Stress-Energy Tensor of Gravitational Waves in $f(R)$-gravity

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$$g_{ik} = g_{ik}^{(0)} + h_{ik},$$

$$h_{ik} \ll \partial h_{ik}$$

$$\Gamma^{i\,(1)}_{kl} = \frac{1}{2}(\nabla^{(0)}_l h^i_k + \nabla^{(0)}_k h^i_l - \nabla^{i\,(0)} h_{kl}),$$

$$G^{(0)}_{ik} = -\left\langle G^{(2)}_{ik} \right\rangle,$$

$$G^{(1)}_{ik} = 0,$$

$$t_{ik} = -\left\langle G^{(2)}_{ik} \right\rangle,$$

$$\nabla^{(0)}_i h^i_k = 0, \quad h = 0,$$

$$t_{ik} = \frac{1}{4}\left\langle \partial_i h^n_q \partial_k h^q_n \right\rangle,$$

$$\Box^{(0)} h_{ik} = 0,$$

Isaakson, 1968

Christopher P.L. Berry, Jonathan R. Gair, Phys.Rev.D 83 (2011) 104022, Phys.Rev.D 85 (2012) 089906; 1104.0819

$$f(R) = R + \frac{1}{2}a_2 R^2,$$

$$g^{(0)}_{ik} = \eta_{ik},$$

$$R^{(1)}_{ik} = \frac{1}{2}(\partial_i\partial_n h^n_k + \partial_k\partial_n h^n_i - \partial_i\partial_k h - \Box h_{ik}),$$

$$\bar{G}^{(1)}_{ik} = R^{(1)}_{ik} - \partial_i\partial_k(a_2 R^{(1)}) + \eta_{ik}\Box(a_2 R^{(1)}) - \frac{1}{2}R^{(1)}\eta_{ik},$$

$$\bar{G}^{(1)} = 3\Box(a_2 R^{(1)}) - R^{(1)},$$

$$\bar{h}_{ik} = h_{ik} + A_{ik},$$

$$A_{ik} = -\frac{1}{2} h \eta_{ik},$$

$$\bar{h}_{ik} = h_{ik} - \frac{1}{2} h \eta_{ik} + b a_2 R^{(1)} \eta_{ik},$$

Lorenz or de Donder gauge

$$\nabla_i \bar{h}^i_k = 0, \;\; \bar{h} = 0,$$

$$\bar{G}^{(1)}_{ik} = \frac{2-b}{6}\bar{G}^{(1)}\eta_{ik} - \frac{1}{2}\Box\left(\bar{h}_{ik} - \frac{1}{2}\bar{h}\eta_{ik}\right) - (b+1)\left[\partial_i\partial_k(a_2 R^{(1)}) + \frac{1}{6}R^{(1)}\eta_{ik}\right],$$

$$b = -1,$$

$$\Box\bar{h} = 0,$$

$$\Box\bar{h}_{ik} = 0,$$

$$3\Box(a_2 R^{(1)}) - R^{(1)} = 0, \quad h = -4a_2 R^{(1)},$$

$$G_{ik} = G_{ik}^{B} + G_{ik}^{(1)} + G_{ik}^{(2)} + \dots,$$

$$G_{ik}^{B} = -\left\langle G_{ik}^{(2)} \right\rangle,$$

$$t_{ik} = -\frac{1}{8\pi G}\left\langle G_{ik}^{(2)} \right\rangle,$$

$$g_{ik} = \gamma_{ik} + h_{ik},$$

$$\gamma_{ik} = \eta_{ik} + j_{ik},$$

$$j_{ik} = O(\varepsilon^2),$$

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$$\Gamma^{i\,(1)}_{kl} = \frac{1}{2}\gamma^{in}\Big[\partial_k\big(\bar{h}_{nl} - a_2 R^{(1)}\gamma_{nl}\big) + \partial_l\big(\bar{h}_{nk} - a_2 R^{(1)}\gamma_{nk}\big)$$

$$-\partial_n\big(\bar{h}_{kl} - a_2 R^{(1)}\gamma_{kl}\big),$$

$$R^{i\,(2)}_{klm} = \nabla^{(0)}_l \Gamma^{i\,(2)}_{km} - \nabla^{(0)}_m \Gamma^{i\,(2)}_{kl} + \Gamma^{i\,(1)}_{nl}\Gamma^{n\,(1)}_{km} - \Gamma^{i\,(1)}_{nm}\Gamma^{n\,(1)}_{kl},$$

$$\left\langle G_{ik}^{(2)} \right\rangle$$

covariant derivatives commute

$$\left\langle A\nabla_i\nabla_k B \right\rangle = \left\langle A\nabla_k\nabla_i B \right\rangle,$$

gradients average out to zero

$$\left\langle \nabla_m(A_{ik}) \right\rangle = 0,$$

one can freely integrate by parts

$$\left\langle B\nabla_i A_{lm} \right\rangle = -\left\langle A_{lm}\nabla_i B \right\rangle,$$

$$\left\langle G_{ik}^{(2)} \right\rangle = \left\langle -\frac{1}{4}\partial_i \bar{h}^{pq}\partial_k \bar{h}_{pq} - \frac{3}{2}a_2^2 \partial_i R^{(1)} \partial_k R^{(1)} \right\rangle,$$

$$t_{ik} = \frac{1}{32\pi G}\left\langle \partial_i h^{pq}\partial_k h_{pq} + \frac{1}{8}\partial_i h \partial_k h \right\rangle,$$

$$\bar{G}_{ik} = f' R_{ik} - \frac{1}{2} f g_{ik} + g_{ik} \Box f' - \nabla_i \nabla_k f' = \bar{G}^{(0)}_{ik} + \bar{G}^{(1)}_{ik} + \bar{G}^{(2)}_{ik},$$

$$f'\Big(R^{(0)} + \big(R^{(1)} + R^{(2)}\big)\Big) = f'^{(0)} + f'^{(1)} + f'^{(2)},$$

$$f'^{(0)} = f'(R^{(0)}),$$

$$f'^{(1)} = f''(R^{(0)}) R^{(1)},$$

$$f'^{(2)} = f''(R^{(0)}) R^{(2)} + \frac{1}{2} f'''(R^{(0)}) \Big(R^{(1)}\Big)^2,$$

$$\bar{h}_{ik} = h_{ik} - \frac{1}{2} h \eta_{ik} + b R^{(1)} \eta_{ik},$$

$$\bar{G}^{(1)}_{ik} = -\frac{1}{2} f'(0) \Box \bar{h}_{ik} + \frac{1}{4} \eta_{ik} f'(0) \Box \bar{h} + \frac{1}{6} \eta_{ik} \frac{2f''(0) - bf'(0)}{f''(0)} \bar{G}^{(1)}$$

$$-(bf'(0) + f''(0)) \left[\partial_i \partial_k R^{(1)} + \frac{1}{6} \frac{f'(0)}{f''(0)} R^{(1)} \right],$$

$$b = -\frac{f''(0)}{f'(0)},$$

$$\Box \bar{h}_{ik} = 0,$$

$$3f''(0)\Box R^{(1)} - f'(0)R^{(1)} = 0,$$

$$(32\pi G)t_{ik} = f_0' \left\langle \partial_i h^{lq} \partial_k h_{lq} \right\rangle + \frac{1}{8} f_0' \left\langle \partial_i h \partial_k h \right\rangle,$$

Starobinsky model 0706.2041

$$f_S = R + \lambda R_s \left[\left(1 + \frac{R^2}{R_s^2} \right)^{-n} - 1 \right],$$

$$f_S'(0) = 1,$$

$$f_S'(R_0 \neq 0) = 1 - \frac{2\lambda(1 + R_0^2/R_s^2)^{-n} n R_0}{R_s(1 + R_0^2/R_s^2)},$$

$$f_S'(R_0 = R_s) = 1 - \lambda n 2^{-n},$$

Hu-Sawicki model 0705.1158

$$f_{HS} = R - m^2 \frac{c_1 (R/m^2)^n}{c_2 (R/m^2)^n + 1},$$

$$f'_{HS}(0) = 1,$$

$$f'_{HS}(R_0 \neq 0) = 1 - \frac{m^2 c_1 (R_0/m^2)^n n}{(c_2 (R_0/m^2)^n + 1)^2 R_0},$$

$$f'_{HS}(R = m^2) = 1 - \frac{c_1 n}{(c_2 + 1)^2},$$

de Sitter background

$$\nabla_i f_0' = 0,$$

higher frequency limit

$$h_{ik} \ll \partial_n h_{ik},$$

$$(32\pi G)t_{ik} = f_0' \left\langle \partial_i h^{lq} \partial_k h_{lq} \right\rangle + \frac{1}{8} f_0' \left\langle \partial_i h \partial_k h \right\rangle,$$