New slow-roll equations for inflationary models with the Gauss-Bonnet term

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based on E.O. Pozdeeva, M.A. Skugoreva, A.V. Toporensky, S.Yu. Vernov, *arXiv:2403.06147*

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We consider models with the Gauss–Bonnet term, described by the following action:

$$S = \int d^4x\sqrt{-g}\left[U_0 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi) - \frac{1}{2}\xi(\phi)\mathcal{G}\right], \qquad (1)$$

where $U_0 = \frac{M_{\rm Pl}^2}{2} = \frac{1}{16\pi G}$,
the functions $V(\phi)$ and $\xi(\phi)$ are differentiable ones,
R is the Ricci scalar and

$$\mathcal{G} = R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

is the Gauss–Bonnet term.

The perturbation theory for such types of models has been developed in
C. Cartier, J.c. Hwang, E.J. Copeland, Phys. Rev. D **64** (2001) 103504
J. c. Hwang and H. Noh, Phys. Rev. D **71** (2005) 063536

STRING THEORY MOTIVATED GRAVITY

Einstein–Gauss–Bonnet gravity models are motivated by α' corrections in string theories. The most general Lagrangian density at the next to leading order in the parameter α' reads[1]:

$$L_{string} = -\frac{\lambda}{2}\alpha'\xi(\phi)\left[c_1\mathcal{G} + c_2 G^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + c_3\Box\phi\phi^{;\mu}\phi_{;\mu} + c_4(\phi^{;\mu}\phi_{;\mu})^2\right],$$

- $\mathcal{G}$ is the Gauss–Bonnet term:

$$\mathcal{G} = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta},$$

- $G^{\mu\nu} \equiv R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$ is the Einstein tensor,
- $\alpha' = \lambda_s^2$, where λ_s is the fundamental string length scale;
- c_i are constants (we will consider the case $c_k = 0$, $k = 2, 3, 4$);
- λ is an additional parameter allowing for different species of string theories, $\lambda = -1/4$ for the Bosonic string and $\lambda = -1/8$ for Heterotic string respectively.

[1] D.J. Gross and J.H. Sloan, Nucl. Phys. B **291** (1987) 41;
R.R. Metsaev and A.A. Tseytlin, Nucl. Phys. B **293** (1987) 385.

INFLATIONARY MODELS

Inflationary models with the Gauss–Bonnet term have been studied in many papers:

Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81**, 123520 (2010)
A. De Felice, S. Tsujikawa, J. Elliston, R. Tavakol, JCAP **08** (2011) 021
M. De Laurentis, M. Paolella and S. Capozziello, Phys. Rev. D **91** (2015) 083531,
G. Hikmawan, J. Soda, A. Suroso, and F.P. Zen, Phys. Rev. D **93**, 068301 (2016)
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) 063519
S. Koh, B.H. Lee and G. Tumurtushaa, Phys. Rev. D **95** (2017) 123509,
K. Nozari and N. Rashidi, Phys. Rev. D **95** (2017) 123518
S.D. Odintsov and V.K. Oikonomou, Phys. Rev. D **98** (2018) 044039
Z. Yi and Y. Gong, Universe **5** (2019) 200
E.O. Pozdeeva, *Eur. Phys. J.* C **80** (2020) 612
E.O. Pozdeeva, S.Yu. Vernov, Eur. Phys. J. C **81** (2021) 633
R. Kawaguchi and S. Tsujikawa, Phys. Rev. D **107** (2023) 063508
S.D. Odintsov, V.K. Oikonomou, F.P. Fronimos, Phys. Rev. D **107** (2023) 08
Yogesh, I.A. Bhat and M.R. Gangopadhyay, [arXiv:2408.01670].

EVOLUTION EQUATIONS IN THE FLRW METRIC

In the spatially flat Friedmann–Lemaître–Robertson–Walker metric with

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2),$$

we obtain the following system of evolution equations

$$12H^2\left(U_0 - 2\xi_{,\phi}\psi H\right) = \psi^2 + 2V, \tag{2}$$

$$4\dot{H}\left(U_0 - 2\xi_{,\phi}\psi H\right) = 4H^2\left(\xi_{,\phi\phi}\psi^2 + \xi_{,\phi}\dot{\psi} - H\xi_{,\phi}\psi\right) - \psi^2, \tag{3}$$

$$\dot{\psi} + 3H\psi = -V_{,\phi} - 12H^2\xi_{,\phi}\left(\dot{H} + H^2\right), \tag{4}$$

where $H = \dot{a}/a$ is the Hubble parameter, $a(t)$ is the scale factor, $\psi = \dot{\phi}$, dots denote the derivatives with respect to the cosmic time t, and $A_{,\phi} \equiv \frac{dA}{d\phi}$ for any function $A(\phi)$.

As usually for inflationary model construction, the e-folding number $N = \ln(a/a_e)$, where a_e is a constant, is considered as a measure of time during inflation.
Using the relation $\frac{d}{dt} = H\,\frac{d}{dN}$ and introducing $\chi = \frac{\psi}{H}$, one get the following system:

$$
\begin{aligned}
\frac{d\phi}{dN} &= \chi, \\
\frac{d\chi}{dN} &= \frac{1}{H^2\left(B - 2\xi_{,\phi}H^2\chi\right)}\left\{3\left[3 - 4\xi_{,\phi\phi}H^2\right]\xi_{,\phi}H^4\chi^2 \right. \\
&\quad \left. + \left[3B + 2\xi_{,\phi}V_{,\phi} - 6U_0\right]H^2\chi - \frac{V^2}{U_0}X\right\} - \frac{\chi}{2H^2}\frac{dH^2}{dN}, \qquad (5) \\
\frac{dH^2}{dN} &= \frac{H^2}{2\left(B - 2\xi_{,\phi}H^2\chi\right)}\left[\left(4\xi_{,\phi\phi}H^2 - 1\right)\chi^2 - 16\xi_{,\phi}H^2\chi - 4\frac{V^2}{U_0^2}\xi_{,\phi}X\right],
\end{aligned}
$$

where $B = 12\xi_{,\phi}^2H^4 + U_0$ and $X = \frac{U_0^2}{V^2}\left(12\xi_{,\phi}H^4 + V_{,\phi}\right)$.

Following
Z. K. Guo and D. J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897],
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) no.6, 063519 arXiv:1512.04768],
E. O. Pozdeeva, M. R. Gangopadhyay, M. Sami, A. V. Toporensky and S. Y. Vernov, Phys. Rev. D **102** (2020) no.4, 043525 [arXiv:2006.08027],
S. D. Odintsov and T. Paul, Phys. Dark Univ. **42** (2023), 101263 [arXiv:2305.19110],
we consider the slow-roll parameters:

$$\varepsilon_1=-\frac{\dot{H}}{H^2}=-\frac{d\ln(H)}{dN},\qquad \varepsilon_{i+1}=\frac{d\ln|\varepsilon_i|}{dN},\quad i\geqslant 1, \tag{6}$$

$$\delta_1=\frac{2}{U_0}\xi_{,\phi}H\psi=\frac{2}{U_0}\xi_{,\phi}H^2\chi,\qquad \delta_{i+1}=\frac{d\ln|\delta_i|}{dN},\quad i\geqslant 1. \tag{7}$$

The spectral index n_s and the tensor-to-scalar ratio r are connected with the slow-roll parameters as follows[2],

$$n_s = 1 - 2\varepsilon_1 - \frac{2\varepsilon_1\varepsilon_2 - \delta_1\delta_2}{2\varepsilon_1 - \delta_1} = 1 - 2\varepsilon_1 - \frac{d\ln(r)}{dN} = 1 + \frac{d}{dN}\ln\left(\frac{H^2}{U_0 r}\right), \quad (8)$$

$$r = 8|2\varepsilon_1 - \delta_1|. \quad (9)$$

The scalar perturbations amplitude

$$A_s = \frac{H^2}{\pi^2 U_0\, r}\,. \quad (10)$$

The inflationary parameters are constrained by the combined analysis of Planck, BICEP/Keck and other observations as follows[3]:

$$A_s = (2.10 \pm 0.03) \times 10^{-9}, \qquad n_s = 0.9654 \pm 0.0040\,, \qquad r < 0.028\,.$$

[2] Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897]

[3] G. Galloni, N. Bartolo, S. Matarrese, M. Migliaccio, A. Ricciardone and N. Vittorio, JCAP **04** (2023) 062 [arXiv:2208.00188].

Using system (5), we obtain the parameter $\varepsilon_1(N)$ in the following form:

$$\varepsilon_1 = -\frac{1}{2H^2}\frac{dH^2}{dN} = \frac{3}{\psi^2+2V}\left[\psi^2 - 4H^2\left(\xi_{,\phi\phi}\psi^2 + \xi_{,\phi}\dot{\psi} - H\xi_{,\phi}\psi\right)\right] \tag{11}$$

Using definitions of the slow-roll parameters, we get

$$V = U_0 H^2 \left[6 - 2\varepsilon_1 - 5\delta_1 - \delta_1\left(\delta_2 - \varepsilon_1\right)\right]. \tag{12}$$

and

$$\chi^2 = 2U_0\left[2\varepsilon_1 - \delta_1 + \delta_1\left(\delta_2 - \varepsilon_1\right)\right]. \tag{13}$$

It is useful, to rewrite evolution equations in terms of the slow-roll parameters. Equations (2) and (3) are equivalent to

$$12U_0H^2\left(1-\delta_1\right) = \psi^2 + 2V = \frac{U_0^2\delta_1^2}{4\xi_{,\phi}^2H^2} + 2V\,, \tag{14}$$

$$4U_0\dot{H}\left(1-\delta_1\right) = -\psi^2 + 2U_0H^2\delta_1\left(\delta_2+\varepsilon_1-1\right).$$

THE STANDARD SLOW-ROLL APPROXIMATION

The standard approximate equations have been proposed in
Z.K. Guo, D.J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897]
and described via the effective potential in
E.O. Pozdeeva, M.R. Gangopadhyay, M. Sami, A.V. Toporensky, S.Yu. Vernov, Phys. Rev. D **102** (2020) 043525 [arXiv:2006.08027].
This way assumes that all inflationary parameters are negligibly small and can be removed from equations. In this slow-roll approximation, the leading order equations have the following form:

$$H^2 \simeq \frac{V}{6U_0}, \tag{15}$$

$$\dot{H} \simeq -\frac{\dot{\phi}^2}{4U_0} - \frac{\xi_{,\phi} H^3 \dot{\phi}}{U_0}, \tag{16}$$

$$\dot{\phi} \simeq -\frac{V_{,\phi} + 12\xi_{,\phi} H^4}{3H}. \tag{17}$$

THE EFFECTIVE POTENTIAL

To analyze the stability of de Sitter solutions in model (1) the effective potential has been proposed[4]:

$$V_{eff}(\phi) = -\frac{U_0^2}{V(\phi)} + \frac{1}{3}\xi(\phi). \tag{18}$$

Using slow-roll approximation and the effective potential, we get the following useful expressions:

$$\frac{dH}{dN} \simeq -\frac{H}{U_0}V_{,\phi}V_{eff\,,\phi}\,, \quad \chi = \frac{d\phi}{dN} \simeq -2\frac{V}{U_0}V_{eff\,,\phi}, \quad \frac{dN}{d\phi} \simeq -\frac{U_0}{2V\,V_{eff\,,\phi}}\,.$$

In terms of the effective potential, the slow-roll parameters are as follows:

$$\varepsilon_1 = \frac{V_{,\phi}}{U_0}V_{eff\,,\phi}\,, \quad \varepsilon_2 = -\frac{2V}{U_0}V_{eff\,,\phi}\frac{d}{d\phi}\ln\left(V_{,\phi}V_{eff\,,\phi}\right),$$

$$\delta_1 = -\frac{2V^2}{3U_0^3}\xi_{,\phi}V_{eff\,,\phi}\,, \qquad \delta_2 = -\frac{2V}{U_0}V_{eff\,,\phi}\frac{d}{d\phi}\ln\left(V^2\xi_{,\phi}V_{eff\,,\phi}\right).$$

[4] E.O. Pozdeeva, M. Sami, A.V. Toporensky and S.Yu. Vernov, Phys. Rev. D **100** (2019) 083527 [arXiv:1905.05085].

So, $|\epsilon_1| \ll 1$ and $|\delta_1| \ll 1$ if the function $V_{eff\,,\phi}$ is sufficiently small. It allows us to use the effective potential for construction of inflationary scenarios.

Inflationary parameters are:

- the tensor-to-scalar ratio r

$$r = 16\frac{V^2}{U_0^3}\left(V_{eff\,,\phi}\right)^2. \tag{19}$$

- the scalar perturbations amplitude A_s

$$A_s \approx \frac{V}{6\pi^2 U_0^2\, r} = \frac{U_0}{96\pi^2 V\left(V_{eff\,,\phi}\right)^2}. \tag{20}$$

- the spectral index n_s

$$n_s = 1 + \frac{2}{U_0}\left(2V\,V_{eff\,,\phi\phi} + V_{,\phi}V_{eff\,,\phi}\right) = 1 + \frac{d}{dN}\ln\left(A_s\right). \tag{21}$$

PROBLEMS OF THE STANDARD SLOW-ROLL APPROXIMATION

It has been shown by numerical calculations in
C. van de Bruck and C. Longden, Phys. Rev. D **93** (2016) no.6, 063519 [arXiv:1512.04768]
that the model with a fourth degree monomial potential $V = V_0\phi^4$ and $\xi = \xi_0/V$, where V_0 and ξ_0 are some positive constants, has no exit from inflation, whereas the standard slow-roll approximation shows that this exit does exist, so the approximation proposed in
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Z.K. Guo and D.J. Schwarz, Phys. Rev. D **81** (2010), 123520 [arXiv:1001.1897]
is not accurate at the end of inflation.

It is important to improve the slow-roll approximation and compare approximate results with results of numerical calculations without any approximation.

We propose two new slow-roll approximations.

Multiplying (14) to H^2 and substituting ψ in terms of the slow-roll parameter δ_1, we obtain:

$$12\,U_0\,(1-\delta_1)\,H^4 - 2VH^2 - \frac{\delta_1^2 U_0^2}{4\,\xi_{,\phi}^2} = 0\,.$$

We consider the positive H^2 at $\delta_1 < 1$:

$$H^2 = \frac{V}{12\,U_0\,(1-\delta_1)} + \frac{\sqrt{V^2\xi_{,\phi}^2 + 3\,U_0^3\delta_1^2\,(1-\delta_1)}}{12\,U_0\,(1-\delta_1)\,|\xi_{,\phi}|}\,. \qquad (22)$$

NEW APPROXIMATION I

We expand the obtained expression (22) to series with respect to the slow-roll parameter $\delta_1 \ll 1$:

$$H^2 \approx \frac{V}{6\,U_0} + \frac{V}{6\,U_0}\delta_1 + \mathcal{O}(\delta_1^2)\,. \quad (23)$$

For $\dot{H}$, we get

$$\dot{H} \simeq \, -\frac{H^2\delta_1}{2} - \frac{U_0\delta_1^2}{16\xi_{,\phi}^2 H^2} - \frac{H^2\delta_1^2}{2}\,. \quad (24)$$

We neglect terms proportional to $\ddot{\phi}$ and $\dot{\phi}^2$ in the field equation and get the following approximate equation:

$$\frac{3U_0\delta_1}{2\xi_{,\phi}} = \, -V_{,\phi} - 12H^2\xi_{,\phi}\left(\dot{H} + H^2\right). \quad (25)$$

Substituting H^2 and $\dot{H}$ into here and neglecting terms, proportional to δ_1^n, where $n \geqslant 2$, we get

$$\delta_1(\phi) = -\frac{2\,V^2\xi_{,\phi}\,V_{eff\,,\phi}}{V^2\xi_{,\phi}^2 + 3\,U_0^3}\,.$$

The knowledge of $\delta_1(\phi)$ allows us to obtain $H(\phi)$ and $\chi(\phi)$.

$$H^2 \simeq \frac{V}{6U_0}\left[1 - \frac{2V^2\xi_{,\phi}V_{eff\,,\phi}}{V^2\xi_{,\phi}^2 + 3U_0^3}\right] = \frac{V\left(9U_0^3 - 6U_0^2\xi_{,\phi}\,V_{,\phi} + \xi_{,\phi}^2V^2\right)}{18U_0\left(3U_0^3 + \xi_{,\phi}^2V^2\right)}. \tag{26}$$

$$\chi = \frac{d\phi}{dN} = \frac{U_0\delta_1}{2\xi_{,\phi}H^2} \simeq -\frac{6U_0^2VV_{eff\,,\phi}}{V^2\xi_{,\phi}^2 + 3U_0^3 - 2V^2\xi_{,\phi}V_{eff\,,\phi}}\,. \tag{27}$$

We get the slow-roll parameters as functions of ϕ:

$$\varepsilon_1(\phi) = -\frac{1}{2}\frac{d\phi}{dN}\frac{d\ln(H^2)}{d\phi}, \quad \varepsilon_2(\phi) = \frac{U_0\delta_1}{2\xi_{,\phi}H^2\varepsilon_1}\varepsilon_{1,\phi}\,, \quad \delta_2 = \frac{U_0}{2H^2\xi_{,\phi}}\delta_{1,\phi}.$$

NEW APPROXIMATION II

The second way to get $\delta_1(\phi)$ is the following.
We neglect the term proportional to δ_1^2 and get a nonzero solution:

$$H^2 = \frac{V}{6U_0(1-\delta_1)}. \qquad (28)$$

Considering the differential of H^2 and using the definition of the slow-roll parameters, we get

$$\frac{dH^2}{dN} = \frac{V_{,\phi}\,\delta_1}{12\xi_{,\phi}H^2(1-\delta_1)} + \frac{V\delta_1\delta_2}{6U_0(1-\delta_1)^2} = \frac{U_0\;V_{,\phi}\,\delta_1}{2\xi_{,\phi}V} + \frac{V\delta_1\delta_2}{6U_0(1-\delta_1)^2}\,.$$

and

$$\varepsilon_1 = -\frac{3\,U_0^2\,V_{,\phi}}{2V^2\xi_{,\phi}}\delta_1(1-\delta_1) - \frac{\delta_1\delta_2}{2\,(1-\delta_1)}\,. \qquad (29)$$

From definition of slow-roll parameters, we get

$$\dot{\psi} \approx \frac{U_0\delta_1}{2\xi_\phi}\left(\delta_2+\varepsilon_1-\frac{3U_0^2\xi_{,\phi\phi}\delta_1}{V\,\xi_{,\phi}^2}\right)\,. \qquad (30)$$

Substituting H^2, ϵ_1, $\dot{\psi}$ into the field equation, multiplying it to $(1-\delta_1)^2$, and supposing that any products of the slow-roll parameters are negligible, we get

$$\delta_1(\phi) = -\frac{2\xi_{,\phi}\left(3U_0^2 V_{,\phi} + V^2\xi_{,\phi}\right)}{9U_0^2\left(U_0 - \xi_{,\phi}V_{,\phi}\right)}. \tag{31}$$

Now we can express H^2, χ, $N_{,\phi}$, and ϵ_1 via ϕ:

$$H^2(\phi) \simeq \frac{3U_0V(U_0 - \xi_{,\phi}V_{,\phi})}{2(9U_0^3 - 3U_0^2\xi_{,\phi}V_{,\phi} + 2\xi_{,\phi}^2V^2)}, \tag{32}$$

$$\chi = \frac{U_0\delta_1}{2\xi_{,\phi}H^2} \simeq -\frac{2(3U_0^2V_{,\phi} + \xi_{,\phi}V^2)(9U_0^3 - 3U_0^2\xi_{,\phi}V_{,\phi} + 2\xi_{,\phi}^2V^2)}{27U_0^2V(U_0 - \xi_{,\phi}V_{,\phi})^2}, \tag{33}$$

$$\frac{dN}{d\phi} = \chi^{-1}, \quad \varepsilon_1(\phi) = -\frac{\chi}{2}\frac{d\ln(H^2)}{d\phi}, \tag{34}$$

MODELS WITH MONOMIAL POTENTIALS

We propose models with the potential $V = V_0\phi^n$, where $n = 2$ or $n = 4$ and

$$\xi = \frac{CU_0^2}{V + \Lambda}, \qquad (35)$$

where C and Λ are positive constants. Such a modification is natural in general, removing a singular behavior at $\phi = 0$ and gives us an exit from inflation when ϕ becomes small enough.
The initial value of the scalar field ϕ is positive and it tends to zero during inflation.
Calculating the derivative of the effective potential (18),

$$V_{eff\,,\phi} = \frac{U_0^2 n\left(V_0^2\,(3 - C)\,\phi^{2\,n} + 6\,\Lambda\,V_0\phi^n + 3\,\Lambda^2\right)}{3V_0\phi^{n+1}\,(V_0\,\phi^n + \Lambda)^2}\,, \qquad (36)$$

we find that $V_{eff\,,\phi} > 0$ for any $\phi > 0$ at $C < 3$.
It is a sufficient condition that a de Sitter solution does not exist at any $\phi > 0$.
This condition allows us to get an inflationary model without any fine-tuning of the initial data.

QUADRATIC POTENTIAL

For the model with the potential $V = V_0\,\phi^2$ and the following values of parameters:

$$U_0 = \frac{M_{\rm Pl}^2}{2}, \quad C = 2.754, \quad V_0 = 4.05{\times}10^{-11}M_{\rm Pl}^2, \quad \Lambda = 1.0125{\times}10^{-12}M_{\rm Pl}^4,$$

numerical integration gives the following values of the inflationary parameters:

$$A_s = 2.0968 \times 10^{-9}, \qquad n_s = 0.9654, \qquad r = 0.0102.$$

The inflationary parameters are calculated at $\phi_0 = 2.7565$ that corresponds to $N = 0$. The inflation finishes at $N_{end} = 65$, that corresponds to $\phi_{end} = 0.0286$. The constructed inflationary scenario does not contradict to the observation data[5]

[5] Y. Akrami *et al.* [Planck], Astron. Astrophys. **641** (2020), A10 [arXiv:1807.06211]. P.A.R. Ade *et al.* [BICEP and Keck], Phys. Rev. Lett. **127** (2021) 151301 [arXiv:2110.00483].

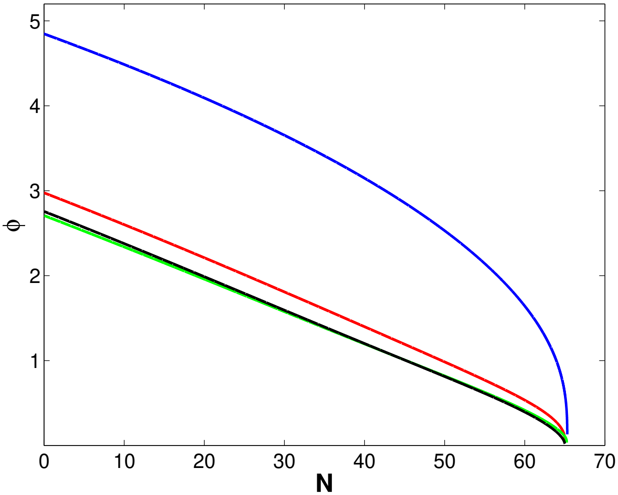

Рис.: *1.* The inflationary model with $V(\phi) = V_0\phi^2$. Values of the function $\phi(N)$ in units of $M_{\rm Pl}$. The black line is the result of the numerical integration. The blue curve is obtained in the standard approximation, red — in the approximation I , green — in the approximation II by. The initial values $\phi(0) = \phi_0$ are given in Table 1.

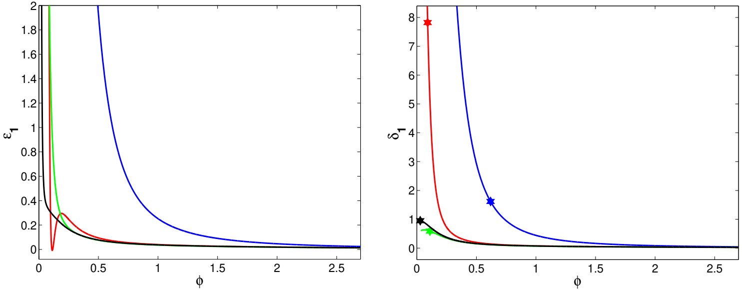

Рис.: *2.* The slow-roll parameters $\varepsilon_1(\phi)$ (left panel) and $\delta_1(\phi)$ (right panel) for the model with $V(\phi) = V_0\phi^2$. The black line is the result of the numerical integration, blue curves are obtained in the standard approximation, red curves in the approximation I, and green curves in the approximation II. The stars denote the end of the inflation (when $\varepsilon_1 = 1$). Values of ϕ are given in units of $M_{\rm Pl}$.

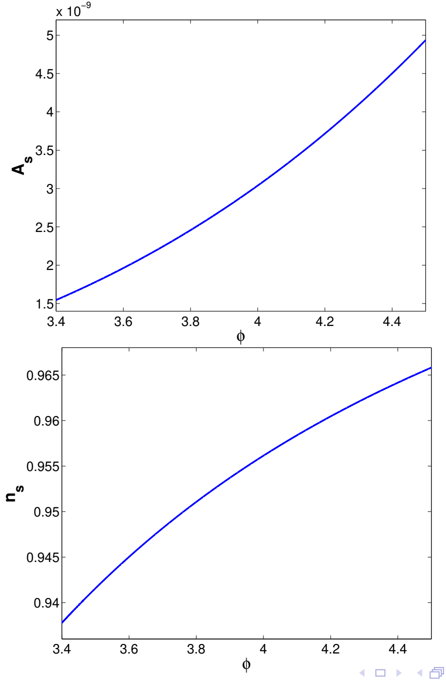
x 10^-9
5
4.5
4
3.5
3
2.5
2
1.5
A_s
3.4
3.6
3.8
4
4.2
4.4
ϕ
0.965
0.96
0.955
0.95
0.945
0.94
n_s
3.4
3.6
3.8
4
4.2
4.4
ϕ

Таблица: **1. Numerical and approximate values of parameters, characterizing the inflationary dynamic in the model with the quadratic potential.**

Parameter	Numeric result	Standard Approx	Approx I	Approx II
$\phi_0/M_{\rm Pl}$	2.7565	4.8472	2.9757	2.7082
$10^9 A_s(\phi_0)$	2.097	6.696	2.491	1.985
$n_s(\phi_0)$	0.965	0.971	0.967	0.965
$r(\phi_0)$	0.0102	0.0096	0.0099	0.0104
$\phi_{end}/M_{\rm Pl}$	0.0286	0.6184	0.0906	0.1097
$\delta_1(\phi_{end})$	0.950	1.62	7.82	0.590
$N(\phi_{end})$	65.0	65.0	65.0	65.0

Таблица: **2. Values of the inflationary parameters for the model with the quadratic potential in different approximations.**

Parameter	Standard Approx	Approx I	Approx II
$\phi_{in}/M_{\mathrm{Pl}}$	3.6589	2.7912	2.7676
$10^9 A_s(\phi_{in})$	2.10	2.10	2.10
$n_s(\phi_{in})$	0.947	0.965	0.966
$r(\phi_{in})$	0.0174	0.0104	0.0102
$N(\phi_{end}) - N(\phi_{in})$	35.1	60.0	66.6

FOURTH-ORDER POTENTIAL

The situation is similar for the model with the fourth-order potential $V = V_0\phi^4$. For parameters

$$V_0 = 3.4 \times 10^{-11}, \quad C = 2.856, \quad \Lambda = 5.95 \times 10^{-13} M_{\mathrm{Pl}}^4 .$$

numeric calculations show that the inflation scenario does not contradict the current observation data. We fix the number of e-folding to be equal $N = 60.6$ and get unappropriated results for the standard approximations. New approximations, as in the previous example, work essentially better (see Table 3).

Таблица: **3. Numerical and approximate values of parameters, characterizing the inflationary dynamic in the model with the quartic potential.**

Parameter	Numeric result	Standard Approx	Approx I	Approx II
ϕ_0/M_{Pl}	1.4019	4.9705	1.4898	1.3974
$10^9 A_s(\phi_0)$	2.096	117.2	2.599	2.017
$n_s(\phi_0)$	0.965	0.953	0.965	0.965
$r(\phi_0)$	0.0044	0.0120	0.0045	0.0045
$\phi_{end}/M_{\mathrm{Pl}}$	0.2000	0.8899	0.3048	0.3037
$\delta_1(\phi_{end})$	0.885	1.80	4.23	0.577
$N(\phi_{end})$	60.6	60.6	60.6	60.6

Таблица: **4. Values of the inflationary parameters for the model with the quartic potential in different approximations.**

Parameter	Standard Approx	Approx I	Approx II
$\phi_{in}/M_{\mathrm{Pl}}$	2.5555	1.4104	1.4116
$10^9 A_s(\phi_{in})$	2.10	2.10	2.10
$n_s(\phi_{in})$	0.817	0.964	0.965
$r(\phi_{in})$	0.0466	0.0045	0.0045
$N(\phi_{end}) - N(\phi_{in})$	13.5	54.6	61.8

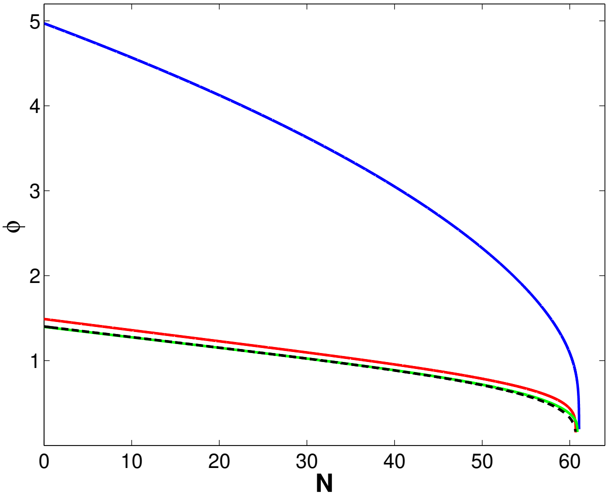

Рис.: *4*. The inflationary model with $V(\phi) = V_0\phi^4$. Values of the function $\phi(N)$ in units of $M_{\rm Pl}$. The black line is the result of the numerical integration. The blue curve is obtained in the standard approximation, red — in the approximation I , green — in the approximation II. The initial values $\phi(0) = \phi_0$ are given in Table 3.

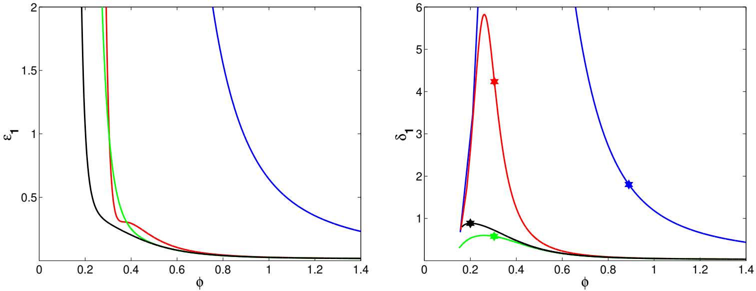

Рис.: 5. The slow-roll parameters $\varepsilon_1(\phi)$ (left panel) and $\delta_1(\phi)$ (right panel) for the model with $V(\phi) = V_0\phi^4$. The black line is the result of the numerical integration, blue curves are obtained in the standard approximation, red curves — in the approximation I, and green curves — in the approximation II. The stars denote the end of the inflation (when $\varepsilon_1 = 1$). Values of ϕ are given in units of $M_{\rm Pl}$.

CONCLUSIONS

- We propose new slow-roll approximations for inflationary models with the Gauss–Bonnet term. We find more accurate expressions of the standard slow-roll parameters as functions of the scalar field. The construction of a higher accuracy slow-roll approximation is based on the use of not the function $H(\phi)$, but the function $H(\phi, \delta_1)$. To get $H(\phi)$ we need to obtain $\delta_1(\phi)$.
- To check the accuracy of approximations considered we construct inflationary models with quadratic and quartic monomial potentials and the $V = V_0\phi^n$ and the function $\xi = \frac{CU_0^2}{V+\Lambda}$. Numerical analysis of these models indicates that the proposed inflationary scenarios do not contradict to the observation data.
- The obtained numerical solutions have been compared with slow-roll approximations. As for the standard approximation, we show that it is not accurate enough to get correct values of inflationary parameters and correct number of e-folding during inflation. On the contrary, the proposed approximations give the results close enough to the numerical solutions. Observational parameters calculated using these approximations are still within the allowed regions.

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Thank for your attention