Confinement of ghosts and stability in higher derivative quantum gravity

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Based (mainly) on

F. Salles and I.Sh., PRD and 1401.4583.

P. Peter, F. Salles, I.Sh., PRD and 1801.00063.

W.C. Silva, I. Sh., JHEP and 2301.13291.

M. Asorey, G. Krein, I.Sh. arXiv: 2408.16514

General Relativity and Quantum Theory

General Relativity (GR) is a complete theory of classical gravitational phenomena. It proved valid in the wide range of energies and distances.

There are covariant equations for the matter (fields and particles, fluids etc) and Einstein equations for the gravitational field $g_{\mu\nu}$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G \, T_{\mu\nu}.$$

The most important solutions of GR have specific symmetries.

1) Spherically-symmetric solution: Planets, Stars, Black holes.
2) Isotropic and homogeneous metric: Universe.

And in both cases there are unavoidable singularities (Penrose and Hawking theorems).

Applicability of GR

- Singularities are significant, because they emerge in the most important solutions, in the main areas of application of GR.

In cosmology, extrapolating backward in time, we find that the use of GR leads to a problem, while at the late Universe GR provides a consistent basis for cosmology and astrophysics.

The solution of the problem of singularities is to assume that

- GR is not valid at all scales.

At the very short distances and/or when the curvature becomes very large, the gravitational phenomena must be described by some other theory, more general than GR.

But, due to the success of GR, we expect that this unknown theory coincides with GR at the large distance & weak field limit.

- The deviations from GR may be owing to quantum effects.

Dimensional arguments.

The expected scale of the quantum gravity effects is associated to the Planck units of length, time and mass. The idea of Planck units is based on the existence of the 3 fundamental constants:

$$c = 3 \cdot 10^{10}\, cm/s\,,$$

$$\hbar = 1.054 \cdot 10^{-27}\, erg \cdot sec\,;$$

$$G = 6.67 \cdot 10^{-8}\, cm^3/sec^2\, g\,.$$

One can use them uniquely to construct the dimensions of

length $l_P = G^{1/2}\,\hbar^{1/2}\,c^{-3/2} \,\approx\, 1.4 \cdot 10^{-33}\, cm;$

time $t_P = G^{1/2}\,\hbar^{1/2}\,c^{-5/2} \,\approx\, 0.7 \cdot 10^{-43}\, sec;$

mass $M_P = G^{-1/2}\,\hbar^{1/2}\,c^{1/2} \,\approx\, 0.2 \cdot 10^{-5} g \,\approx\, 10^{19}\, GeV.$

Three choices for Quantum Gravity (QG)

The existence of fundamental Planck units ($M_{\mathrm{P}} \sim 10^{19}\, GeV$) indicates new fundamental physics at this very high energy scale. How to interpret this result of dimensional analysis?

General classification of possible approaches two Quantum Gravity (QG). Three distinct groups:

- **Quantize both gravity and matter fields. This is, definitely, the most fundamental possible approach.**

- **Quantize only matter fields on classical curved background (semiclassical approach).**

- **Quantize "something else." E.g., in case of (super)string theory both matter and gravity are induced.**

Which approach is "better"?
Indeed, they have something important in common.

- **QFT and Curved space-time are well-established notions, which passed many experimental/observational tests.**

Therefore, our first step should be to consider QFT of matter fields on classical curved background.

Different from quantum theory of gravity, QFT of matter fields in curved space is renormalizable and free of conceptual problems.

Moreover, understanding renormalization of semiclassical theory can be very helpful in order to impose some constrains on a complete QG.

Semiclassical approach: metric enters the generating functional of the Green functions as external parameters,

$$Z(J,\, g_{\mu\nu}) \,=\, \int d\Phi\, \exp\big\{iS(\Phi,\, g_{\mu\nu}) + i\Phi J\big\}\,.$$

Effective Action depends on the mean field and metric, $\Gamma(\Phi, g_{\mu\nu})$.

The renormalizable QFT in curved space requires introducing a generalized form of the gravity (external field), "vacuum action".

$$S_{vac} = S_{EH} + S_{HD}$$

where $$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,\{R + 2\Lambda\}\,.$$

is the Einstein-Hilbert action with the cosmological constant.

S_{HD} **includes higher derivative terms. The most useful form is**

$$S_{HD} = \int d^4x\sqrt{-g}\,\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\}\,,$$

where $$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + \frac{1}{3}R^2$$

is the square of the Weyl tensor and

$$E = R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4\,R_{\alpha\beta}R^{\alpha\beta} + R^2$$

is the integrand of the Gauss-Bonnet topological term.

Quantum gravity (QG): covariant renormalization

As the first example consider quantum GR.

$$S_{EH} = -\frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R+2\Lambda)\,.$$

Power counting: $\quad D+d = 2+2p\,.$

At the 1-loop level we can expect the divergences like

$$\mathcal{O}(R^2_{...}) = R^2_{\mu\nu\alpha\beta}\,,\;\; R^2_{\mu\nu}\,,\;\; R^2\,.$$

't Hooft and Veltman; Deser and van Nieuwenhuisen, (1974); ...

At the 2-loop level we have

$$\mathcal{O}(R^3_{...}) = R_{\mu\nu}\Box R^{\mu\nu}\,,\,...R^3\,,\;\; R_{\mu\nu}R^{\mu}_{\alpha}R^{\alpha\nu}\,,\;\; R_{\mu\nu\alpha\beta}R^{\mu\nu}{}_{\rho\sigma}R^{\mu\nu\rho\sigma}\,.$$

M.H. Goroff and A. Sagnotti, NPB 266 (1986).

Since the last structure does not vanish on-shell, the theory is non-renormalizable.

Within the standard perturbative approach non-renormalizability means the theory has no predictive power.

Every time we introduce a new type of a counterterm, it is necessary to fix renormalization condition and this means a measurement. So, before making a single predictions, it is necessary to have an infinite amount of experimental data.

What are the possible solutions?

- **Change standard perturbative approach to something else.**

There are many options, but their consistency or their relation to the QG program are not clear, in all cases.

- **Change the theory, i.e., take another theory to construct QG.**

The first option is widely explores in the asymptotic safety scenarios, in the effective approaches to QG, induced gravity approach (including string theory) and so on.

Let us concentrate on the second idea.

The most natural choice is four derivative model, because we need four derivatives anyway for the quantum matter fields.

Already known action: $S_{gravity} = S_{EH} + S_{HD}$

where S_{HD} **includes square of the Weyl tensor and** R

$$S_{HD} = -\int d^4x\sqrt{-g}\left\{\frac{1}{2\lambda}C^2 + \frac{\omega}{3\lambda}R^2 + \text{ surface terms}\right\},$$

$$C^2(4) = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + R^2/3\,,$$

Propagators of metric and ghosts behave like $\mathcal{O}(k^{-4})$ **and we have** K_4, K_2, K_0 **vertices.**

The superficial degree of divergence

$$D + d = 4 - 2K_2 - 4K_0.$$

This theory is definitely renormalizable. Dimensions of counterterms are 4, 2, 0.

K. Stelle, Phys. Rev. D (1977).

Well, there is a price to pay: massive ghosts

$$G_{\text{spin}-2}(k) \sim \frac{1}{m^2}\left(\frac{1}{k^2} - \frac{1}{k^2+m^2}\right), \quad m \propto M_P\,.$$

The tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and a huge mass.

Particle with negative energy means instability of vacuum state.

The Minkowski space is not protected from the spontaneous creation of massive ghost and many gravitons from vacuum.

Different sides of the HDQG problems with massive ghosts:

- **In classical systems higher derivatives generate exploding instabilities at the non-linear level** *(M.V. Ostrogradsky, 1850).*

- **Interaction between ghost and gravitons may violate energy conservation in the massless sector** *(M.J.G. Veltman, 1963).*

- **Ghost produce violation of unitarity of the S -matrix.**

One can include more than four derivatives,

$$S = S_{EH} + \sum_{n=0}^{N} \int d^4x \sqrt{-g} \Big\{ \omega_n^C C_{\mu\nu\alpha\beta} \Box^n C_{\mu\nu\alpha\beta} + \omega_n^R R \Box^n R \Big\} + \mathcal{O}\big(R^3_{\dots}\big).$$

Simple analysis shows that in this theory massive ghost-like states are still present.

For the real poles case:

$$G_2(k) = \frac{A_0}{k^2} + \frac{A_1}{k^2 + m_1^2} + \frac{A_2}{k^2 + m_2^2} + \cdots + \frac{A_{N+1}}{k^2 + m_{N+1}^2}\,.$$

For any sequence $0 < m_1^2 < m_2^2 < m_3^2 < \cdots < m_{N+1}^2$, **the signs of the corresponding terms alternate,** $A_j \cdot A_{j+1} < 0$.

M. Asorey, J.-L. Lopez & I. Sh., IJMPhA (1997), hep-th/9610006.

$$S = S_{EH} + \int d^4x\sqrt{-g}\left\{\omega_N^C C_{\mu\nu\alpha\beta}\Box^N C_{\mu\nu\alpha\beta} + \omega_N^R R\Box^N R + \cdots\right\}.$$

Again, let us consider only vertices with a maximal $K_\nu = 2k+4$.

Then we have $r_l = K_\nu = 2k+4$ **and, combining**

$$D + d = \sum_{l_{int}}(4 - r_l) - 4n + 4 + \sum_\nu K_\nu$$

$$\textbf{with} \qquad l_{int} = p + n - 1\,,$$

we can easily arrive at the estimate of d for $D = 0$

$$d = 4 + k(1-p)\,.$$

For $k = 0$ **we meet the standard HDQG result,** $d \equiv 4$. **Starting from** $k = 1$ **we have superrenormalizable theory, where the divergences exist only for** $p = 1, 2, 3$.

For $k \geq 3$ **we have superrenormalizable theory, where divergences exist only for** $p = 1$, **that is at the one-loop level.**

Renormalization group in fourth-derivative QG

In the seminal paper by Fradkin and Tseytlin were done several outstanding theoretical discoveries about quantum gravity:

E.S. Fradkin and A.A. Tseytlin, Nucl. Phys. B201 (1982) 469.

- **Systematic analysis of gauge dependence of the renormalization group running in QG. In quantum GR, there is unique on-shell renormalization group equation for the dimensionless combination** $\gamma = 16\pi G\Lambda$**:**

$$\mu\frac{d\gamma}{\mu} = -\frac{29}{5}\,\frac{\gamma^2}{(4\pi)^2}.$$

- **Heat-kernel formalism for fourth derivative operators.**
- **Conceptually consistent derivation of one-loop beta functions in fourth-derivative gravity.**
- **Clear distinction between conformal and non-conformal QG's.**
- **Discussion of effective approach in higher derivative QG.**
- **AF in QG and separation of essential charges.**

Exact β-functions in QG

In the superrenormalizable QG one can derive exact and universal RG equations by working at the one-loop level !

M. Asorey, J.-L. Lopez & I. Sh., IJMPhA (1997), hep-th/9610006.

$$\beta_\Lambda = \mu \frac{d\rho_\Lambda}{d\mu} = \frac{1}{(4\pi)^2} \left(\frac{5\omega_{N-2,C}}{\omega_{N,C}} + \frac{\omega_{N-2,R}}{\omega_{N,R}} - \frac{5\omega^2_{N-1,C}}{2\omega^2_{N,C}} - \frac{\omega^2_{N-1,R}}{2\omega^2_{N,R}} \right).$$

L. Modesto, L. Rachwal & I.Sh., arXiv:1704.03988; 2104.13980

$$\beta_G = \mu \frac{d}{d\mu} \left(-\frac{1}{16\pi G} \right) = -\frac{1}{6(4\pi)^2} \left(\frac{5\omega_{N-1,C}}{\omega_{N,C}} + \frac{\omega_{N-1,R}}{\omega_{N,R}} \right).$$

Different from four-derivative quantum gravity these β-functions do not depend on the choice of gauge-fixing conditions.

For $N \geq 3$ they are exact.

Two sides of higher derivatives in QG.

A consistent theory working at arbitrary energy scale cannot be constructed without at least fourth derivatives.

If the higher derivative terms are included, then the tree-level spectrum includes massless graviton and massive spin-2 "ghost" with negative kinetic energy and huge mass.

If we do not include the higher derivative terms into classical action, they will emerge with infinite coefficients and (most relevant) with logarithmically running parameters. In any case, the unphysical ghosts come back.

No way to live with ghosts and no way to live without ghosts.

Since we live, there should be an explanation of this.

Standard (for some people, at least) logic

The idea is to consider all higher derivative terms to be small perturbations by definition.

J.Z. Simon, Phys. Rev. **D41** *(1990);*
L. Parker and J.Z. Simon, Phys. Rev. **D47** *(1993), gr-qc/9211002.*

In this approach all higher derivative terms, including the renormalized terms in the classical action, quantum corrections, running parameter etc, are regarded as small perturbations over the much greater Einstein-Hilbert term.

Certainly, this approach is a kind of ad hoc one and it can work only for energies much below M_P scale, that is not what we expect from the "theory of everything", such as QG.

Also, there is a lot of ambiguity.

Should we treat R^2 as perturbation? Why? And even if so, what we have to do with the Starobinsky model? Forbid it?

What to do with R^3, $RR_{\mu\nu}R^{\mu\nu}$ and other terms like these?

Should we treat all such terms as perturbations? Why? Because they have higher derivatives? Even regardless of the fact they do not produce ghosts?

What is the rule of splitting the action into main part and perturbation?

Moreover, if the procedure is considered as part of effective approach to QFT, this means we assume that QG phenomena are relevant only far below the Planck scale.

This is something opposite to what we expect from quantum gravity, after all. The original motivation for quantum gravity is to deal with the Planck energies.

Ghost-free HD models of gravity

Suggested as an alternative to Zweibach transformation in string theory: In the non-local theory

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,\Big\{R + G_{\mu\nu}\,\frac{a(\Box)-1}{\Box}\,R^{\mu\nu}\Big\}, \quad a(\Box) = e^{-\Box/m^2}\,.$$

A. Tseytlin, PLB, hep-th/9509050.

In this and similar theories propagator of metric perturbations has a single massless pole, corresponding to gravitons.

With this choice there are no ghosts!

The idea is to use Zweibach-like transformation, but arrive at the non-local theory which is non-polynomial in derivatives, instead of “killing” all higher derivatives that one can kill.

Can it be a basis for a QG theory?

There was a proposal to use the same kind of non-local models to construct superrenormalizable and unitary models of QG.

N.V. Krasnikov, T.M.Ph. **73** *(1987) 1184.*

E.T. Tomboulis, hep-th/9702146; arXiv:1507.00981 – PRD. ...

L. Modesto, L. Rachwal, NPB (2014), arXiv:1407.8036.

The propagator is defined by the terms bilinear in curvature's,

$$S = \int_x \Big\{ -\frac{1}{\kappa^2} R + \frac{1}{2} C_{\mu\nu\alpha\beta}\, \Phi(\Box)\, C^{\mu\nu\alpha\beta} + \frac{1}{2} R\, \Psi(\Box)\, R \Big\}.$$

The equation for defining the poles:

$$p^2 \Big[1 + \kappa^2 p^2 \Phi(-p^2) \Big] = p^2\, e^{\alpha p^2} = 0.$$

In this particular case there is only a massless pole corresponding to gravitons. **But unfortunately, it is impossible to preserve the ghost-free structure at the quantum level.**

I.Sh., Counting ghosts ... in the "ghost-free" .. 1502.00106, PLB.

Typically there are infinitely many poles on the complex plane.

No way to live without ghosts!

One can conclude that in all three approaches to QG, namely semiclassical, legitimate QG, induced gravity/strings, there is no reasonable way to get rid of massive ghost-like states.

What we can really do is to make all the ghosts complex, in the sense of complex "massive" poles in the propagator.

The complex poles always come in complex conjugate pair, which opens interesting possibilities, related to Lee-Wick quantization.

This is coherent with the previous attempts to solve the problem of higher derivative massive ghosts.

E. Tomboulis (1977,1980,1984), A. Salam and J. Strathdee (1978), I. Antonidis & E. Tomboulis (1986), D. A. Johnston (1988), S. Hawking et al (1990, . . .), ...

Loop effects and complex poles resolve the issue?

According to the works done in 70-ies and 80-ies, the main hope to have unitary & renormalizable fourth-derivative QG is related to the splitting of real massive pole of a fourth-derivative theory into a couple of complex conjugate poles, at the quantum level.

E. Tomboulis (1977,1980,1984),
A. Salam and J. Strathdee (1978), . . .

In this case one has to consider always a scattering of a pair of the conjugate particles, it opens the way to have unitary theory.

S. Hawking et al (1990, . . .), ...

The main problem is that the definite resolution of the problem of unitarity in the fourth-derivative model requires complete information about the dressed propagator.

D. A. Johnston, NPB (1988).

Do we really need so much?

Complex ghosts and Lee-Wick unitarity in QG

Starting from *Tomboulis (1977)* **and** *Salam and Strathdee (1978)* **the main hope in the "minimal" fourth-derivative QG was that the real ghost pole splits into a couple of complex conjugate poles under the effect of quantum corrections.**

One-loop effects, large-*N* approximation and lattice-based considerations indicated an optimistic picture, but unfortunately all of them are not conclusive, as shown by *Johnston (1988)*.

However, for six- or more- derivative theory of QG, one can just start from the theory which has only complex massive poles.

L. Modesto, and I.Sh. PLB (2016), arXiv:1512.07600.

It turns out that such a theory is unitary and, moreover, this property may probably hold even at the quantum level.

Let us write the six-derivatives action in a slightly different form:

$$S = -\frac{2}{\kappa^2}\int d^4x\sqrt{-g}R \;-\int d^4x\sqrt{-g}\,\Big\{\frac{\alpha}{2}\,C_{\mu\nu\alpha\beta}\Pi_2 C^{\mu\nu\alpha\beta} + \alpha\omega\,R\Pi_0 R\Big\},$$

where $\Pi_{0,2} = \Pi_{0,2}(\Box) = 1 + \,...$ **are some polynomials of order** k.
After the Wick rotation the equations for the poles are

$$\alpha\Pi_2(p^2)p^2 \;=\; 2M_P^2\,, \quad \alpha\omega\,\Pi_0(p^2)p^2 \;=\; M_P^2\,.$$

Let us consider the six-order theory,

$$\Pi_2(p^2) \;=\; 1 + \frac{p^2}{2A_2}, \quad \Pi_0(p^2) \;=\; 1 + \frac{p^2}{2A_0}, \quad A_{0,2} \sim [\mathrm{mass}]^2.$$

The solution for the tensor part: $p^2 = m_2^2 = -A_2 \pm \sqrt{A_2^2 + \frac{4A_2M_P^2}{\alpha}}$.

Possible cases:

- **Two real positive solutions:** $0 < m_{2+}^2 < m_{2-}^2$ **.**
- **Two pairs of complex conjugate solutions.**

In QFT theory of the field $h_{\alpha\beta}$ the condition of unitarity of the S - matrix can be formulated in a usual way,

$$S^\dagger S = 1\,, \quad \text{or} \quad S = 1 + iT \quad \text{and} \quad -i(T - T^\dagger) = T^\dagger T\,.$$

By defining the scattering amplitude as

$$\langle f|T|i\rangle \;=\; (2\pi)^D\;\delta^D(p_i - p_f)\,T_{fi}.$$

we arrive at

$$-i\,(T_{fi} - T^*_{if}) = \sum_k T^*_{kf}T_{ki}\,.$$

Assuming that for the forward scattering amplitude $i = f$, previous equation simplifies to

$$2\,\mathrm{Im}\,T_{ii} = \sum_k T^*_{ik}\,T_{ik} > 0\,.$$

The detailed analysis of tree-loop, one-loop and multi-loop diagrams shows that this relation is satisfied because massive poles always show up in a complex conjugate pairs.

The main issue is stability

Certainly, the unitarity of the S- matrix is not the unique condition of consistency of the quantum gravity theory.

The most important feature is the stability of physically relevant solutions of classical general relativity in the presence of higher derivatives and massive ghosts.

The problem is well explored for the cosmological backgrounds. Gravitational waves on de Sitter space (energy $\ll M_p$):

A. A. Starobinsky, Let. Astr. Journ. (in Russian) (1983).

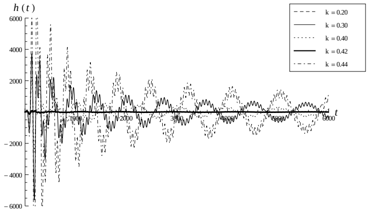
S. Hawking, T. Hertog, and H.S. Real, PRD (2001).
J. Fabris, A. Pelinson and I.Sh., NPB (2001).

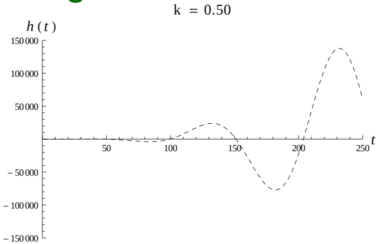
J. Fabris, A. Pelinson, F. Salles and I.Sh., JCAP, arXiv:1112.5202.

More general FRW-backgrounds:

F. Salles and I.Sh., PRD, arXiv:1401.4583.

More general cosmological backgrounds





Example: radiation-dominated Universe. There are no growing modes until the frequency k achieves the value ≈ 0.5 in Planck units. Starting from this value, we observe instability as an effect of massive ghost.

The anomaly-induced quantum correction is $\mathcal{O}(R^3_{....})$. Until the energy is not of the Planck order of magnitude, these corrections can not compete with classical $\mathcal{O}(R^2_{....})$ - terms.

Massive ghosts are present only in the vacuum state. We just do not observe them “alive” until the energy scale M_P .

What can we do with (trans)Planck frequencies?

Let us take a look at the simplest possible equation for the fourth-derivative gravity without quantum or semiclassical corrections,

$$\frac{1}{3}h^{(IV)} + 2Hh^{(III)} + \Big(H^2 + \frac{M_P^2}{32\pi a_1}\Big)\ddot{h} + \frac{1}{6}\frac{\nabla^4 h}{a^4} - \frac{2}{3}\frac{\nabla^2\ddot{h}}{a^2} - \frac{2H}{3}\frac{\nabla^2\dot{h}}{a^2}$$
$$-\Big(H\dot{H} + \ddot{H} + 6H^3 - \frac{3M_P^2 H}{32\pi a_1}\Big)\dot{h} - \Big[\frac{M_P^2}{32\pi a_1} - \frac{4}{3}\Big(\dot{H} + 2H^2\Big)\Big]\frac{\nabla^2 h}{a^2}$$
$$-\Big[24\dot{H}H^2 + 12\dot{H}^2 + 16H\ddot{H} + \frac{8}{3}H^{(III)} - \frac{M_P^2}{16\pi a_1}\Big(2\dot{H} + 3H^2\Big)\Big]h = 0.$$

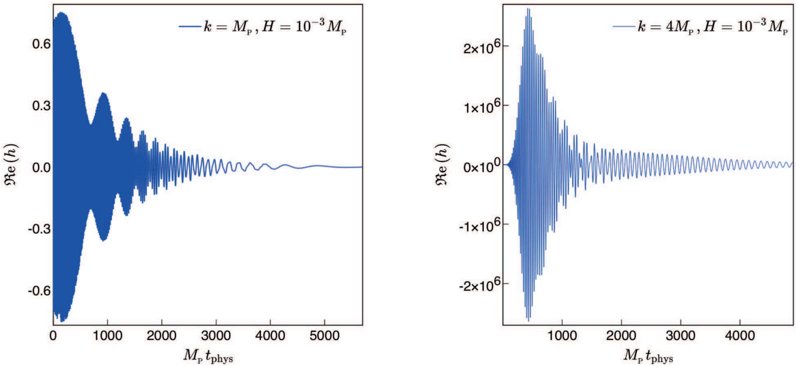
It is easy to note that the space derivatives ∇ and hence the wave vector $\vec{k}$ enter this equation only in the combination

$$\vec{q} = \frac{\vec{k}}{a(t)}.$$

When the universe expands, the frequency becomes smaller!

Patrick Peter, Filipe de O. Salles, I.Sh., On the ghost-induced instability on de Sitter background. PRD (2018), arXiv:1801.00063

The qualitative conclusion is perfectly well supported by numerical analysis, including the case when the semiclassical corrections are taken into account.



The growth of the waves really stops at some point. At least in the cosmological setting this may be a solution of the problem.

Confinement of ghosts?

The idea of confinement of ghosts with complex poles and residues, is in the air. The inspiration comes from QCD.

*N. Nakanishi, Prog. Theor. Phys. **54** (1975) 1213.*

Complex poles and higher derivative QG:

E. Tomboulis (1977,1980,1984),
A. Salam and J. Strathdee (1978),
I. Antonidis & E. Tomboulis (1986),
D. A. Johnston (1988),
S. Hawking et al (1990, . . .), ...

Confinement in higher derivative QG:

B. Holdom and J. Ren, arXiv:1512.05305 and PRD.
M. Frasca, A. Ghoshal and N. Okada, arXiv:2106.07629 and PRD.
M. Frasca, A. Ghoshal and A. Koshelev, arXiv:2207.06394 and PLB.
J. Liu, L. Modesto and G. Calcagni, arXiv:2208.13536 and JHEP.
G.P. de Brito, arXiv:2309.03838 and PRD.

Confinement of ghosts?

Is it possible to form normal bound states out of massive ghosts? The answer is positive in the complex case.

M. Asorey, G. Krein, I.Sh. arXiv:2408.16514

Consider the six-derivative Euclidean Lagrangian

$$\mathcal{L}_{6der} \,=\, \frac{1}{2}\psi\,(-\partial^2)(-\partial^2+m^2)(-\partial^2+m^{*2})\,\psi+V(\psi)\,.$$

There is a massless mode and two complex conjugate modes.

The auxiliary fields representation of the ghost part has the form

$$\mathcal{L}_{\rm gh}=\frac{i}{2}\varphi_1\big(-\partial^2+m^2\big)\varphi_1-\frac{i}{2}\varphi_2\big(-\partial^2+m^{*2}\big)\varphi_2+U(\varphi_1,\varphi_2)\,.$$

For out toy model, we choose

$$U(\varphi_1,\varphi_2)\,=\,-\frac{\lambda_{12}}{4}\,\varphi_1^2\varphi_2^2, \qquad \textbf{where} \qquad g\,=\,\frac{\lambda_{12}}{(2\pi)^4}.$$

The scheme which we use for exploring the possibility of a bound state is what is commonly utilized in QCD.

M. Asorey, G. Krein, I.Sh. arXiv:2408.16514

Let us make the simplest possible computation.

Let $O_{\varphi_1\varphi_2}(x)$ be the composite operator

$$O_{\varphi_1\varphi_2}(x) \,=\, \varphi_1(x)\varphi_2(x).$$

Consider the correlation function

$$C(x,y) \,=\, \int \frac{d^4p}{(2\pi)^4}\, e^{-ip\cdot(x-y)}\, C(p) \,=\, \langle O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\rangle$$
$$= \frac{1}{Z_{\rm gh}} \int \mathcal{D}\varphi_1 \mathcal{D}\varphi_2\; O_{\varphi_1\varphi_2}(x)\, O_{\varphi_1\varphi_2}(y)\, e^{-S_{\rm gh}}.$$

It is straightforward to get, using the bubble diagram $C(p) = G_B(p) + G_B(p)\big[\lambda_{12}\, G_B(p)\big]$**, with**

$$G_B(p) = \int \frac{d^4k}{(2\pi)^4} D_{\varphi_1}(p-k) D_{\varphi_2}(k), \qquad D_{\varphi_{1/2}}(p) = \frac{\pm\, i}{p^2 + m^2/m^{*2}}\,.$$

Iterate the one-loop result to obtain a Dyson's type of equation:

$$C(p) \,=\, G_B(p)\sum_{k=0}^{\infty}\big[\lambda_{12}\,G_B(p)\big]^k \,=\, \frac{G_B(p)}{1-\lambda_{12}\,G_B(p)}\,.$$

There is a physical bound state if $C(p)$ has a pole at a value $p^2=-\mathcal{M}^2$ and the residue at the pole is positive. This means

$$1-\lambda_{12}\,G_B(p)\Big|_{p^2=-\mathcal{M}^2} = 0.$$

Direct calculation gives the representation

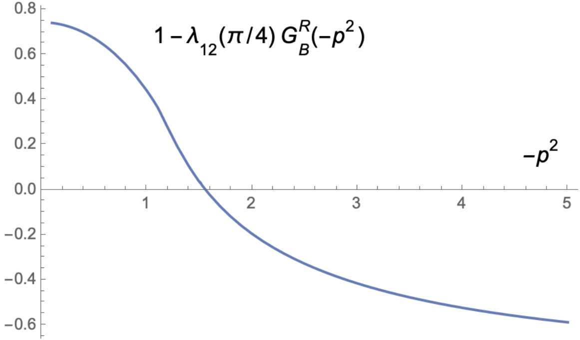
$$G_B(p) \,=\, \int_0^{\infty}\frac{dk\,k}{4(2\pi)^2\,p^2}\Big\{(1-i)\mu^2+k^2\Big\}^{-1}\,\Big\{(1+i)\mu^2 + k^2+p^2-\sqrt{\big[(1+i)\mu^2+(k+p)^2\big]\big[(1+i)\mu^2+(k-p)^2)\big]}\Big\},$$

where $m^2=(1+i)\mu^2$ **and** $m^{*2}=(1-i)\mu^2$.

The last integral is logarithmically divergent, but this divergence can be easily renormalized by momentum substraction

$$G_B^R(p) \,=\, G_B(p)\,-\,G_B(p_0).$$

The result can be plotted as shown in Figure.



The denominator as a function of $\mathcal{M}^2 = -p^2$, where we have chosen $g = \pi/16$ and $\mu^2 = 1$. We can clearly observe the pole and its position is free from ambiguities, regardless the detailed analysis is technically not very simple.

Finally, to be a physical pole, its residue should be positive.

Expanding $C(p)$ around the point $p^2 = -\mathcal{M}^2$, we get

$$C(p)\Big|_{p^2\approx-\mathcal{M}^2} = \frac{G_B^R(-\mathcal{M}^2)+\cdots}{1-\lambda_{12}G_B^R(-\mathcal{M}^2)-(p^2+\mathcal{M}^2)\,\lambda_{12}\;G_B^R{}'(-\mathcal{M}^2)+\cdots}$$
$$= \frac{R_G^R}{p^2+\mathcal{M}^2}+\cdots\;,$$

with the residue R_G^R given by

$$R_G^R = -\frac{1}{G_B^R{}'(-\mathcal{M}^2)} > 0 \quad \textbf{at the pole} \quad \mathcal{M}^2 \approx 1.56\,.$$

The existence of a physical solution with a bound state of ghosts depends on the complex masses of the pair $(1\pm i)\mu$, on the coupling constant g and the momentum substraction point p_0.

We found that there is a window where the condensation of ghosts is possible, $g^- \leq g \leq g^+$, with $g^- = 0.11$ and $g^+ = 0.79$.

Similar constraints are possible in superrenormalizable QG under the special choice of the action. Such a choice will be compatible with renormalizability and running of couplings.

Cosmological implications of ghost confinement

There were general discussions of the theories with Planck-order cut-off on the energy density of gravitons, e.g.,

G. Dvali, S. Folkerts, C. Germani, PRD (2011), arXiv:1006.0984

The first effect of ghost confinement is imposing a Planck cut-off on the energy of the gravitational perturbations. In early cosmology, looking back in time, as a cosmic perturbation becomes trans-Planckian, the pair of complex conjugate ghosts is created and gets confined into a bound state.

This situation rules out the observation of the trans-Planckian physics, which was discussed in the literature, starting from

J. Martin and R. Brandenberger, hep-th/0005209; and PRD.
A.A. Starobinsky, astro-ph/0104043 and Letters to ZhETP.
L. Barbado, C. Barcelo, L. Garay and G. Jannes, 1109.3593 & JHEP.

In the case of cosmological perturbations, Planck-order cut-off may be detected by existing or future observational facilities.

Are these particles, with the masses of the Planck order of magnitude, which interact only gravitationally, realistic candidates to be Dark Matter? To address this question, let us make a numerical estimate using super-optimistic approach.

At the moment of creating the bound states, assume that the energy scale is $\mathcal{E}_{in} = M_P$ and the initial energy density of these composite particles is $\rho_{in}^{BS}(M_P) = M_P^4$. Consider that right after this point the inflation starts. Using $\mathcal{E} \sim 1/a(t)$ we get

$$\rho^{BS}(\mathcal{E}) \;\propto\; M_P^4 \Big(\frac{a_{in}}{a}\Big)^3 \quad\Longrightarrow\quad \rho^{BS}(\mathcal{E}_{end}) \;\propto\; M_P^4\, e^{-3N}.$$

For the critical density using the Friedmann equation

$$\rho_c(\mathcal{E}_{end}) \,=\, \frac{3}{8\pi G}\, H_{end}^2 \,=\, \frac{3M_P^2}{8\pi}\, H_{end}^2 .$$

Taking $H_{end} \approx 10^{12}\,\mathrm{GeV}$ for the Hubble parameter at the end of inflation and $N = 70$ we get

$$\rho^{BS}(\mathcal{E}_{end}) \;\propto\; \rho_c(\mathcal{E}_{end}) \,\times\, 10^{-78} .$$

No chance to explain DM with the bound states of ghosts.

Conclusions

- **The construction of QG theory which is not restricted to the IR region, is impossible without higher derivative terms. The same concerns a consistent formulation of semiclassical theory, i.e. quantization of matter fields.**

- **Including more than four derivatives provides theoretical advantages: superrenormalizable QG and well-defined renormalization group flow, free from gauge-fixing ambiguities.**

- **The theories with higher derivatives may be classically stable when Planck-order cut-off is imposed on perturbations.**

- **In the theory with $6+$ derivatives and complex ghosts it is possible to meet a confinement of ghosts into normal bound states, providing aforementioned cut-off.**

- **There is no definite solution to the problem of ghosts. But it looks like we know in which corner the solution will be found.**

Ghost-free HD models of gravity

Consider two examples of ghost-free HD models of gravity.

- **In the (super)string theory, the object of quantization is a kind of non-linear sigma-model in two space-time dimensions.**

Both metric and matter fields are induced, implying unification of all fundamental forces.

The σ-model approach is close to QFT in curved space,

$$S_{str} = \int d^2\sigma\sqrt{g}\,\Big\{\frac{1}{2\alpha'}g^{\mu\nu}G_{ij}(X)\partial_\mu X^i\partial_\nu X^j$$

$$+\frac{1}{\alpha'}\,\frac{\varepsilon^{\mu\nu}}{\sqrt{g}}A_{ij}(X)\partial_\mu X^i\partial_\nu X^j + B(X)R + T(X)\Big\}\,, \quad i,j = 1,2,...,D\,.$$

The Polyakov approach: conditions of anomaly cancellation order by order in α'. Critical dimensions:

D=26 **for bosonic string,** D=10 **for superstrings.**

At the first order in α' the effective equations give GR !

E.S. Fradkin & A. Tseytlin (1985);
C. Callan, D. Friedan, E. Martinec, M. Perry, (1985).

- **Metric reparametrization remove ghosts at all orders in α' .**

In the torsionless case the effective action can be written as

$$S_M = \frac{2}{\kappa^2}\int d^Dx\sqrt{G}\,e^{-2\phi}\Big\{-R+4\,(\partial\phi)^2$$

$$+\alpha'\left(a_1R_{\lambda\mu\nu\rho}R^{\lambda\mu\nu\rho}+a_2R_{\mu\nu}R^{\mu\nu}+a_3R^2\right)\Big\}+\ ...$$

In order to remove ghosts one performs reparametrization of the background metric $G_{\mu\nu}$

$$G_{\mu\nu}\longrightarrow G'_{\mu\nu}=G_{\mu\nu}+\alpha'\,(x_1\,R_{\mu\nu}+x_2\,R\,G_{\mu\nu})+...$$

where $x_{1,2,...}$ are specially tuned parameters.

B. Zweibach, S. Deser & A.N. Redlich, ... A. Tseytlin (1985-1987).

Ghost-killing reparametrization doesn't affect string S-matrix,

$$G_{\mu\nu} \longrightarrow G'_{\mu\nu} = G_{\mu\nu} + \alpha' \left(x_1 R_{\mu\nu} + x_2 R\, G_{\mu\nu} \right) + ...$$

At the same time, Zweibach reparametrization is ambiguous and this actually produce ambiguous physical solutions.

A. Maroto & I.Sh., PLB, hep-th/9706179.

- **Even more subtle point is that the effectively working ghost-killing transformation must be absolutely precise!**

Any infinitesimal change produce a ghost with a huge mass. Moreover, smaller violation of fine-tuning leads to a greater mass of the ghost, hence (according to a "standard wisdom") smaller violation of fine-tuning produce greater gravitational instability.

At low energies we know that the quantum effects are described by QFT, not string theory. Hence, string theory is ghost-free and unitary only if it completely controls QFT, even in the deep IR.

An alternative to Zweibach transformation

In the non-local theory

$$S = -\frac{1}{2\kappa}\int d^4x\sqrt{-g}\,\Big\{R + G_{\mu\nu}\,\frac{a(\Box)-1}{\Box}\,R^{\mu\nu}\Big\}, \quad a(\Box) = e^{-\Box/m^2}\,.$$

A. Tseytlin, PLB, hep-th/9509050.

In this and similar theories propagator of metric perturbations has a single massless pole, corresponding to gravitons.

With this choice there are no ghosts!

The idea is to use Zweibach-like transformation, but arrive at the non-local theory which is non-polynomial in derivatives, instead of “killing” all higher derivatives that one can kill.

One more ambiguity in the (super)string theory.

IR effects of higher derivatives. Gravitational see-saw?

A.Accioly, B.L. Giacchini, I.Sh., EJPC (2017), arXiv:1604.07348.

Simplest superrenormalizable action,

$$S_{grav} = \int d^4x\sqrt{-g}\Big\{ \frac{2}{\kappa^2}R + \frac{\alpha}{2}R^2 + \frac{\beta}{2}R_{\mu\nu}^2 + \frac{A}{2}R\Box R + \frac{B}{2}R_{\mu\nu}\Box R^{\mu\nu}\Big\},$$

Here $\kappa^2 = 32\pi G = 2M_P^{-2}$, and α, β, A, B are free parameters, where the first two are dimensionless, A, $B \sim (\mathrm{mass})^{-2}$ and we assume this mass has Planck order of magnitude.

In the weak-field limit, $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ and $|\kappa h_{\mu\nu}| \ll 1$ one can identify the masses through the poles of the propagator,

$$m_{2\pm}^2 = \frac{\beta \pm \sqrt{\beta^2 + \frac{16}{\kappa^2}B}}{2B}, \quad m_{0\pm}^2 = \frac{\sigma_1 \pm \sqrt{\sigma_1^2 - \frac{8\sigma_2}{\kappa^2}}}{2\sigma_2},$$

with $\sigma_1 \equiv 3\alpha + \beta$ **and** $\sigma_2 \equiv 3A + B$.

The see-saw requires a relation

$$m_{2+}^2 \ll m_{2-}^2 \implies 16|B| \ll \kappa^2\beta^2\,,$$

In the theory where this condition is satisfied the masses can be approximated by

$$m_{2+}^2 \approx \frac{4}{\kappa^2|\beta|} \;\ll\; m_{2-}^2 \approx \frac{\beta}{B}.$$

As in the original neutrino's seesaw mechanism one of the masses depends mainly on only one parameter, while the other depends on both. And this is a very general situation, indeed.

$$\frac{1}{m_0^4}\,k^6 - \frac{3}{m_1^2}\,k^4 + 3\beta\,k^2 - m_2^2 = 0\,.$$

The lightest mass depends only on β, while the largest one depends on both parameters. No see-saw in HDQG !

Is it a good news? There is no threat to the Planck protection against ghosts, if such protection exist. But it will be certainly difficult to observe the effect of higher derivatives.