Spin as detector for axion

Yuri N. Obukhov

Theoretical Physics Laboratory, IBRAE, Russian Academy of Sciences

Talk at E. S. Fradkin Centennial Conference
Lebedev Institute, Moscow, 2-6 September 2024

Outline

Dynamics of spin and fundamental physics

- Study of spin motion of particle with dipole moments (anomalous magnetic and electric one) is important for search of new physics beyond SM (cf: 2408.15691)
- Tests of foundations (Lorentz symmetry, equivalence principle, spacetime structure beyond Riemann, etc)
- Colella-Overhauser-Werner (1975) and Bonse-Wroblewski experiments - equivalence principle for quantum systems: Measured phase shift due to inertial and gravitational force
- Modern applications: heavy ion collisions physics, search for gravitational waves (new type detectors)
- *Challenge: Probe axion physics via spin effects!*
- Possible new role of precessing spin as an “axion antenna” to establish the nature of dark matter in the Universe.

Magnetoelectric effect

- Classical Maxwell's electrodynamics of local linear medium
- Constitutive law for isotropic matter at rest

$$\boldsymbol{D} = \varepsilon\varepsilon_0 \boldsymbol{E} \quad \text{and} \quad \boldsymbol{H} = \frac{1}{\mu\mu_0}\boldsymbol{B}$$

- ε_0 and μ_0 are electric and magnetic constants of vacuum, ε and μ are (relative) permittivity and permeability
- Vacuum admittance and speed of light:

$$Y_0 = \frac{1}{\Omega_0} = \sqrt{\frac{\varepsilon_0}{\mu_0}}, \quad c = \frac{1}{\sqrt{\varepsilon_0\mu_0}}$$

with Ω_0 as vacuum impedance of $\approx 377\Omega$

- Magnetoelectric effect (ME) is characterized by

General local and linear constitutive law reads

$$\begin{aligned} D^a &= \varepsilon_0\, \varepsilon^{ab}\, E_b + Y_0\, {\alpha_1}^a_b\, B^b \\ H_a &= Y_0\, {\alpha_2}_a^b\, E_b + \mu_0^{-1}\, (\mu^{-1})_{ab}\, B^b \end{aligned}$$

- ε_0, Y_0, and μ_0 are required for dimensional consistency.
- ε^{ab}, $(\mu^{-1})_{ab}$, ${\alpha_1}^a_b$, and ${\alpha_2}^a_b$ dimensionless 3×3 matrices = 36 permittivity, permeability and magnetoelectric moduli.
- Nontrivial ${\alpha_1}^a_b$ and ${\alpha_2}^a_b$ predicted by Landau and Lifshitz for certain magnetic crystals.
- Dzyaloshinskii (1959) pointed to antiferromagnet Cr_2O_3. Astrov (1961, for an electric field, ME_E) and Rado & Folen (1962, for a magnetic field, ME_H) confirmed his predictions experimentally for uniaxial crystals of Cr_2O_3.

- In electrical engineering, in theory of two ports (four poles), Tellegen (1948) defined *gyrator* via

$$v_1 = -\,s\,i_2\,, \qquad v_2 = s\,i_1\,,$$

v are voltages and i currents of ports 1 and 2, respectively. Gyrator is nonreciprocal network element.

- Tellegen: *"The ideal gyrator has the property of 'gyrating' a current into a voltage, and vice versa. The coefficient s, which has the dimension of a resistance, we call the gyration resistance; $1/s$ we call the gyration conductance."*
- In terms of electromagnetic field: quantities related to i_1, i_2 are $\boldsymbol{D}$, $\boldsymbol{H}$ and to v_1, v_2 fields $\boldsymbol{E}$, $\boldsymbol{B}$. Thus, with $s = 1/\alpha$,

$$\boldsymbol{E} = -\,s\,\boldsymbol{H}\,, \qquad \boldsymbol{B} = s\,\boldsymbol{D}\,.$$

Gyrator 'rotates' currents into voltages and axion 'rotates' excitations into field strengths.

- Lindell & Sihvola (2005) introduced concept of *perfect electromagnetic conductor* (PEMC). It obeys axion law:

$$\boldsymbol{D} = \alpha\,\boldsymbol{B}\,, \quad \boldsymbol{H} = -\alpha\,\boldsymbol{E}$$

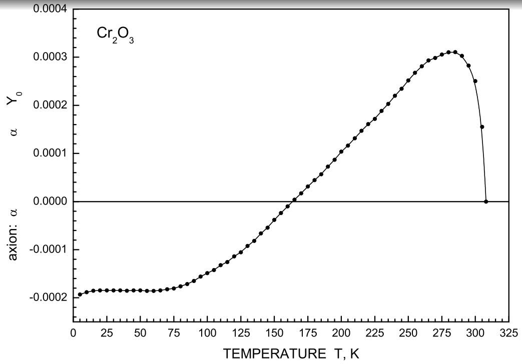
PEMC is a generalization of perfect electric and perfect magnetic conductor. Tretyakov et al (2003) demonstrated possibility to realize PEMC as metamaterial. No energy would propagate therein (⇒ stealth technology).

- *Axion electrodynamics* [Ni(1977)-Wilczek(1987)-Itin(2004)]. Add to vacuum Maxwell-Lorentz an axion piece, then we have constitutive law for axion electrodynamics:

$$H = Y_0 \star F + \alpha\,F, \qquad Y_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

For Cr_2O_3 the axion value was measured $\alpha \approx 10^{-4} Y_0$.

Axion measured in condensed matter



Extracted from Astrov (1961), Rado-Folen (1962) for Cr_2O_3: α in units of Y_0 as a function of temperature T; it is negative for $T < 163$K, positive for $T > 163$K, until it vanishes at Néel temperature ≈ 308K. [See Hehl,YNO, et al, PRA **77** (2008)]

- In high-energy physics, one adds in Lagrangian also the kinetic term of axion $\sim g^{ij}\partial_i\alpha\,\partial_j\alpha$ and the corresponding mass term $\sim m^2_{(a)}\,\alpha^2$. However, such a hypothetical P-odd and T-odd particle has not been found so far, in spite of considerable experimental efforts.
- We see that same properties are shared by
  - α in condensed matter (measured for Cr_2O_3)
  - gyrator concept
  - PEMC metamaterial
  - axion particle
- Frank Wilczek commented on these 4 structures: *"It's a nice demonstration of the unity of physics."*
- Could a detector made of some suitable matter (such as Cr_2O_3 crystals, e.g.) enhance probability of finding axions?

Gravitoelectromagnetism in Einstein's general relativity

- Let t be time, $\boldsymbol{x} = \{x^a\}$ $(a = 1, 2, 3)$ be spatial coordinates:

$$ds^2 = \left(1 - \frac{\mathit{\Phi}}{c^2}\right)^2 c^2 dt^2 + \frac{4}{c}(\boldsymbol{\mathcal{A}} \cdot d\boldsymbol{x})dt - \left(1 + \frac{\mathit{\Phi}}{c^2}\right)^2 d\boldsymbol{x} \cdot d\boldsymbol{x},$$

  with gravitoelectric $\mathit{\Phi}$ and the gravitomagnetic $\boldsymbol{\mathcal{A}}$ potentials.
- Notation: distinguish gravitoelectromagnetic potentials $(\mathit{\Phi}, \boldsymbol{\mathcal{A}})$ them from electromagnetic potentials $A_i = (\Phi, \boldsymbol{A})$.
- For a body with mass M and angular momentum $\boldsymbol{J}$, the gravitoelectromagnetic fields are (Lense-Thirring, 1918):

$$\mathit{\Phi} = \frac{GM}{r}, \qquad \boldsymbol{\mathcal{A}} = \frac{G\,\boldsymbol{J} \times \boldsymbol{r}}{c\,r^3}$$

  Here G is Newton's gravitational constant.

Electrodynamics in curved spacetime

- Gravity is universal: affects also electromagnetism.
- Basic objects: field strength F, excitation H and current J

Maxwell's theory – without coordinates and frames

$$dF = 0, \qquad dH = J$$

- Decompose 2-forms $H = (\boldsymbol{H}, \boldsymbol{D})$ and $F = (\boldsymbol{E}, \boldsymbol{B})$ into 3-vector components $\Longrightarrow$ recast Maxwell equations into

$$\boldsymbol{\nabla} \times \boldsymbol{E} + \dot{\boldsymbol{B}} = 0, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{B} = 0,$$
$$\boldsymbol{\nabla} \times \boldsymbol{H} - \dot{\boldsymbol{D}} = \boldsymbol{J}^{\rm e}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{D} = \rho^{\rm e}$$

Influence of inertia and gravity is encoded in constitutive relation between electric and magnetic fields $\boldsymbol{E}, \boldsymbol{B}$ and electric and magnetic excitations $\boldsymbol{D}, \boldsymbol{H}$.

Axion electrodynamics: constitutive law

$$H = Y_0 \star F + \alpha\, F, \qquad Y_0 = \sqrt{\frac{\varepsilon_0}{\mu_0}}$$

- Specializing to gravitoelectromagnetic geometry, explicit constitutive relation of the axion electrodynamics reads

$$\begin{aligned} \boldsymbol{D} &= \varepsilon_0\varepsilon_g\,\boldsymbol{E} + \frac{Y_0}{c^2}\,\boldsymbol{\mathcal{A}}\times\boldsymbol{B} + \alpha\,\boldsymbol{B}, \\ \boldsymbol{H} &= \frac{1}{\mu_0\mu_g}\,\boldsymbol{B} + \frac{Y_0}{c^2}\,\boldsymbol{\mathcal{A}}\times\boldsymbol{E} - \alpha\,\boldsymbol{E}. \end{aligned}$$

- Effective "medium" is determined by gravity: gravitoelectric Φ describes effective permittivity and permeability

$$\varepsilon_g = \mu_g = \left(1 + \frac{\Phi}{c^2}\right)^2,$$

gravitomagnetic $\boldsymbol{\mathcal{A}}$ responsible for magnetoelectric effects.

Dirac particle in external fields

- Fermion with rest mass m, charge q, EDM & AMM

$$L = \frac{i\hbar}{2}\left(\overline{\Psi}\gamma^\mu D_\mu\Psi - D_\mu\overline{\Psi}\gamma^\mu\Psi\right) - mc\,\overline{\Psi}\Psi$$
$$+ \frac{\mu'}{2c}\overline{\Psi}\sigma^{\mu\nu}\Psi F_{\mu\nu} + \frac{\delta'}{2}\overline{\Psi}\sigma^{\mu\nu}\Psi\widetilde{F}_{\mu\nu} - \frac{\hbar\,g_f}{2f_{(a)}}\,\overline{\Psi}\gamma^\mu\gamma_5\Psi\left(e^i_\mu\partial_i\alpha\right)$$

- Spinor covariant derivative (with $\sigma_{\alpha\beta} = i\gamma_{[\alpha}\gamma_{\beta]}$)

$$D_\mu = e^i_\mu D_i, \qquad D_i = \partial_i - \frac{iq}{\hbar}A_i + \frac{i}{4}\sigma_{\alpha\beta}\Gamma_i{}^{\alpha\beta}$$

  describes minimal coupling with gauge fields $(A_i, e^\alpha_i, \Gamma_i{}^{\beta\gamma})$.
- Pauli terms with $F_{\alpha\beta}$ and dual $\widetilde{F}_{\alpha\beta} = \frac{1}{2}\eta_{\alpha\beta\mu\nu}F^{\mu\nu}$ describe non-minimal coupling to AMM and EDM of fermion

$$\mu' = a\,\frac{q\hbar}{2m}, \qquad \delta' = b\,\frac{q\hbar}{2mc}, \qquad a = \frac{g-2}{2}$$

- Axion coupling $g_f \sim 1$ and $f_{(a)}m_{(a)} \approx f_\pi m_\pi \frac{\sqrt{m_u m_d}}{m_u + m_d}$.

Dirac Hamiltonian $\mathcal{H} = \mathcal{H}^{\rm GEM} + \mathcal{H}^{\rm ax}$ (with $\boldsymbol{\pi} = -i\hbar\boldsymbol{\nabla} - q\boldsymbol{A}$)

$$\mathcal{H}^{\rm GEM} = mc^2\beta^g + q\Phi + \frac{c}{2}\left(\boldsymbol{\pi}\cdot\boldsymbol{\alpha}^g + \boldsymbol{\alpha}^g\cdot\boldsymbol{\pi}\right)$$
$$+ \frac{\hbar}{2c}\,\boldsymbol{\Sigma}\cdot(\boldsymbol{\nabla}\times\boldsymbol{\mathcal{A}}) - \beta^g\left(\boldsymbol{\Sigma}\cdot\boldsymbol{\mathcal{M}} + i\boldsymbol{\alpha}\cdot\boldsymbol{\mathcal{P}}\right),$$
$$\mathcal{H}^{\rm ax} = \frac{\hbar}{2}\,\frac{g_f}{f_{(a)}}\left[\frac{c}{\mu_g}\boldsymbol{\Sigma}\cdot\boldsymbol{\nabla}\alpha - \gamma_5\left(\partial_t\alpha + \frac{2}{c}\boldsymbol{\mathcal{A}}\cdot\boldsymbol{\nabla}\alpha\right)\right]$$

- Here $\beta^g := \frac{\beta}{1+\frac{\Phi}{c^2}}$, $\boldsymbol{\alpha}^g := \frac{\boldsymbol{\alpha}}{\mu_g} + \frac{2}{c^2}\boldsymbol{\mathcal{A}}$, and $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$,

$$\boldsymbol{\alpha} = \begin{pmatrix} 0 & \boldsymbol{\sigma} \\ \boldsymbol{\sigma} & 0 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\sigma} & 0 \\ 0 & \boldsymbol{\sigma} \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & -I \\ -I & 0 \end{pmatrix}$$

- Last term in $\mathcal{H}^{\rm GEM}$ accounts for polarization&magnetization

$$\boldsymbol{\mathcal{M}} = \mu'\boldsymbol{\mathfrak{B}} + \delta'\boldsymbol{\mathfrak{E}} = \frac{\hbar q}{2m}\left(a\,\boldsymbol{\mathfrak{B}} + \frac{b}{c}\,\boldsymbol{\mathfrak{E}}\right),$$
$$\boldsymbol{\mathcal{P}} = c\delta'\boldsymbol{\mathfrak{B}} - \frac{\mu'}{c}\,\boldsymbol{\mathfrak{E}} = \frac{\hbar q}{2m}\left(b\,\boldsymbol{\mathfrak{B}} - \frac{a}{c}\,\boldsymbol{\mathfrak{E}}\right),$$
$$\boldsymbol{\mathfrak{E}} = \boldsymbol{E} + \frac{2}{c}\boldsymbol{\mathcal{A}}\times\boldsymbol{B}, \qquad \boldsymbol{\mathfrak{B}} = \frac{1}{\mu_g}\,\boldsymbol{B}$$

Quantum dynamics of spinning particle

Foldy-Wouthuysen representation: Semiclassical Hamiltonian

$$\mathcal{H}_{FW} = \frac{1}{1+\frac{\Phi}{c^2}}\left[mc^2 + \frac{1}{2m}\left(\boldsymbol{\pi} + \frac{2m}{c}\boldsymbol{\mathcal{A}}\right)^2\right] + q\Phi + \frac{\hbar}{2}\boldsymbol{\sigma}\cdot\boldsymbol{\Omega}$$

- Evolution of physical spin

$$\frac{d\boldsymbol{s}}{dt} = \boldsymbol{\Omega}\times\boldsymbol{s}, \qquad \boldsymbol{\Omega} = \boldsymbol{\Omega}^{\rm em} + \boldsymbol{\Omega}^{\rm dip} + \boldsymbol{\Omega}^{\rm GEM} + \boldsymbol{\Omega}^{\rm ax}$$

- Precession angular velocity is the sum of four terms:

$$\boldsymbol{\Omega}^{\rm em} = \frac{q}{m}\left[-\frac{1}{\gamma}\mathfrak{B} + \frac{1}{\gamma+1}\frac{\widehat{\boldsymbol{v}}\times\mathfrak{E}}{c^2}\right],$$

$$\boldsymbol{\Omega}^{\rm dip} = -\frac{q}{m}\left\{\left[a\left(\mathfrak{B} - \frac{\widehat{\boldsymbol{v}}\times\mathfrak{E}}{c^2} - \frac{\gamma}{\gamma+1}\frac{\widehat{\boldsymbol{v}}\,(\widehat{\boldsymbol{v}}\cdot\mathfrak{B})}{c^2}\right) + \frac{b}{c}\left(\mathfrak{E} + \widehat{\boldsymbol{v}}\times\mathfrak{B} - \frac{\gamma}{\gamma+1}\frac{\widehat{\boldsymbol{v}}\,(\widehat{\boldsymbol{v}}\cdot\mathfrak{E})}{c^2}\right)\right]\right\},$$

$$\boldsymbol{\Omega}^{\rm GEM} = \frac{1}{c}\boldsymbol{\nabla}\times\boldsymbol{\mathcal{A}} + \frac{(2\gamma+1)}{(\gamma+1)c^2}\widehat{\boldsymbol{v}}\times\boldsymbol{\nabla}\Phi,$$

$$\boldsymbol{\Omega}^{\rm ax} = \frac{g_f}{f_{(a)}}\frac{1}{\left(1+\frac{\Phi}{c^2}\right)}\left\{\frac{c}{\gamma}\boldsymbol{\nabla}\alpha + \frac{\widehat{\boldsymbol{v}}}{c}\left[\mu_g\left(\partial_t\alpha + \frac{2}{c}\boldsymbol{\mathcal{A}}\cdot\boldsymbol{\nabla}\alpha\right) + \frac{\gamma}{\gamma+1}\widehat{\boldsymbol{v}}\cdot\boldsymbol{\nabla}\alpha\right]\right\}$$

Spin as an "axion detector"

- Particular case: rotating massive body $\boldsymbol{\Omega}^{\rm GEM} = \boldsymbol{\Omega}^{\rm dS} + \boldsymbol{\Omega}^{\rm LT}$ ("de Sitter precession" plus "Lense-Thirring precession"):

$$\boldsymbol{\Omega}^{\rm dS} = \frac{(2\gamma+1)}{(\gamma+1)}\,\frac{GM\,\boldsymbol{r}\times\widehat{\boldsymbol{v}}}{c^2\,r^3}, \qquad \boldsymbol{\Omega}^{\rm LT} = \frac{G}{c^2\,r^3}\left[\frac{3(\boldsymbol{J}\cdot\boldsymbol{r})\,\boldsymbol{r}}{r^2} - \boldsymbol{J}\right]$$

  Validity of this result confirmed in Gravity Probe B mission.
- General: "mixing" of axion effects with inertial/gravitational.
- For experiments in accelerators on Earth that rotates $\boldsymbol{\omega}_\oplus$, one has $\Phi = 1$ and $\mathcal{A} = -\,c\boldsymbol{v}^{\rm rot}/2$, with $\boldsymbol{v}^{\rm rot} = \boldsymbol{\omega}_\oplus \times \boldsymbol{r}$:

$$\mathcal{H}^{\rm ax}_{FW} = \frac{\hbar c g_f}{2f_{(a)}}\,\boldsymbol{\sigma}\cdot\left[\boldsymbol{\nabla}\alpha + \frac{\boldsymbol{p}}{mc^2}\,\left(\frac{\partial\alpha}{\partial t} + \boldsymbol{v}^{\rm rot}\cdot\boldsymbol{\nabla}\alpha\right)\right]$$

  This extends flat space results (Silenko, Nikolaev, 2022).
- Expect 10^3 times larger axion wind effect in storage rings.
- "Axion antenna" search planned at NICA, COSY, and PTR.

Classical spin in external fields

- Classical theory of spin was developed (Frenkel, Thomas, 1926) soon after spin concept was proposed. This model is used for dynamics of polarized particles in accelerators.
- Neglecting second-order spin effects, dynamical equations

$$
\begin{aligned}
\frac{DU^\alpha}{d\tau} &= -\frac{q}{m}\,F^\alpha{}_\beta\,U^\beta,\\
\frac{DS^\alpha}{d\tau} &= -\frac{q}{m}\,F^\alpha{}_\beta S^\beta + \frac{g_f}{f_{(a)}}\,\eta^{\alpha\beta\gamma\delta}U_\delta\left(e^i_\gamma\,\partial_i\alpha\right)S_\beta\\
&\quad -\frac{2}{\hbar}\left[M^\alpha{}_\beta + \frac{1}{c^2}U^\gamma\left(U^\alpha M_{\beta\gamma} - U_\beta M^\alpha{}_\gamma\right)\right]S^\beta
\end{aligned}
$$

- U^α velocity, S^α spin, polarization tensor $M_{\alpha\beta} = \mu' F_{\alpha\beta} + c\delta'\widetilde{F}_{\alpha\beta}$
- Full agreement established between quantum-mechanical theory and classical Frenkel-Thomas-BMT model of spin.
- See also: Balakin-Popov (2015), Dvornikov (2019)

Conclusions and Outlook

- Dynamics of spin in external electromagnetic, gravitational, and axion fields is analyzed in the gravitoelectromagnetism approach in Einstein's general relativity theory.
- Possible extension: interaction mechanism of axion with particle's spin via EDM Pauli term $\frac{\delta'}{2}\overline{\Psi}\sigma^{\alpha\beta}\Psi\widetilde{F}_{\alpha\beta}$ amounts to shift of EDM parameter $b = b_0 + \kappa_d\alpha/f_{(a)}$, with b_0 for constant EDM, and dimensionless model-dependent factor $\kappa_d \approx 10^{-2}$. This produces an oscillating contribution in the precession angular velocity for the classical axion field $\alpha = \alpha_0 \cos(\omega_{(a)} - \boldsymbol{k}_{(a)} \cdot \boldsymbol{x})$ in the invisible halo of our Galaxy.
- Deuteron: Karanth et al, *Phys. Rev. X* **13** (2023) 031004
- Details: YNO, *Int. J. Mod. Phys. A* **38** (2023) 2342002 ; Vergeles, Nikolaev, YNO, Silenko, Teryaev, *Phys. Usp.* **66** (2023) 109

Thanks !