Nonfactorizable charm loop in radiative leptonic FCNC decays

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Ilia Belov[1], Dmitri Melikhov[2], Aleksandr Berezhnoy[2]

[1]INFN, Sezione di Genova, Italy
[2]SINP of Lomonosov Moscow State University, Russia

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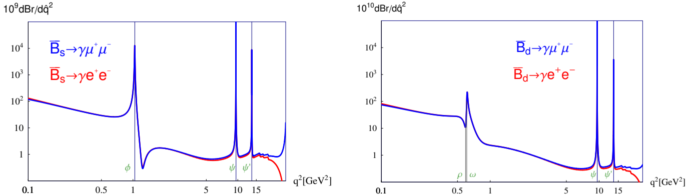
Rare radiative leptonic decays $B_{s(d)} \to \gamma\, l^+ l^-$

- Induced by the weak flavor-changing neutral currents (FCNC) $b \to s(d)$
- Forbidden at the tree level of SM $\Longrightarrow$ occur only via the diagrams with loops
  New particles might contribute to the loops $\Longrightarrow$ potential New Physics
- Small branchings of the order of $10^{-8} \div 10^{-10}$ are predicted in SM
- Searched for with the same signature as $B_{s(d)} \to l^+ l^-$, i.e. without reconstructing a photon. Currently, from [LHCb (2022)], [LHCb (2022)] the upper limit is at

$$\mathrm{Br}(B_s^0 \to \gamma\, \mu^+ \mu^-) < 2.0 \cdot 10^{-9} \qquad [m_{\mu\mu} > 4.9 \text{ GeV}]$$

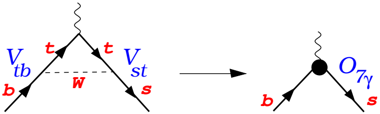
- Analogously to $B_s \to \{K^{(*)}, \phi\}\, l^+ l^-$, can be tested for the Lepton Flavor Violation
  See discussion in [D. Guadagnoli, M. Reboud and R. Zwicky (2017)].
- Theoretically, $B_{s(d)} \to \gamma\gamma$ decay has the same topology but its branching is enhanced by the factor $1/\alpha$.
  Currently, from [Belle (2015)] the upper limit is $\mathrm{Br}(B_s^0 \to \gamma\gamma) < 3.1 \cdot 10^{-6}$



Pic.: Differential branching fractions for $B_s \to \gamma l^+ l^-$ (left) and $B_d \to \gamma l^+ l^-$ (right) decays. The figures are taken from [A. Kozachuk, D. Melikhov and N. Nikitin (2018)].

FCNC $b \to s(d)\,\gamma$ and $b \to s(d)\,l^+l^-$ transitions in SM

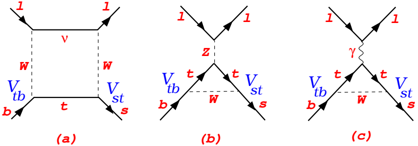
- The dominant contribution to $b \to s\,\gamma$ amplitude comes from a penguin with top quark. At scale $\mu \sim m_b$ the heavy degrees of freedom (t-quark, W-boson) are integrated out, thus leading to the local operator $\mathcal{O}_7$



$$H_{\rm top}^{(b\to s\gamma)} = -\frac{G_F}{\sqrt{2}}\, V_{tb}V_{ts}^*\, \frac{e}{8\pi^2}\, C_7(\mu)\, \mathcal{O}_{7\gamma}\,,$$

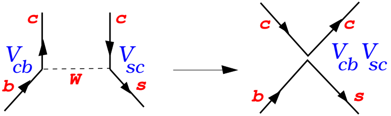
$$\mathcal{O}_{7\gamma} = \bar{s}\,\sigma_{\mu\nu}\,(1+\gamma_5)\,b\cdot F^{\mu\nu}$$

- The top-quark contribution to $b \to s\,l^+l^-$ amplitude is generated not only by the electromagnetic penguin operator $\mathcal{O}_{7\gamma}$, but also by the operators $\mathcal{O}_{9V}$ and $\mathcal{O}_{10A}$ described by the box (a) and penguin (b) diagrams



$$H_{\rm top}^{(b\to s\,ll)} = \frac{G_F}{\sqrt{2}}\,\frac{e^2}{8\pi^2}\,V_{tb}V_{ts}^*\,\Big[- 2im_b\,\frac{C_7(\mu)}{q^2}\,\bar{s}\sigma_{\mu\nu}q^\nu\,(1+\gamma_5)\,b\cdot\bar{l}\gamma^\mu l + C_{9V}(\mu)\,\mathcal{O}_{9V} + C_{10A}(\mu)\,\mathcal{O}_{10A}\Big]$$

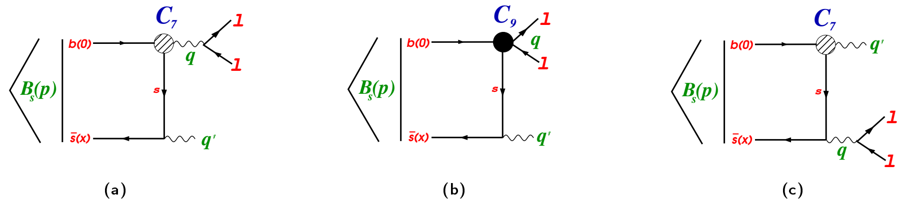
- The subleading contribution to $b \to s\,\gamma$ and $b \to s\,l^+l^-$ amplitudes comes from a charm-quark loop. The four-fermion interaction is described by the linear combination of $\mathcal{O}_1$ and $\mathcal{O}_2$ local operators, that can be rearranged into the color singlet-singlet and octet-octet operators



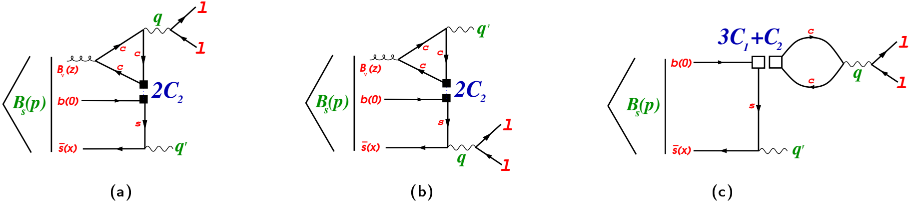
$$H_{\rm eff}^{(b\to s\bar{c}c)} = H_{\rm fact\ charm}^{[1\times1]} + H_{\rm nf\ charm}^{[8\times8]}$$

$$\begin{cases} H_{\rm fact\ charm}^{[1\times1]} = -\frac{G_F}{\sqrt{2}}\,V_{cb}V_{cs}^*\,\left(C_1 + \frac{C_2}{3}\right)\,4\bar{s}_L\gamma_\mu b_L\cdot\bar{c}_L\gamma_\mu c_L \\ H_{\rm nf\ charm}^{[8\times8]} = -\frac{G_F}{\sqrt{2}}\,V_{cb}V_{cs}^*\,(2C_2)\,4\bar{s}_L\gamma_\mu t^a b_L\cdot\bar{c}_L\gamma_\mu t^a c_L \end{cases}$$

Top and charm contributions to $\bar{B}_s \to \gamma\, l^+ l^-$ amplitude



Pic.: Diagrams describing the top-quark contributions. Dashed circle denotes the O_7 operator, solid circle — O_9 operator. In diagrams (a) and (b) the real photon is emitted by spectator s-quark; in diagram (c) the real photon is emitted from the penguin. We do not show $1/m_b$-suppressed diagrams where real or virtual photon is emitted by spectator b-quark.



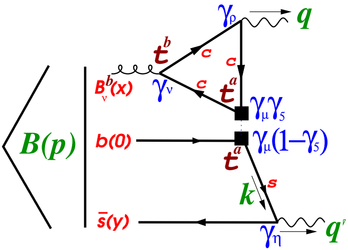
Pic.: Diagrams describing the charm-quark contributions: (a) and (b) — Nonfactorizable contributions induced by the $[8 \times 8]$ part of the Hamiltonian (solid squares), (c) Factorizable contribution induced by the $[1 \times 1]$ part of the Hamiltonian (empty squares); a similar factorizable contribution with the real photon emitted from the charm-quark loop vanishes and is not shown.

$$\mathcal{A}_{\text{nf charm}}^{(\bar{B}_s\to\gamma\gamma)} = \Big\{H_{\rho\eta}(q,q')\varepsilon_\rho(q)\,\varepsilon_\eta(q') + H_{\rho\eta}(q',q)\varepsilon_\rho(q')\,\varepsilon_\eta(q)\Big\}$$

$$\mathcal{A}_{\text{nf charm}}^{(\bar{B}_s\to\gamma ll)} = \frac{e}{Q^2}\,\Big\{H_{\rho\eta}(q,q')\bar{l}\gamma_\rho l\,\varepsilon_\eta(q') + H_{\rho\eta}(q',q)\varepsilon_\rho(q')\,\bar{l}\gamma_\eta l\Big\}$$

Since the top-quark and the charm-quark amplitudes,
$\mathcal{A}_{\text{top}}^{(\bar{B}_s\to\gamma\gamma)}$, $\mathcal{A}_{\text{top}}^{(\bar{B}_s\to\gamma ll)}$, $\mathcal{A}_{\text{nf charm}}^{(\bar{B}_s\to\gamma\gamma)}$, $\mathcal{A}_{\text{nf charm}}^{(\bar{B}_s\to\gamma ll)}$,
have the similar structure, it is convenient to describe the effect of charm as a (non-universal) addition to the Wilson coefficient C_7,

$$\boxed{C_7^{\text{eff}} = C_7 + \Delta_{V(A)}^{\text{NF}} C_7.}$$



Pic.: One of the diagrams describing charm loop contribution to $B_s \to \gamma\,\gamma^{(*)}$ decay via nonfactorizable soft gluon exchange.

- $H_{\rho\eta}$ tensor in a T-product form:

$$H_{\rho\eta}(q',q) = i\int dz e^{iq'z}\langle 0|T\{\bar{e}Q_c c(z)\gamma_\rho c(z), i\int dy\, L_{\textbf{weak}}^{b\to s\bar{c}c[8\times 8]}(y), i\int dx\, L_{\mathbf{Gcc}}(x), eQ_s\,\bar{s}(0)\gamma_\eta s(0)\}|\bar{B}_s(p)\rangle$$

- Derived expression for the $H_{\rho\eta}$ tensor in Standard Model:

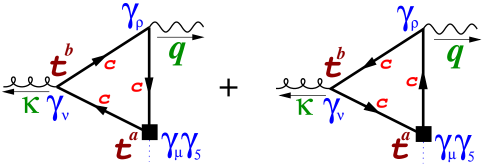
$$H_{\rho\eta}(q',q) = -\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^* e^2(2C_2)Q_sQ_c \times \frac{1}{(2\pi)^8}\int dkdye^{-i(k-q')y}dxd\kappa e^{-i\kappa x}$$
$$\Gamma_{cc}^{\mu\nu\rho(ab)}(\kappa,q)\langle 0|\bar{s}(y)\gamma^\eta\frac{\not{k}+m_s}{m_s^2-k^2}\gamma^\mu(1-\gamma^5)t^a B_\nu^b(x)b(0)|\bar{B}_s(p)\rangle.$$

- In the case of at least one real photon ($q'^2 = 0$ or $q^2 = 0$), $H_{\rho\eta}$ contains only 2 form factors $\mathcal{H}_{V,A}^{\text{NF}}(q^2,q'^2)$,

$$H_{\rho\eta}(q',q) = -\frac{G_F}{\sqrt{2}}V_{cb}V_{cs}^* e^2(2C_2)Q_sQ_c\left[\epsilon_{\rho\eta q'q}\boxed{\mathcal{H}_V^{\text{NF}}} - i\,\boxed{\mathcal{H}_A^{\text{NF}}}\left(g_{\eta\rho}\,qq' - q'_\eta q_\rho\right)\right].$$

• Two one-loop diagrams with the $\langle VVA\rangle$ structure give equal contributions to the three-point function $\Gamma^{\mu\nu\rho\,(ab)}_{cc}$.

• $\Gamma^{\mu\nu\rho\,(ab)}_{cc}$ can be parametrized with three form factors $F_{0,1,2}(\kappa, q)$. This representation works in the region of the external momenta far below the thresholds, $q^2, \kappa^2, (\kappa+q)^2 \ll 4m_c^2$.



Pic.: The $\langle VVA\rangle$ triangle one-loop diagrams for $\Gamma^{\mu\nu\rho\,(ab)}_{cc}$.

For the parametrization

$$\Gamma^{\mu\nu\rho}_{cc}(\kappa, q) = -i\left(\kappa^\mu + q^\mu\right)\epsilon^{\nu\rho\kappa q}\,F_0 - i\left(q^2\epsilon^{\mu\nu\rho\kappa} - q^\rho\epsilon^{\mu\nu q\kappa}\right)\,F_1 - i\left(\kappa^2\epsilon^{\mu\rho\nu q} - \kappa^\nu\epsilon^{\mu\rho\kappa q}\right)\,F_2$$

the convolution with the gluon field $B_\nu(x)$ might be fully given in terms of the gluon field strength $G_{\nu\alpha}(x)$,

$$\int d\kappa e^{-i\kappa x}\;\Gamma^{\mu\nu\rho\,(ab)}_{cc}(\kappa, q)B^b_\nu(x)dx = \frac{1}{4}\int d\kappa e^{-i\kappa x}\;\overline{\Gamma}^{\mu\nu\rho\alpha}_{cc}(\kappa, q)G^a_{\nu\alpha}(x)dx,$$

and therefore $\boxed{\text{no explicit use of any specific gauge for the gluon field is necessary.}}$

$$\overline{\Gamma}^{\mu\nu\rho\alpha}_{cc}(\kappa, q) = \left(\kappa^\mu + q^\mu\right)\epsilon^{\nu\rho\alpha q}\,F_0 + \left(q^\rho\epsilon^{\mu\nu\alpha q} + q^2\epsilon^{\mu\nu\rho\alpha}\right)\,F_1 + \left(\kappa^\mu\epsilon^{\alpha\nu\rho q} + \kappa^\rho\epsilon^{\alpha\mu\nu q} - \kappa q\,\epsilon^{\alpha\mu\nu\rho}\right)\,F_2,$$

where the form factors $F_{0,1,2}$ are functions of three independent invariant variables q^2, κ^2, and κq:

$$F_i\left(\kappa^2, \kappa q, q^2\right) = \frac{1}{\pi^2}\int\limits_0^1 d\xi \int\limits_0^{1-\xi} d\eta\,\frac{\Delta_i(\xi,\eta)}{m_c^2 - 2\,\xi\eta\;\kappa q - \xi(1-\xi)q^2 - \eta(1-\eta)\kappa^2}, \qquad i = 0, 1, 2,$$

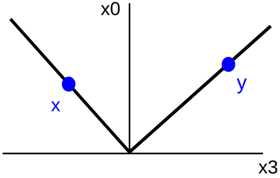
$$\Delta_0 = -\xi\eta, \quad \Delta_1 = \xi(1-\eta-\xi), \quad \Delta_2 = \eta(1-\eta-\xi).$$

Double collinear LC configuration

- In the HQ limit the double collinear kinematics dominates the NF charm-loop contribution to FCNC B-decay amplitudes.
- Applied for the calculation of NF charm-loop form factors in $B_s \to \gamma\gamma$ decay. It was shown that the leading contribution is proportional to the following combination of the 3DAs:

$$\Psi_A + \Psi_V + 2\,(W + Y_A - \tilde{Y}_A) \sim (\lambda_E^2 + \lambda_H^2).$$

[Q. Qin, Y.-L. Shen, C. Wang and Y.-M. Wang (2023)]



$x_\mu \sim \bar{n}_\mu$ [gluon coordinate],
$y_\mu \sim n_\mu$ [light quark coordinate],
$n^2 = \bar{n}^2 = 0$ and $n\bar{n} = 2$,
$v_\mu = p_\mu / M_B = \frac{1}{2}(n_\mu + \bar{n}_\mu)$.

The B-meson three-particle BS amplitude is parametrized by

$$\langle 0|\bar{s}(\tau_1 n) G_{\nu\alpha}(\tau_2 \bar{n}) \Gamma\, b(0)|\bar{B}_s(p)\rangle = \frac{f_B M_B^3}{4} \int D(\omega,\lambda)\, e^{-i(\lambda\tau_1 + \omega\tau_2)M} \mathrm{Tr}\Big\{\gamma_5 \Gamma (1 + \not{v})$$
$$\times \Big[(v_\nu\gamma_\alpha - v_\alpha\gamma_\nu)[\Psi_A - \Psi_V] - i\sigma_{\nu\alpha}\Psi_V + (n_\nu\gamma_\alpha - n_\alpha\gamma_\nu)\,(Y_A + W) + i\epsilon_{\nu\alpha\mu\beta}\, n^\mu \gamma^\beta \gamma^5 \tilde{Y}_A$$
$$+ (\bar{n}_\nu\gamma_\alpha - \bar{n}_\alpha\gamma_\nu)\,(Y_A + W) + i\epsilon_{\nu\alpha\mu\beta}\, \bar{n}^\mu \gamma^\beta \gamma^5 \tilde{Y}_A + \ldots \Big]\Big\},$$

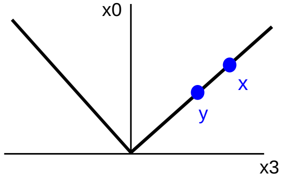
where $D(\omega,\lambda) = d\omega\, d\lambda\, \theta(\omega)\theta(\lambda)\theta(1-\omega-\lambda)$.

Collinear LC configuration

- The amplitude differs by terms $\mathcal{O}(\lambda_{B_s}/M_B)$ from the amplitude in the double-collinear approximation.
- Applied for calculation of NF charm-loop form factors in $B \to K^{(*)} l^+ l^-$, $B \to K^* \gamma$ and $B_s \to \phi l^+ l^-$ decays.

[A. Khodjamirian, T. Mannel and N. Offen (2007)]
[A. Khodjamirian, T. Mannel, A. Pivovarov and Y.-M. Wang (2010)]
[N. Gubernari, D. van Dyk and J. Virto (2020)]

- $\{\Psi_A, \Psi_V\}$ + six 3DAs $\left\{X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, W, Z\right\}$ parametrizing the kinematics-dependent part.



$x^2 = y^2 = 0$ and $x \sim y$

$x = uy$ with $u \neq 0 \Longrightarrow xy = 0$

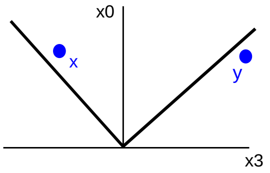
$$\langle 0|\bar{s}(y) G_{\nu\alpha}(u\,y)\Gamma\, b(0)|\bar{B}_s(p)\rangle = \frac{f_B M_B^3}{4}\int D(\omega,\lambda)\, e^{-i\lambda y p - i\omega u y p}\mathrm{Tr}\Bigg\{\gamma_5\Gamma(1+\not{v})$$
$$\times\Bigg[(p_\nu\gamma_\alpha - p_\alpha\gamma_\nu)\frac{1}{M_B}[\Psi_A - \Psi_V] - i\sigma_{\nu\alpha}\Psi_V - \frac{(y_\nu p_\alpha - y_\alpha p_\nu)}{yp}\left(X_A + \frac{\not{y}}{yp}M_B W\right)$$
$$+\frac{(y_\nu\gamma_\alpha - y_\alpha\gamma_\nu)}{yp}M_B\left(Y_A + W + \frac{\not{y}}{yp}M_B Z\right) - i\epsilon_{\nu\alpha\mu\beta}\frac{y^\mu p^\beta}{yp}\gamma^5\tilde{X}_A + i\epsilon_{\nu\alpha\mu\beta}\frac{y^\mu\gamma^\beta}{yp}\gamma^5 M_B\tilde{Y}_A\Bigg]\Bigg\},$$

where the continuity and regularity of 3BS requires the following constraints on 3DAs:

$$\int D(\omega,\lambda)\,\{X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, Z, W\} = 0, \qquad \int D(\omega,\lambda)\,\omega\,\{Z, W\} = 0, \qquad \int D(\omega,\lambda)\,\lambda\,\{Z, W\} = 0.$$

Noncollinear LC configuration

- Generic kinematics. Proposed recently in [D. Melikhov (2022), D. Melikhov (2023)].
- An accurate account for the $\mathcal{O}(\lambda_{B_s}/M_B)$ terms. $\{\Psi_A, \Psi_V\}$ + 2×6 3DAs $\left\{X_A^{(x,y)}, Y_A^{(x,y)}, \tilde{X}_A^{(x,y)}, \tilde{Y}_A^{(x,y)}, W^{(x,y)}, Z^{(x,y)}\right\}$ parametrizing the kinematics-dependent part



x [gluon coordinate] and y [light quark coordinate] are independent coordinates

$$\langle 0|\bar{s}(y)G_{\nu\alpha}(x)\Gamma\, b(0)|\bar{B}_s(p)\rangle = \frac{f_B M_B^3}{4}\int D(\omega,\lambda)\, e^{-i\lambda yp - i\omega xp}\,\mathrm{Tr}\Big\{\gamma_5\Gamma\,(1+\not{v})$$

$$\times\Big[(p_\nu\gamma_\alpha - p_\alpha\gamma_\nu)\frac{1}{M_B}[\Psi_A-\Psi_V] - i\sigma_{\nu\alpha}\Psi_V - \frac{(x_\nu p_\alpha - x_\alpha p_\nu)}{xp}\left(X_A^{(x)} + \frac{\not{x}}{xp}M_B W^{(x)}\right) +$$

$$\frac{(x_\nu\gamma_\alpha - x_\alpha\gamma_\nu)}{xp}M_B\left(Y_A^{(x)} + W^{(x)} + \frac{\not{x}}{xp}M_B Z^{(x)}\right) - i\epsilon_{\nu\alpha\mu\beta}\frac{x^\mu p^\beta}{xp}\gamma^5\tilde{X}_A^{(x)} + i\epsilon_{\nu\alpha\mu\beta}\frac{x^\mu\gamma^\beta}{xp}\gamma^5 M_B\tilde{Y}_A^{(x)} + \dots\Big]\Big\}$$

where the continuity and regularity of the 3BS requires the stronger constraints on 3DAs:

$$\int_0^{2\omega_0-\lambda} d\omega\, X^{(x)}(\omega,\lambda) = 0 \qquad \forall\lambda, \qquad\qquad \int_0^{2\omega_0-\omega} d\lambda\, X^{(y)}(\omega,\lambda) = 0 \qquad \forall\omega,$$

$$\int_0^{2\omega_0-\lambda} d\omega\,\omega^n Z^{(x)}(\omega,\lambda) = 0 \qquad \forall\lambda, \qquad\qquad \int_0^{2\omega_0-\omega} d\lambda\,\lambda^n Z^{(y)}(\omega,\lambda) = 0 \qquad \forall\omega, \quad n=0,1.$$

Model for the B-meson 3DAs

- Each of 3DAs is a function of ω and λ, which are the fractions of B-meson momentum carried by gluon and light quark, respectively. The power scaling in ω and λ is related to the conformal spins of the fields and remains the key property of the model.

Local Duality model from [V. Braun, Y. Ji and A. Manashov (2017)]

Each of DAs is written as an expansion in functions with definite twist. The twists 3 and 4 are complemented by the higher twists 5 and 6 as following:

$$\left.\begin{aligned}
\phi_3 &= \frac{105(\lambda_E^2-\lambda_H^2)}{32\omega_0^7 M_B^2}\,\lambda\omega^2\,(2\omega_0-\omega-\lambda)^2\,\theta\,(2\omega_0-\omega-\lambda)\\
\phi_4 &= \frac{35(\lambda_E^2+\lambda_H^2)}{32\omega_0^7 M_B^2}\,\omega^2\,(2\omega_0-\omega-\lambda)^3\,\theta\,(2\omega_0-\omega-\lambda)
\end{aligned}\right\}\Psi_{A,V}(\omega,\lambda)=\frac{\phi_4\pm\phi_3}{2},$$

$$\left.\begin{aligned}
\psi_4 &\sim \lambda_E^2\,\lambda\omega,\\
\widetilde{\psi}_4 &\sim \lambda_H^2\,\lambda\omega,\\
\phi_5 &\sim (\lambda_E^2+\lambda_H^2)\,\lambda,\\
\psi_5 &\sim \lambda_E^2\,\omega,\\
\widetilde{\psi}_5 &\sim \lambda_H^2\,\omega,\\
\phi_6 &\sim (\lambda_E^2-\lambda_H^2),
\end{aligned}\right\}\text{Together with } \phi_3,\phi_4 \text{ contribute to } \left\{X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, W, Z\right\},$$

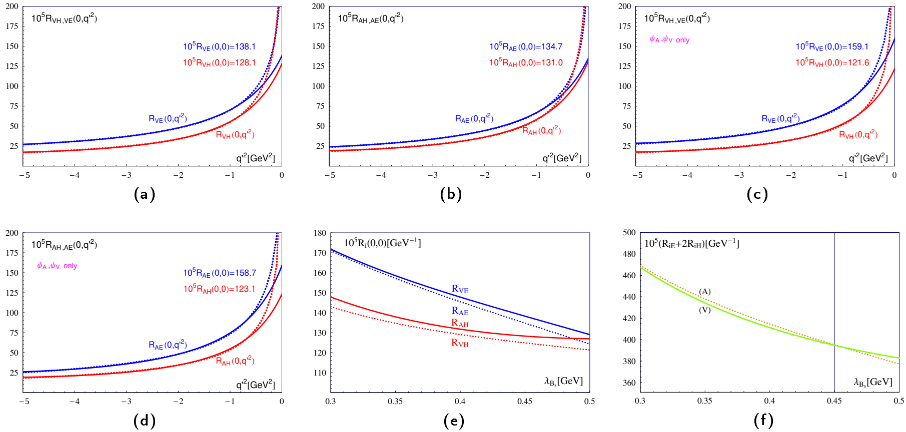
$$\omega_0 = \frac{5}{2}\frac{\lambda_{B_s}}{m_{B_s}}.$$

From QCD sum rules, $\lambda_H^2 \simeq 2\lambda_E^2$

[A. Grozin, M. Neubert (1997)]
[T. Nishikawa, K. Tanaka (2014)]

- For $\{X_A, Y_A, \tilde{X}_A, \tilde{Y}_A, W, Z\}$ the correction at large ω and λ is applied $\Longrightarrow$ our corrected model reproduces well the collinear DA magnitudes and power behaviour at small ω and λ.

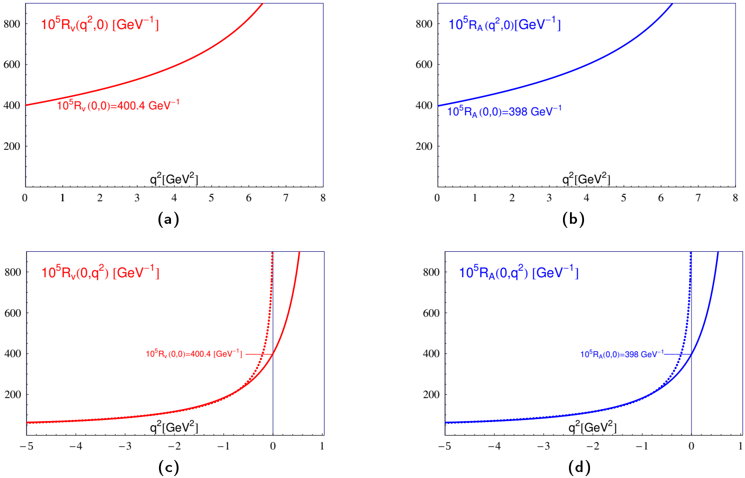
Calculated form factors. $\mathcal{H}_i^{\rm NF}(0,q'^2)=R_{iE}(0,q'^2)\lambda_E^2+R_{iH}(0,q'^2)\lambda_H^2,\quad i=A,V$



Pic.: (a—d) panels. The contributions R_{iE} and R_{iH} to the form factors H_i $[i=A,V]$ as functions of q'^2. Dashed lines show the calculation results and solid lines show the fits. (a,b): the appropriate modifications of the 3DAs X_A, Y_A, *etc.* are taken into account. (c,d): Only the contributions of Ψ_A and Ψ_V are taken into account.

Pic.: (e,f) panels. The dependence on parameter λ_{B_s} of the form factors: (e) $R_{iE,iH}(0,0)$, $i=A,V$; (f) Linear combination $R_{iE}(0,0)+2R_{iH}(0,0)$ that determines ΔC_7 taking into account approximate relation $\lambda_H^2\simeq 2\lambda_E^2$.

Calculated form factors. $\mathcal{H}_i^{\rm NF}(q^2,q'^2)=R_i(q^2,q'^2)\lambda_E^2$, $R_i(q^2,q'^2)=R_{iE}(q^2,q'^2)+2R_{iH}(q^2,q'^2),\quad i=A,V$



Pic.: $R_i(q^2,0)$ dependence (a,b) and $R_i(0,q'^2)$ dependence (c,d) for $i=V,A$. Solid lines are the fits, and dashed lines present the results of direct calculations in the case of $R_i(0,q'^2)$.

Results for the $\delta^{\rm NF} C_7$ correction

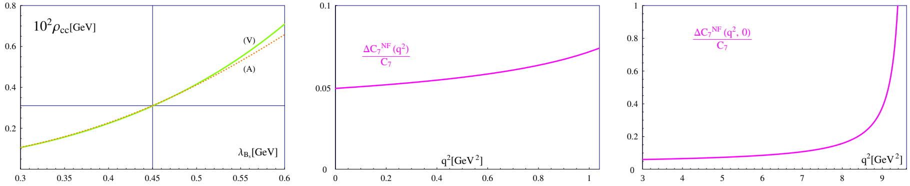
Adding charm contributions to the top contributions leads to the following sum in the form factors $A_i(q^2)$ $[i=A,V]$ parametrizing the total $B_s \to \gamma l^+ l^-$ amplitude,

$$\mathtt{A_i(q^2)} = \underbrace{\frac{2C_7}{q^2} m_b \left(F_{Ti}(q^2,0) + F_{Ti}(0,q^2)\right)}_{\text{Penguin with } t\text{-quark}} + \underbrace{C_9 \frac{F_i(q^2,0)}{M_B}}_{\text{Box with } t\text{-quark}} + \underbrace{8\pi^2 \frac{\mathcal{H}_i^{\rm F}(q^2,0)}{q^2}}_{\text{Factorizable charm}} + \underbrace{16\pi^2 Q_s Q_c \frac{\mathcal{H}_i^{\rm NF}(q^2,0) + \mathcal{H}_i^{\rm NF}(0,q^2)}{q^2}}_{\text{Nonfactorizable charm}},$$

from which the relative correction is defined as

$$\delta_i^{\rm NF} C_7 = 8\pi^2\, Q_s Q_c \frac{C_2}{C_7 m_b} \rho_{cc}^{(i)}, \quad \text{where} \quad \rho_{cc}^{(i)} = \begin{cases} \frac{\mathcal{H}_i^{\rm NF}(0,0)}{F_T(0,0)} & \text{for the case of } B_s \to \gamma\gamma \\ \frac{\mathcal{H}_i^{\rm NF}(q^2,0)+\mathcal{H}_i^{\rm NF}(0,q^2)}{F_{Ti}(q^2,0)+F_{Ti}(0,q^2)} & \text{for the case of } B_s \to \gamma l^+ l^- \end{cases}$$

- Numerically, $\delta_A^{\rm NF} C_7(q^2) \simeq \delta_V^{\rm NF} C_7(q^2)$.
- The nearest hadron singularities are at $4M_K^2$ and $M_{J/\psi}^2$.



(a) (b) (c)

Pic.: (a) The functions $\rho_{cc}^{(i)}$ at $q^2=0$ versus λ_{B_s}, $i=V,A$. (b–c) The relative NF correction $\delta_V^{\rm NF} C_7(q^2)$: (b) the full result at $0<q^2<4M_K^2$; (c) $\delta_V^{\rm NF} C_7(q^2,0)$ which dominates in $\delta_V^{\rm NF} C_7(q^2)$ at $q^2 > 3$ GeV^2.

Conclusions

(i) We derived and made use of the expression for the $\langle VVA\rangle$ quark loop that is fully given in terms of the gluon field strength $G_{\mu\nu}(x)$. This has an advantage that no explicit use of any specific gauge for gluon field is necessary.

(ii) We studied the generic noncollinear 3BS of the B-meson. This quantity contains new Lorentz structures and new 3DAs compared to collinear and double-collinear 3BS. We took into account constraints from the requirement of analyticity and continuity and implemented proper modifications of the corresponding 3DAs $X_i(\omega,\lambda)$ at large values of their arguments.

• We derived analytical expressions for the form factors $\mathcal{H}_i^{\rm NF}(q^2,q'^2)$, $i=A,V$, describing NF contribution of charm loops to the amplitude of the B_s meson transition into two photons

$$\mathcal{H}_i^{\rm NF}(q^2,q'^2)=\lambda_E^2 R_{iE}(q^2,q'^2)+\lambda_H^2 R_{iH}(q^2,q'^2),\qquad i=A,V$$

We interpolated the results of $\mathcal{H}_i^{\rm NF}(q^2,q'^2)$ calculations in the (q^2,q'^2) rectangular region [sufficiently far from the quark thresholds] with a formula, which takes into account the presence of the poles at $q^2=M^2_{J/\psi}$ and $q'^2=M^2_\phi$.

• The contribution of NF charm in $B_s\to\gamma ll$ decay can be conveniently treated as the q^2-correction to the Wilson coefficient C_7, while the contribution of F charm — as the q^2-dependent correction to the Wilson coefficient C_9, such that both relative corrections are positive:

$$\boxed{\Delta^{\rm NF}C_7(q^2)/C_7>0 \text{ at } q^2<4M_K^2,}\qquad \Delta^{\rm F}C_9(q^2)/C_9>0 \text{ at } q^2<M^2_{J/\psi}.$$

• Our numerical results for the form factors $\mathcal{H}_i^{\rm NF}(q^2,q'^2)$ depend sizeably on the precise value of the parameter λ_{B_s} and exhibit about 10% accuracy for a fixed value of λ_{B_s}. For the $B\to\gamma\gamma$ amplitude an explicit $\delta C_7(\lambda_{B_s})$ dependence was calculated, from which for our benchmark point $\lambda^0_{B_s}=0.45$ GeV we found

$$\boxed{\delta^{\rm NF}C_7(\lambda^0_{B_s})=0.045\pm0.004.}$$

For λ_{B_s} in the range $0.3<\lambda_{B_s}(\mathrm{GeV})<0.6$, δC_7 covers the range 2÷10 %.

Thank you for your attention!

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