cdot \frac{\sigma_{\text{2D}}}{\tilde{\sigma}_{\text{2D}}} = 3.44 \text{ pb.} \\ \tilde{\sigma} &= 3.57 \text{ pb} \quad (10^{-3}) \end{aligned}$$

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`SuperChic2`:

$$\sigma = 3.45 \pm 0.05 \text{ pb.}$$

`SuperChic2` uses the dipole form factor approximation with $\Lambda^2 = 0.71$ GeV^2.
`libepa` cross section σ with this Λ is 3.50 pb.

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`SuperChic2`:

$$\sigma = 3.45 \pm 0.05 \text{ pb.}$$

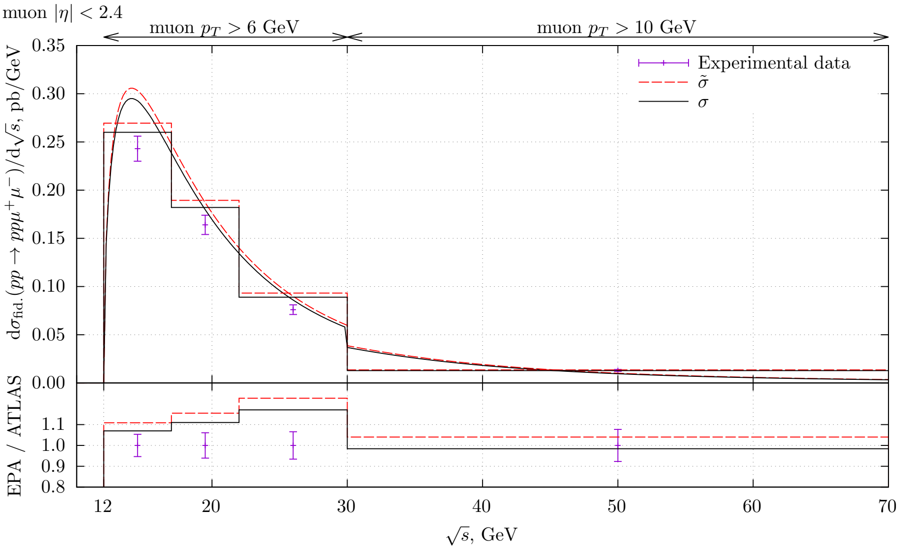
> **`SuperChic2`** uses the dipole form factor approximation with $\Lambda^2 = 0.71$ GeV^2.
> **`libepa`** cross section σ with this Λ is 3.50 pb.

`HERWIG`:

$\tilde{\sigma} = 3.56 \pm 0.05$ pb,

$\sigma = 3.06 \pm 0.05$ pb   with the help of corrections from PLB 741, 66 (2015)

libepa vs experimental data



Byproduct: convenient GSL wrappings

$$I(a) \equiv \frac{15}{a} \int\limits_0^a \mathrm{d}x\, x \int\limits_0^{\sqrt{1-\left(\frac{x}{a}\right)^2}} \mathrm{d}y\, y \int\limits_0^{\sqrt{1-\left(\frac{x}{a}\right)^2-y^2}} \mathrm{d}z\, \frac{z}{\sqrt{x^2+y^2+z^2}} = \frac{a(a+\frac{1}{2})}{(a+1)^2}.$$

```
#include <epa/epa.hpp>

int main(void) {
  // build the computation function
  auto I = [
    // precompute integrators
    integrate_x = epa::default_integrator(0),
    integrate_y = epa::default_integrator(1),
    integrate_z = epa::default_integrator(2)
  ](double a) -> double {
    return 15.0 / a * integrate_x([&](double x) -> double {
        return x * integrate_y([&](double y) -> double {
            return y * integrate_z([&](double z) -> double {
                return z / sqrt(x*x + y*y + z*z);},
                0, sqrt(1 - pow(x / a, 2) - y * y));},
            0, sqrt(1 - pow(x / a, 2)));},
        0, a);
  };

  for (double a = 1; a <= 100; a += 1)
    printf("%3.0f\t%.5f\t%.5f\n", a, I(a), a * (a + 0.5) / pow(a + 1, 2));
  return 0;
};
```

Here `I` is a closure computing $I(a)$. It captures variables `integrate_x`, `integrate_y` and `integrate_z` which are integrators used to calculate the integrals with respect to x, y and z.

Conclusions

- UPC are a great source of events for studying physics in $\gamma\gamma$ fusion, and `libepa` provides tools for it.

- `libepa` takes into account survival factor and allows to impose experimental cuts. These features are necessary for comparison with experimental data.

- Results are consistent with existing Monte Carlo codes.

`libepa` is quite different from other programs used to calculate UPC cross sections:

- `libepa` is a library rather than a standalone program.

- `libepa` relies on deterministic one-dimensional integration rather than the Monte Carlo approach.

- `libepa` is designed in the functional programming paradigm.

Conclusions

- `libepa` is a library rather than a standalone program.

  It provides a set of tools for the user to create their own computation rather than a set of pre-programmed computations with variable numerical parameters. At the same time common computations are kept simple, and cross sections for proton-proton collisions can be obtained by a single call to `libepa`.

- `libepa` relies on deterministic one-dimensional integration rather than the Monte Carlo approach.

  The fact that `libepa` uses deterministic integration rather than Monte Carlo may be an advantage or a disadvantage depending on the problem at hand and the approach to solve it. An explicit representation of the computation function in terms of mathematical expressions possibly involving recurring one-dimensional integrals over well-defined domains is required.

- `libepa` is designed in the functional programming paradigm.

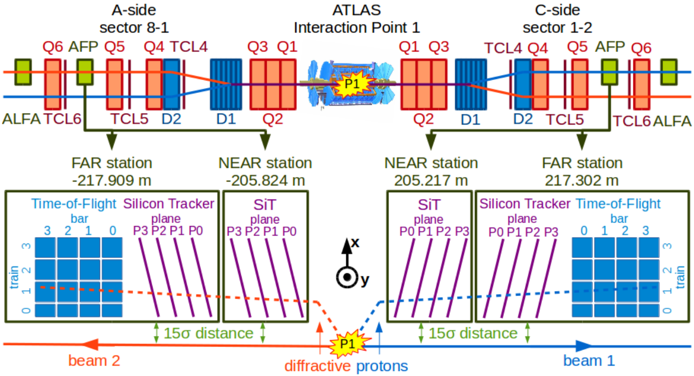
  The functional programming approach allowed for the interface when the user can replace part of a common computation with their own function, e.g., by changing the spectrum of a colliding particle, tweak the integration algorithm, or build a computation for a function not explicitly supported by the library.

  When combined with CFFI bindings to a language that features a read-evaluate-print loop (REPL), it gives the user a powerful calculator that can quickly evaluate various values of interest to the research at hand.

Backup slides

Forward detectors

[1909.10827]



	200	420
Distance from the IP, m	200	420
ξ range	0.015–0.15	0.002–0.02
6.5 TeV p energy loss, GeV	97.5–975	13–130
in the center-of-mass frame, MeV	14–141	1.9–19
0.5 PeV ^{208}Pb energy loss, TeV	7.8–78	1.0–10
in the center-of-mass frame, GeV	2.9–29	0.37–3.7