Restricted Formalism and Gravity Modifications

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Basic ideas
A.O. Barvinsky and D.V. Nesterov, "Restricted gauge theory formalism and unimodular gravity," Phys. Rev. D **108** (2023) no.6, 065004, arXiv:2212.13539 [hep-th].

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Restriction of gauge theories

Parental gauge theory

Consider generic gauge theory $\overset{\curlywedge}{S}[\,\varphi^I\,]$ subject to a closed algebra of *irreducible* local gauge generators $\overset{\curlywedge}{\mathcal{R}}{}^I_\alpha$,

$$\overset{\curlywedge}{S}_{,I}\overset{\curlywedge}{\mathcal{R}}{}^I_\alpha = 0, \qquad \overset{\curlywedge}{\mathcal{R}}{}^I_{\alpha,J}\overset{\curlywedge}{\mathcal{R}}{}^J_\beta - \overset{\curlywedge}{\mathcal{R}}{}^I_{\beta,J}\overset{\curlywedge}{\mathcal{R}}{}^J_\alpha = \overset{\curlywedge}{\mathcal{R}}{}^I_\gamma \overset{\curlywedge}{C}{}^\gamma_{\alpha\beta}. \tag{1}$$

⊛ Irreducibility: rank $\overset{\curlywedge}{\mathcal{R}}{}^I_\alpha = \text{range}\,\alpha \equiv m_0 \quad (m_0 < \overset{\curlywedge}{n} \equiv \text{range}\, I)$

Restricted theory

Restricted theory, originating from the *parental* one, is the theory with configuration space constrained by the equations

$$\theta^a(\varphi^I) = 0. \tag{2}$$

⊛ Consider functions $\theta^a(\varphi^I)$ to be independent: rank $\theta^a_{,I} = \text{range}\, a \equiv m_1$

Representations of the restricted theory

Representation on extended configuration space

$$S^{\lambda}[\varphi,\lambda] = \overset{\curlywedge}{S}[\varphi] - \lambda_a \theta^a(\varphi), \tag{3}$$

Reduced configuration space representation

Let restriction surface, $\Sigma_\theta :\ \theta^a[\varphi] = 0$, be parametrized via fields ϕ^i

$$\varphi^I = e^I(\phi), \qquad \theta^a\big(e^I(\phi)\big) \equiv 0 \tag{4}$$

Then action of the restricted theory in equivalent reduced representation

$$S^{\lambda}[\varphi,\lambda] \ \leftrightarrow\ S^{\rm red}[\phi] \tag{5}$$

is just

$$S^{\rm red}[\phi] = \overset{\curlywedge}{S}[e(\phi)] \tag{6}$$

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Residual gauge symmetry

$$\begin{array}{c}\text{Symmetry of } \overset{\curlywedge}{S}\text{:}\\ \overset{\curlywedge}{\delta}_\xi \overset{\curlywedge}{S} = \overset{\curlywedge}{S}_{,I}\overset{\curlywedge}{\mathcal{R}}^I_\alpha \xi^\alpha = 0\end{array} \longrightarrow \begin{array}{c}\text{Residual symmetry of } S^\lambda\text{:}\\ \overset{\curlywedge}{\delta}_\xi \overset{\curlywedge}{S} - \lambda_a \overset{\curlywedge}{\delta}_\xi \theta^a = 0 - \lambda_a \theta^a_{,I}\overset{\curlywedge}{\mathcal{R}}^I_\alpha \xi^\alpha := 0\end{array} \tag{7}$$

Thus residual symmetry of S^λ is defined by restriction on gauge parameters

$$\xi^\alpha = \xi^\alpha(\varepsilon): \qquad \theta^a_{,I}\overset{\curlywedge}{\mathcal{R}}^I_\alpha \xi^\alpha(\varepsilon) \equiv Q^a_\alpha \xi^\alpha(\varepsilon) \equiv 0 \tag{8}$$

ε^ρ — some set of infinitesimal parameters of the residual gauge symmetry

$$\boxed{Q^a_\alpha \equiv \theta^a_{,I}\overset{\curlywedge}{\mathcal{R}}^I_\alpha} \text{ — } \underline{\text{gauge-restriction operator}} \tag{9}$$

Its rank properties define basic dynamical and gauge properties of the restricted theory

Iff Q^a_α — full-rank (rank $Q^a_\alpha = m_1$), then *restriction* is just *gauge fixing*.

⊛ Regularity assumption

$$\text{rank } Q^a_\alpha\big|_{\Sigma_\theta} \equiv m_1 - m_2 = const, \qquad m_2 \text{ — rank deficit (at } \Sigma_\theta) \tag{10}$$

This imply range $\rho = m_0 - m_1 + m_2$ for the space of residual gauge parameters ε^ρ

Residual gauge symmetry

Symmetry of $\hat{S}$: $\longrightarrow$ Residual symmetry of S^λ:

$$\hat{\delta}_\xi \hat{S} = \hat{S}_{,I} \hat{\mathcal{R}}^I_\alpha \xi^\alpha = 0 \quad \longrightarrow \quad \hat{\delta}_\xi \hat{S} - \lambda_a \hat{\delta}_\xi \theta^a = 0 - \lambda_a \theta^a_{,I} \hat{\mathcal{R}}^I_\alpha \xi^\alpha := 0 \tag{7}$$

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Restricted theory dynamics

Generically physical space of the restricted theory is not the subspace of that of the parental theory.

$$\text{EoM for } \overset{\curlywedge}{S}: \quad \overset{\curlywedge}{S}_{,I} \cong 0 \quad \longrightarrow \quad \text{EoM for } S^{\lambda}: \quad \begin{cases} \overset{\curlywedge}{S}_{,I} - \lambda_a \, \theta^a_{,I} \cong 0, \\ \theta^a \cong 0. \end{cases} \tag{11}$$

Theories are classically equivalent if on shell $\lambda_a \cong 0$ and $Q^a_{\ \alpha}$— full-rank.

The latter two conditions are not independent. They are equivalent.

On shell behavior of λ_a

Gauge restriction operator $Q^a_{\ \alpha}$ defines equations of motion for λ_a

$$\lambda_a Q^a_{\ \alpha} = 0 \tag{12}$$

On shell λ_a — left kernel of $Q^a_{\ \alpha}$.

If $Q^a_{\ \alpha}$ — full-rank, then on shell $\lambda_a \cong 0$, solutions of restricted theory are subfamily of solutions of the parental theory.

If $Q^a_{\ \alpha}$ — rank-deficient, then under regularity conditions on θ^a, for $\lambda_a \neq 0$ there are new solutions absent in the parental theory.

Effective action

Suppose the parental gauge theory $\overset{\curlywedge}{S}[\varphi^I]$ is well-elaborated. In particular we know expressions for its quantum effective action (EA).

Can we use this knowledge to construct EA of the restricted theory $S^{\text{red}}[\phi^i]$?

Yes, We Can!

Morover,
we can preserve configuration-space and gauge covariance of the parental theory

Configuration space reduction: $\varphi^I \to \phi^i$. Field operators a_{ij} in the restricted theory
Residual gauge symmetry: $\xi^\alpha \to \varepsilon^\rho$. Ghost-sector operators b^ρ_σ in the restricted theory

We can construct one-loop EA in terms of determinants of operators A_{IJ} and B^α_β

Price to be paid: reducible “parental-covariant” gauge generator set

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Price to be paid: reducible "parental-covariant" gauge generator set

Reducible gauge generators of the restricted theory

Residual symmetries

Instead of working with irreducible set of residual parameters ε^ρ

$$\xi^\alpha = \xi^\alpha(\varepsilon): \qquad Q^a_\alpha \xi^\alpha(\varepsilon) \equiv 0 \tag{13}$$

where $Q^a_\alpha \equiv \theta^a_{,I}\, \overset{\curlywedge}{\mathcal{R}}{}^I_\alpha$ one can use the *reducible* set of gauge parameters

$$\xi^\alpha_{reducible} = T^\alpha_\beta\, \xi^\beta, \qquad T^\alpha_\beta \text{ — projector: } \; Q^a_\alpha\, T^\alpha_\beta = 0 \tag{14}$$

Convenient alternative

Even more convenient: to formulate the residual gauge symmetries

$$\delta_\xi \varphi^I = \overset{\curlywedge}{\mathcal{R}}{}^I_\alpha T^\alpha_\beta\, \xi^\beta \tag{15}$$

with *free* gauge parameters ξ^α, projecting instead the parental generators $\overset{\curlywedge}{\mathcal{R}}{}^I_\alpha$

$$\boxed{\mathcal{R}^I_\beta \equiv \overset{\curlywedge}{\mathcal{R}}{}^I_\alpha\, T^\alpha_\beta} \quad \text{— } \textit{reducible} \text{ set of gauge generators} \tag{16}$$

Projector

$$\boxed{T^\alpha_\beta = \delta^\alpha_\beta - k^\alpha_a\, (Qk)^{-1\,a}_{\;\;\;\,b}\, Q^b_\beta} \tag{17}$$

k^α_a — arbitrary structure with rank condition $\quad \mathrm{rank}\ (Qk)^a_b = \mathrm{rank}\ Q^a_\alpha = \mathrm{rank}\ k^\beta_b$

Theorem

Provided the parental theory had gauge algebra

$$\hat{\mathcal{R}}^I_{\alpha,J}\hat{\mathcal{R}}^J_\beta - \hat{\mathcal{R}}^I_{\beta,J}\hat{\mathcal{R}}^J_\alpha = \hat{\mathcal{R}}^I_\gamma \hat{C}^\gamma_{\alpha\beta} + \hat{E}^{IJ}_{\alpha\beta}\hat{S}_{,J}, \qquad (18)$$

the projected gauge generators of the restricted theory satisfy

$$\mathcal{R}^I_{\alpha,J}\mathcal{R}^J_\beta - \mathcal{R}^I_{\beta,J}\mathcal{R}^J_\alpha = \mathcal{R}^I_\gamma C^\gamma_{\alpha\beta} + E^{IJ}_{\alpha\beta}\hat{S}_{,J} \qquad (19)$$

with new structure functions $C^\gamma_{\alpha\beta}$ *and* $E^{IJ}_{\alpha\beta}$

$$C^\gamma_{\alpha\beta} \equiv T^\gamma_\zeta \hat{C}^\zeta_{\delta\epsilon} T^\delta_{\ \alpha} T^\epsilon_{\ \beta} + N^\gamma_{\ \beta\alpha} - N^\gamma_{\ \alpha\beta}, \qquad (20)$$

$$E^{IJ}_{\alpha\beta} \equiv D^I_{\ K} D^J_{\ L} \hat{E}^{KL}_{\gamma\,\delta} T^\gamma_{\ \alpha} T^{\ \delta}_{\beta}, \qquad (21)$$

where

$$N^\gamma_{\ \alpha\beta} \equiv T^\gamma_{\ \delta} k^{\ \delta}_{a,J} (Qk)^{-1\,a}_{\ \ \ b} Q^b_{\ \alpha} \mathcal{R}^J_{\ \beta}, \qquad (22)$$

$$D^I_{\ J} \equiv \delta^I_{\ J} - \hat{\mathcal{R}}^I_{\ \epsilon} k^\epsilon_{\ a} (Qk)^{-1\,a}_{\ \ \ b} \theta^b_{\ ,J}. \qquad (23)$$

- For $E^{KL}_{\gamma\,\delta} \sim \hat{E}^{KL}_{\gamma\,\delta} \neq 0$ (21) is not an open algebra of generators for the restricted theory due to the last term.
- If the parental algebra (18) is *closed*, $\hat{E}^{KL}_{\gamma\,\delta} = 0$, one gets the *closed* algebra of the residual generators $\mathcal{R}^I_\alpha$.

Restricted gauge theory setup with reducible generators

- Initial gauge invariant action

$$S^{\lambda}[\,\varphi^I, \lambda_a\,] \qquad \Big| \qquad S^{\text{red}}[\,\phi^i\,] \tag{24}$$

- Reducible gauge generators $(\alpha, \beta ... = 1, \ldots, m_0)$

$$\mathcal{R}^I_\alpha = \overset{\curlywedge}{\mathcal{R}}{}^I_\beta T^\beta_\alpha \;:\quad S^\lambda_{,I}\mathcal{R}^I_\alpha = 0 \qquad \Big| \qquad \mathcal{R}^i_\alpha = e^i\mathcal{R}^I_\alpha \;:\quad S^{\text{red}}_{,i}\mathcal{R}^i_\alpha = 0 \tag{25}$$

- First-stage reducibility generators $(a, b ... = 1, \ldots, m_1 < m_0)$

$$\mathcal{Z}^\alpha_{\ a} = k^\alpha_{\ b}\mu^b_{\ a} \;:\quad \mathcal{R}^I_\alpha \mathcal{Z}^\alpha_{\ a} = \mathcal{R}^i_\alpha \mathcal{Z}^\alpha_{\ a} = 0 \tag{26}$$

$k^\alpha_{\ b}$ — projector parameter from $T^\alpha_{\ \beta} = \delta^\alpha_\beta - k^\alpha_{\ a}\,(Qk)^{-1\,a}_{\ \ \ b}\,Q^b_{\ \beta}$, (17)

$\mu^b_{\ a}$ — arbitrary operator so that rank $k^\alpha_{\ a}$ = rank $k^\alpha_{\ b}\mu^b_{\ a}$ (will be fixed later)

- Second-stage reducibility generators $(A, B ... = 1, \ldots, m_2 < m_1)$
  For generic restriction, which is not a gauge fixing, there exist

$$\mathcal{Z}^a_{\ A} \;:\quad \mathcal{Z}^\alpha_{\ a}\mathcal{Z}^a_{\ A} = 0 \tag{27}$$

Restricted gauge theory setup with reducible generators

- Initial gauge invariant action

$$S^{\lambda}[\varphi^I, \lambda_a] \qquad \Big| \qquad S^{\rm red}[\phi^i] \tag{24}$$

- Reducible gauge generators $(\alpha, \beta ... = 1, \dots, m_0)$

$$\mathcal{R}^I_\alpha = \hat{\mathcal{R}}^I_\beta T^\beta_\alpha \;:\; S^\lambda_{,I}\mathcal{R}^I_\alpha = 0 \qquad \Big| \qquad \mathcal{R}^i_\alpha = e^i \mathcal{R}^I_\alpha \;:\; S^{\rm red}_{,i}\mathcal{R}^i_\alpha = 0 \tag{25}$$

- First-stage reducibility generators $(a, b ... = 1, \dots, m_1 < m_0)$

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k^α_b — projector parameter from $T^\alpha_\beta = \delta^\alpha_\beta - k^\alpha_a (Qk)^{-1\,a}_{\;\;\;b} Q^b_\beta$, (17)

μ^b_a — arbitrary operator so that rank k^α_a = rank $k^\alpha_b \mu^b_a$ (will be fixed later)

- Alternatively:
  - Quantities with indices a (except θ^a and its derivatives) consider as defined on the functional space of dimensionality $m_1 - m_2$ (orthogonal to $\mathcal{Z}^a_A$)
  - Inverses of m_2-degenerate operators B^a_b are defined in the Moore-Penrose sence

Modification allows to use 1-stage reducible BV without loosing gauge covariance

Batalin-Vilkovisky procedure for 1-stage reducible gauge theories

Batalin-Vilkovisky configuration space

BV procedure for action $S^{\rm red}[\,\phi^i\,]$ with reducible generators $\mathcal{R}^i_\alpha = e^i_I \mathcal{R}^I_\alpha$ imply configuration space extension

$$\Phi \;=\; (\underbrace{\phi^i, \mathcal{C}^\alpha, \mathcal{C}^a}_{\Phi_{\rm min}},\ \bar{\mathcal{C}}_\alpha, \bar{\mathcal{C}}_a, \mathcal{C}'^a, \pi_\alpha, \pi_a, \pi'^a),$$

$$\Phi^* \;=\; (\underbrace{\phi^*_i, \mathcal{C}^*_\alpha, \mathcal{C}^*_a}_{\Phi^*_{\rm min}},\ \bar{\mathcal{C}}^{*\,\alpha}, \bar{\mathcal{C}}^{*\,a}, \mathcal{C}'^*_a, \pi^{*\,\alpha}, \pi^{*\,a}, \pi'^*_a).$$

Φ	$gh\,(\Phi)$	Φ^*	$gh\,(\Phi^*)$
minimal sector			
ϕ^i	0	ϕ^*_i	-1
$\mathcal{C}^\alpha$	$+1$	$\mathcal{C}^*_\alpha$	-2
$\mathcal{C}^a$	$+2$	$\mathcal{C}^*_a$	-3
auxiliary sector			
$\bar{\mathcal{C}}_\alpha$	-1	$\bar{\mathcal{C}}^{*\,\alpha}$	0
π_α	0		
$\bar{\mathcal{C}}_a$	-2	$\bar{\mathcal{C}}^{*\,a}$	$+1$
π_a	-1		
$\mathcal{C}'^a$	0	$\mathcal{C}'^*_a$	-1
π'^a	$+1$		

BV master action

Proper solution $S^{BV}[\Phi,\Phi^*] = S^{\rm min} + S^{\rm aux}$ of the master equation $\left(S^{BV}, S^{BV}\right) = 0$

$$S^{\rm min} \;=\; S^{\rm red}[\phi] + \phi^*_i \mathcal{R}^i_\alpha \mathcal{C}^\alpha + \mathcal{C}^*_\alpha \mathcal{Z}^\alpha_a \mathcal{C}^a + \dots\,, \qquad S^{\rm aux} \;=\; \pi_\alpha \bar{\mathcal{C}}^{*\,\alpha} + \pi_a \bar{\mathcal{C}}^{*\,a} + \mathcal{C}'^*_a \pi'^a.$$

Gauge fixing

$$\Psi[\,\Phi\,] \;=\; \bar{\mathcal{C}}_\alpha \big(\underbrace{\chi^\alpha(\phi) + \sigma^\alpha_a(\phi)\,\mathcal{C}'^a}_{X^\alpha(\phi,\mathcal{C}'^a)}\big) + \bar{\mathcal{C}}_a\,\omega^a_\alpha(\phi)\,\mathcal{C}^\alpha \;+\; \tfrac{1}{2}\,\big(\bar{\mathcal{C}}_\alpha \varkappa^{\alpha\beta}\pi_\beta + \bar{\mathcal{C}}_a \rho^a_b \pi'^b + \pi_a \rho^a_b \mathcal{C}'^b\big).$$

Gaussian gauge fixing: $\det \varkappa^{\alpha\beta} \neq 0$, " $\det \rho^a_b$ " $\neq 0$.

Gauge-fixed action and generating functional

$$S_\Psi[\Phi] = S^{BV}\left[\Phi, \delta\Psi[\Phi]/\delta\Phi\right] \quad \to \quad Z = \int D\Phi\, e^{i\,S_\Psi[\Phi]} \tag{29}$$

(BV procedure defines generating functional modulo contribution of local measure)

Integration over auxiliary fields $\pi_\alpha, \pi_a, \pi'^a$ gives

$$Z = \int D\Phi_{\rm red}\, \left(\det \varkappa^{\alpha\beta}\right)^{-1/2} \det \rho^a_b\;\, e^{i\,S^{FP}[\Phi_{\rm red}]}, \tag{30}$$

$\Phi_{\rm red}$ — the reduced set of BV fields $\Phi_{\rm red} = (\phi^i, \mathcal{C}'^a, \mathcal{C}^\alpha, \bar{\mathcal{C}}_\alpha, \mathcal{C}^a, \bar{\mathcal{C}}_a)$
$S^{FP}[\Phi_{\rm red}]$ — the Faddeev-Popov action

$$S^{FP}[\Phi_{\rm red}] = \underbrace{S^{\rm red}[\phi] - \tfrac{1}{2} X^\alpha \varkappa_{\alpha\beta} X^\beta}_{S^{\rm gf}[\phi,\mathcal{C}']} + \bar{\mathcal{C}}_\alpha \underbrace{\left(X^\alpha_{,i} \mathcal{R}^i_\beta - \sigma^\alpha_a\, \rho^{-1}{}^a_b\, \omega^b_\beta\right)}_{F^\alpha_\beta} \mathcal{C}^\beta + \bar{\mathcal{C}}_a \underbrace{\left(\omega^a_\alpha \mathcal{Z}^\alpha_b\right)}_{F^a_b} \mathcal{C}^b + \ldots$$

Restricted theory specific — the particular structure of $\mathcal{R}^i_\beta$ and $\mathcal{Z}^\alpha_b$.

The effective action should be k-independent, which suggests

$$\mathcal{Z}^\alpha_b = k^\alpha_a\, (Qk)^{-1}{}^a_b \tag{32}$$

This agrees with canonical (BFV) normalization of the path integral measure.

One-loop effective action

$$Z^{1-\text{loop}}_{restricted} = e^{i\Gamma^{1-\text{loop}}} = \frac{\det \rho^a_b}{(\det \varkappa^{\alpha\beta})^{1/2}} \frac{\det F^\alpha_\beta}{(\det F_{ij})^{1/2} (\det \kappa_{ab})^{1/2} \det F^a_b} \quad (33)$$

inverse propagators

$$F_{ij} \equiv S^{\text{red}}_{,ij} - X^\alpha_{,i} \Pi_{\alpha\beta} X^\beta_{,j},$$
$$F^\alpha_\beta \equiv X^\alpha_{,i} \mathcal{R}^i_\beta - \sigma^\alpha_a \rho^{-1}{}^a_b \omega^b_\beta,$$
$$F^a_b \equiv \omega^a_\gamma \mathcal{Z}^\gamma_b.$$

auxiliary quantities

$$\kappa_{ab} \equiv \sigma^\alpha_a \varkappa_{\alpha\beta} \sigma^\beta_b,$$
$$\kappa^{ba} \equiv (\kappa_{ab})^{-1},$$
$$\Pi_{\alpha\beta} \equiv \varkappa_{\alpha\beta} - \varkappa_{\alpha\gamma} \sigma^\gamma_a \kappa^{ab} \sigma^\delta_b \varkappa_{\delta\beta},$$

This form of EA is covariant w.r.t. parental gauge algebra space, but is built on reduced configuration space.

Parental configuration space covariance

$$Z^{1-\text{loop}}_{restricted} = \frac{\det \rho^a_b}{(\det \varkappa^{\alpha\beta})^{1/2}} \frac{\det F^\alpha_\beta}{(\det F_{IJ})^{1/2} (\det \Theta^{ab})^{1/2} (\det \kappa_{ab})^{1/2} \det F^a_b} \quad (34)$$

$$F_{IJ} = \hat{S}_{,IJ} - \lambda_a \theta^a_{,IJ} - X^\alpha_{,I} \Pi_{\alpha\beta} X^\beta_{,J}, \qquad \Theta^{ab} \equiv \theta^a_{,I} F^{-1\,IJ} \theta^b_{,J}. \quad (35)$$

Gauge fermion components are defined on parental space φ^I, $F_{ij} = e^I_{,i} e^J_{,j} F_{IJ}$, and lifted $F^\alpha_\beta = X^\alpha_{,I} \mathcal{R}^I_\beta - \sigma^\alpha_a \rho^{-1}{}^a_b \omega^b_\beta$ (which is true since $\mathcal{R}^I_\beta$ is tangential to Σ_θ).

One-loop effective action

$$Z^{1-\text{loop}}_{restricted} = e^{i\Gamma^{1-\text{loop}}} = \frac{\det \rho^a_b}{\left(\det \varkappa^{\alpha\beta}\right)^{1/2}} \frac{\det F^\alpha_\beta}{\left(\det F_{ij}\right)^{1/2} \left(\det \kappa_{ab}\right)^{1/2} \det F^a_b} \tag{33}$$

inverse propagators	auxiliary quantities
$F_{ij} \equiv S^{\text{red}}_{,ij} - X^\alpha_{,i} \Pi_{\alpha\beta} X^\beta_{,j},$	$\kappa_{ab} \equiv \sigma^\alpha_a \varkappa_{\alpha\beta} \sigma^\beta_b,$
$F^\alpha_\beta \equiv X^\alpha_{,i} \mathcal{R}^i_\beta - \sigma^\alpha_a \rho^{-1}{}^a_b \omega^b_\beta,$	$\kappa^{ba} \equiv \left(\kappa_{ab}\right)^{-1},$
$F^a_b \equiv \omega^a_\gamma \mathcal{Z}^\gamma_b.$	$\Pi_{\alpha\beta} \equiv \varkappa_{\alpha\beta} - \varkappa_{\alpha\gamma} \sigma^\gamma_a \kappa^{ab} \sigma^\delta_b \varkappa_{\delta\beta},$

This form of EA is covariant w.r.t. parental gauge algebra space, but is built on reduced configuration space.

Parental configuration space covariance

$$Z^{1-\text{loop}}_{restricted} = \frac{\det \rho^a_b}{\left(\det \varkappa^{\alpha\beta}\right)^{1/2}} \frac{\det F^\alpha_\beta}{\left(\det F_{IJ}\right)^{1/2} \left(\det \Theta^{ab}\right)^{1/2} \left(\det \kappa_{ab}\right)^{1/2} \det F^a_b} \tag{34}$$

$$F_{IJ} = \hat{S}_{,IJ} - \lambda_a \theta^a_{,IJ} - X^\alpha_{,I} \Pi_{\alpha\beta} X^\beta_{,J}, \qquad \Theta^{ab} \equiv \theta^a_{,I} F^{-1\,IJ} \theta^b_{,J}. \tag{35}$$

Gauge fermion components are defined on parental space φ^I, $F_{ij} = e^I_{,i} e^J_{,j} F_{IJ}$, and lifted $F^\alpha_\beta = X^\alpha_{,I} \mathcal{R}^I_\beta - \sigma^\alpha_a \rho^{-1}{}^a_b \omega^b_\beta$ (which is true since $\mathcal{R}^I_\beta$ is tangential to Σ_θ).

Parental and restricted theories' EA interrelation

Disentangling of restriction condition

The set of functions θ^a, according to the rank deficit of Q^a_α, can be split into gauge-invariant functions θ^A, range $A = m_2$, and gauge-fixing θ^p, range $p = m_1 - m_2$

$$\theta^a \to (\theta^A, \theta^p): \quad \delta(\theta^a) = \delta(\theta^A)\,\delta(\theta^p)\,Y, \quad Y \equiv \det[Y^A_a\, Y^p_a] \equiv \partial(\theta^A, \theta^p)/\partial\theta^a \quad (36)$$

θ^p — conditions of partial gauge fixing $\chi^p \equiv \theta^p$,

θ^A — gauge invariants, which are forced to vanish in the restricted path integral, are responsible for *inequivalence* of the restricted and parental theories.

Restricted–parental one-loop relation

For theories with the Jacobian Y *independent* of integration fields in the one-loop order there is simple relation of the restricted and parental theories

$$\boxed{Z^{1-\text{loop}}_{\text{restricted}} = \hat{Z}^{1-\text{loop}} \left(\det \varTheta^{AB}\right)^{-1/2}} \quad (37)$$

$$\varTheta^{AB} \equiv \theta^A_{,I} \hat{F}^{-1\,IJ} \theta^B_{,J}, \quad \hat{F}_{IJ} = \hat{S}_{,IJ} - \chi^\alpha_{,I} \varkappa_{\alpha\beta} \chi^\beta_{,J} - \lambda_A \theta^A_{,IJ} \quad (38)$$

– $\varTheta^{AB}$ is defined in terms of the Green's function of $\hat{F}_{IJ}$ of the parental theory with a source λ_A at gauge-invariant observable θ^A.

– the presence of source term may be interpreted as going off shell and calculating on backgrounds $\hat{S}_{,I} = \lambda_A \theta^A_{,I}$, which specify saddle points of restricted theory.

– to compare $Z^{1-\text{loop}}_{\text{restricted}}$ and $\hat{Z}^{1-\text{loop}}$ in (37) these objects should be calculated on the same backgrounds.

Unimodular gravity (UMG)

Parental theory — Einstein general relativity

$$\hat{S}[\varphi] \mapsto S_{\rm E}[g_{\mu\nu}] = \int d^4x \sqrt{g}^{1/2}(x)\, R(g_{\mu\nu}(x)), \qquad \varphi^I \mapsto g_{\mu\nu}(x),\ \ I \mapsto (\mu\nu, x) \tag{39}$$

Gauge transformations $\delta_\xi \varphi^I = \hat{\mathcal{R}}^I_\alpha \xi^\alpha \mapsto$ diffeomorphisms $\delta_\xi g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu$, so that the gauge generators

$$\hat{\mathcal{R}}^I_\alpha \mapsto 2g_{\alpha(\mu}\nabla_{\nu)}\delta(x,y), \qquad I \mapsto (\mu\nu, x),\ \ \alpha \mapsto (\alpha, y) \tag{40}$$

Restriction

Unimodular restriction of (39) — restriction to the subspace of metrics $g_{\mu\nu}(x)$ with a unit determinant, $g(x) \equiv -\det g_{\mu\nu}(x) = 1$

$$\theta^a \mapsto \theta^x \equiv \theta(x) = g^{1/2}(x) - 1, \qquad a \mapsto x, \tag{41}$$

$$\theta^a_{,I} \mapsto \theta^{x,\,\mu\nu,y} = \tfrac{1}{2} g^{1/2} g^{\mu\nu}\delta(x,y), \qquad a \mapsto x,\ \ I \mapsto (\mu\nu, y) \tag{42}$$

The gauge-restriction operator (9)

$$Q^a_\alpha \mapsto Q^x_{\alpha,y} = g^{1/2}\nabla_\alpha \delta(x,y) = \partial_\alpha\big(g^{1/2}(x)\delta(x,y)\big), \tag{43}$$

Restricted theory – Unimodular gravity

Action

$$S^{\lambda}[\,g_{\mu\nu},\lambda\,]=\int d^4x\big(\,g^{1/2}R(g_{\mu\nu})-\lambda(g^{1/2}-1)\big) \tag{44}$$

Equations of motion

$$\frac{\delta S^{\lambda}}{\delta g_{\mu\nu}}=-g^{1/2}\big(R^{\mu\nu}-\tfrac{1}{2}g^{\mu\nu}R+\tfrac{1}{2}\lambda g^{\mu\nu}\big)=0, \qquad \frac{\delta S^{\lambda}}{\delta\lambda}=-(g^{1/2}-1)=0 \tag{45}$$

The Lagrange multiplier $\lambda(x)$ on shell is *constant*

$$\lambda_a Q^a_{\ \alpha}=0 \quad\mapsto\quad -g^{1/2}\partial_\alpha\lambda=0 \tag{46}$$

The *vacuum solution* of equations of motion is a generic Einstein space metric $g_{\mu\nu}$,

$$R_{\mu\nu}=\Lambda\, g_{\mu\nu}, \quad \Lambda=\lambda/2=\mathrm{const} \tag{47}$$

with a unit determinant $g\equiv-\det g_{\mu\nu}=1$.

Physical difference

Left kernel of $Q^a_{\ \alpha}$ of dim. $m_2=1$ is spanned by the zero mode $Y^A_{\ a}\mapsto Y_x=1$

Gauge-invariant physical degree of freedom constrained by the UMG restriction

$$\theta^A=Y^A_{\ a}\theta^a=0 \quad\mapsto\quad \bar\theta=\int d^4x\,(g^{1/2}(x)-1)=0, \tag{48}$$

is the full spacetime volume, $\int d^4x\, g^{1/2}\,|_{\theta=0}=\int d^4x$, not specified in GR.

The projector T^α_β

The following choice of k^α_b provides covariance w.r.t. coordinate change

$$k^\alpha_b \quad \mapsto \quad k^{\alpha,x}_y = \nabla^\alpha \delta(x,y), \qquad \alpha \mapsto (\alpha, x), \;\; b \mapsto y, \tag{49}$$

$$(Qk)^a_b \quad \mapsto \quad (Qk)^x_y = g^{1/2} \Box \delta(x,y) \qquad a \mapsto x, \;\; b \mapsto y \tag{50}$$

where $\nabla^\alpha = g^{\alpha\beta}\nabla_\beta$, $\Box = g^{\alpha\beta}\nabla_\alpha\nabla_\beta$, (acting on scalars).

Thus the projector $T^\alpha_\beta(Q,k) \;\; \mapsto \;\; T^{\alpha,x}_{\beta,y} = T^\alpha_\beta(\nabla)\delta(x,y)$ corresponds to operator

$$T^\alpha_\beta(\nabla) \, = \, \delta^\alpha_\beta - \nabla^\alpha \frac{1}{\Box} \nabla_\beta. \tag{51}$$

$T^\alpha_\beta(\nabla)$ is a projector on the subspace of spacetime transverse vectors.
$T^\alpha_\beta(\nabla)$ is nonlocal, where $\frac{1}{\Box}$ is the Green's function of $\Box$, understood in the Moore-Penrose sense (orthogonal to $\Box$ zero constant mode).

Gauge fixing

To preserve explicit covariance we use background gauge fixing

$$\chi^\alpha \mapsto \chi^\alpha(x) = \mathrm{g}^{1/2}\nabla^\mu h^\alpha_\mu(x) \equiv \mathrm{g}^{1/2}\mathrm{g}^{\mu\beta}\mathrm{g}^{\alpha\nu}\nabla_\beta\big(g_{\mu\nu}(x) - \mathrm{g}_{\mu\nu}(x)\big) \tag{52}$$

(on UMG background is the DeWitt gauge $\mathrm{g}^{1/2}\mathrm{g}^{\alpha\beta}(\nabla^\mu h_{\beta\mu} - \frac{1}{2}\mathrm{g}^{\mu\nu}\nabla_\beta h_{\mu\nu})$)

For reasons of explicit covariance one can choose

$$\varkappa^{\alpha\beta} \mapsto \varkappa^{\alpha,x\;\beta,y} = \mathrm{g}^{1/2}\mathrm{g}^{\alpha\beta}\delta(x,y), \tag{53}$$

$$\sigma^\alpha_a \mapsto \sigma^{\alpha,x}_y = -\mathrm{g}^{1/2}\mathrm{g}^{\alpha\beta}\nabla_\beta\delta(x,y) = -g^{1/2}(x)k^{\alpha,x}_y\big|_{g_{\mu\nu}\to \mathrm{g}_{\mu\nu}}, \tag{54}$$

$$\omega^a_\alpha \mapsto \omega^x_{\alpha,y} = \mathrm{g}^{1/2}\nabla_\alpha\delta(x,y) = Q^x_{\alpha,y}\big|_{g_{\mu\nu}\to \mathrm{g}_{\mu\nu}}, \tag{55}$$

$$\rho^a_b \mapsto \rho^x_y = \mathrm{g}^{1/2}\delta(x,y), \tag{56}$$

(where in (54) ∇ acts on scalar, in (55) ∇ acts on vector)

Since $\sigma^\alpha_{a,I}$ does not depend on dynamic metric, extra ghost $\mathcal{C}'^a$ vanish on shell and

$$X^\alpha_{,I} = \chi^\alpha_{,I} \mapsto \mathrm{g}^{1/2}\mathrm{g}^{\alpha(\mu}\nabla^{\nu)}\delta(x,y). \tag{57}$$

Derived structures

$$\kappa_{ab} \equiv \sigma^\alpha_a\varkappa_{\alpha\beta}\sigma^\beta_b \mapsto \kappa_{xy} = -g^{1/2}\Box\delta(x,y), \tag{58}$$

$$\sigma^a_\beta \equiv \kappa^{ab}\sigma^\alpha_b\varkappa_{\alpha\beta} \mapsto \sigma^x_{\beta,y} = -\frac{1}{\Box}g^{-1/2}\nabla_\beta\delta(x,y), \tag{59}$$

$$\mathit{\Pi}_{\alpha\beta} \equiv \varkappa_{\alpha\beta} - \varkappa_{\alpha\gamma}\sigma^\gamma_a\sigma^a_\beta \mapsto \mathit{\Pi}_{\alpha,x\;\beta,y} = \Big(g^{-1/2}g_{\alpha\beta} - \nabla_\alpha\frac{1}{\Box}g^{-1/2}\nabla_\beta\Big)\delta(x,y) \tag{60}$$

Inverse propagators

$$F_{IJ} \quad \mapsto \quad F^{\mu\nu\;\alpha\beta}(\nabla)\,\delta(x,y),$$

$$F^{\mu\nu\;\alpha\beta}(\nabla) = \tfrac{1}{2}g^{1/2}\Big(g^{\mu(\alpha}g^{\beta)\nu}\Box + 2R^{\mu(\alpha\nu\beta)} - 2\nabla^{(\mu}\nabla^{\nu)}\frac{1}{\Box}\nabla^{(\alpha}\nabla^{\beta)}$$
$$+g^{\mu\nu}\nabla^{(\alpha}\nabla^{\beta)} + g^{\alpha\beta}\nabla^{(\mu}\nabla^{\nu)} - g^{\mu\nu}g^{\alpha\beta}(\Box + \tfrac{1}{4}R)\Big) \qquad (61)$$

$$F^{\alpha}_{\;\beta} \quad \mapsto \quad F^{\alpha}_{\;\beta}(\nabla)\,\delta(x,y),$$

$$F^{\alpha}_{\;\beta}(\nabla) = g^{1/2}\Big((\Box + \tfrac{1}{4}R)\delta^{\alpha}_{\beta} - \tfrac{1}{2}R\,\nabla^{\alpha}\frac{1}{\Box}\nabla_{\beta}\Big), \qquad (62)$$

$$F^{a}_{\;b} = \delta^{a}_{b} \quad \mapsto \quad F^{x}_{\;y} = \text{"}\delta(x,y)\text{"}. \qquad (63)$$

(background equations were used to simplify equations, which is legitimate in one-loop)

The operators acquired nonlocal parts generated due to nonlocal projectors.
In the local part of the tensor operators are not minimal.

Calculation strategies

- via general heat kernel methods on generic backgrounds (the operators should be transformed to forms with local and (preferably) minimal principal symbols)
- via decomposition of the space of tensor and vector fields into irreducible transverse and traceless components

GR and UMG one-loop EA (generic heat kernel)

UMG

$$\boxed{Z^{1-\text{loop}}_{\text{UMG}} = \frac{\det\left(\Box\,\delta^\alpha_\beta + \frac{R}{4}\delta^\alpha_\beta\right)}{\left[\det(\Box\,\delta_{\mu\nu}^{\alpha\beta} + 2R_{\mu\ \nu}^{\ (\alpha\ \beta)})\right]^{1/2}}\left[\frac{\det\left(\Box + \frac{R}{2}\right)}{\det'\left(\Box + \frac{R}{2}\right)}\right]^{1/2}.} \tag{64}$$

Note the nontrivial factor

$$\left[\frac{\det\left(\Box + \frac{R}{2}\right)}{\det'\left(\Box + \frac{R}{2}\right)}\right]^{1/2} = (2\Lambda)^{1/2}, \tag{65}$$

which is a function of the dynamical global degree of freedom Λ in UMG.

GR

Modulo constant extra factor, the result (64) exactly coincides with the one-loop contribution of gravitons in Einstein theory with the action

$$S_\Lambda[\,g_{\mu\nu}] = \int d^4x\, g^{1/2}(R - 2\Lambda) \tag{66}$$

and the on-shell value of the cosmological constant $\Lambda = R/4$,

$$\hat{Z}^{1-\text{loop}}_{\text{E}}(\Lambda) = \frac{\det\left(\Box\,\delta^\alpha_\beta + \frac{R}{4}\delta^\alpha_\beta\right)}{\left[\det(\Box\,\delta_{\mu\nu}^{\alpha\beta} + 2R_{\mu\ \nu}^{\ (\alpha\ \beta)})\right]^{1/2}}\Bigg|_{R_{\mu\nu}=\Lambda g_{\mu\nu}}. \tag{67}$$

GR and UMG one-loop EA (York decomposition)

UMG

Via minimal determinants of operators for irreducible tensor representations

$$\boxed{Z^{1-\text{loop}}_{\text{UMG}} = \left[\frac{\det_{\text{T}} \left(\Box\, \delta^{\mu}_{\nu} + \frac{R}{4} \delta^{\mu}_{\nu} \right)}{\det_{\text{TT}} \left(\Box\, \delta_{\mu\nu}^{\alpha\beta} + 2R_{\mu}{}^{(\alpha}{}_{\nu}{}^{\beta)} \right)} \right]^{1/2}} \tag{68}$$

It manifestly exhibits 5 traceless-tensor modes minus 3 transverse vector modes. Note the disappearance of the additional factor in (64)

GR

On the contrary, in this representation this factor is generated in the GR calculations

$$\hat{Z}^{1-\text{loop}}_{\text{E}} = \left[\frac{\det_{\text{T}} \left(\Box\, \delta^{\mu}_{\nu} + \frac{R}{4} \delta^{\mu}_{\nu} \right)}{\det_{\text{TT}} \left(\Box\, \delta_{\mu\nu}^{\alpha\beta} + 2R_{\mu}{}^{(\alpha}{}_{\nu}{}^{\beta)} \right)} \right]^{1/2} \times \left[\frac{\det' \left(\Box + \frac{R}{2} \right)}{\det \left(\Box + \frac{R}{2} \right)} \right]^{1/2} \tag{69}$$

In terms of minimal determinants on constrained (irreducible) fields the one-loop result for Einstein theory with Λ again differs by the constant-mode contribution of the operator $\Box + \frac{R}{2}$.

Conclusions

- Restricted theory formalism is formulated. It relates the dynamics and quantum properties of two theories, connected by local full-rank restriction on configuration space of the parental theory.
  In particular it gives simple relation between quantum effective actions.
  Particular case: restriction is a pure gauge fixing. Checks part of the results.
  (Note: Related are objects of physically nonequivalent theories!)
- The formalism have been tested for the Unimodular gravity (UMG) as the restricted General relativity.
  These theories differ by one global degree of freedom and the formalism explicitly shows this.
- Limitations:
  – directly applied when restricting theories with closed gauge algebras,
  – reasonable regularity relations for restriction should be checked.
  (Weakening of some rank conditions is possible).
- Nonlocality issue. (Simple criteria for generic theory not found yet.)
  Condensed DeWitt notation hides boundary terms and nonlocality structures which can break part of the residual gauge invariance of restricted theory.
  In particular it enters the game for most of the GUMG models, forcing the change of canonical constraint structure of the theories.