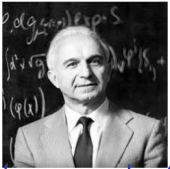
Efim Fradkin Centennial Conference
Lebedev Institute / September 2-6, 2024



KK reduction of Horndeski and the speed of gravity.

based on papers with Volkova, Shtennikova, Valencia-Villegas, Sharov
2405.02281, 2408.01480, 2408.04626, 2408.06329

S. Mironov

INR RAS, ITMP, ITEP (NRCKI)

ESF, LPI, 3 September 2024

Framework

- We consider Horndeski (and beyond Horndeski), subclasses of DHOST theories.

Framework

- We consider Horndeski (and beyond Horndeski), subclasses of DHOST theories.
- These scalar-tensor theories allow to break NEC in a healthy way, without instabilities or other pathologies,

  and are very general

Framework

- We consider Horndeski (and beyond Horndeski), subclasses of DHOST theories.
- These scalar-tensor theories allow to break NEC in a healthy way, without instabilities or other pathologies,

  and are very general

- They are extensively used for
  - early Universe cosmology

Framework

- We consider Horndeski (and beyond Horndeski), subclasses of DHOST theories.
- These scalar-tensor theories allow to break NEC in a healthy way, without instabilities or other pathologies,

  and are very general

- They are extensively used for

– early Universe cosmology

– modern Universe cosmology

Framework

- We consider Horndeski (and beyond Horndeski), subclasses of DHOST theories.
- These scalar-tensor theories allow to break NEC in a healthy way, without instabilities or other pathologies,

  and are very general

- They are extensively used for
  - early Universe cosmology
  - modern Universe cosmology
  - compact objects and other modified gravity solutions

Null Energy Condition $\Longleftrightarrow$ Null Convergence Condition

$T_{\mu\nu} k^\mu k^\nu \geq 0$

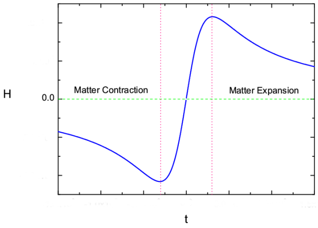
Friedmann equations

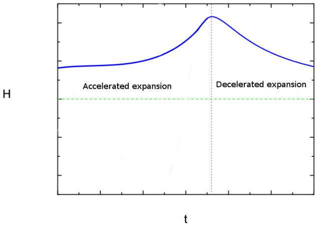
$\dot{H} = -4\pi G(p + \rho) \quad < 0$

$R_{\mu\nu} k^\mu k^\nu \geq 0$

Penrose theorem

no singularity $\Rightarrow$ NEC-violation





Bounce and genesis require NEC-violation

Null Energy Condition $\Longleftrightarrow$ Null Convergence Condition

$T_{\mu\nu}k^{\mu}k^{\nu} \geq 0$

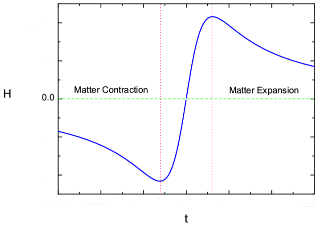
$R_{\mu\nu}k^{\mu}k^{\nu} \geq 0$

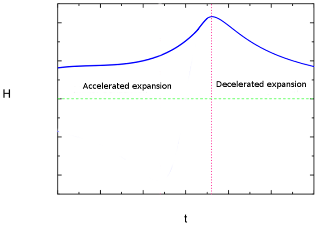
Friedmann equations

$\dot{H} = -4\pi G(p+\rho) \quad < 0$

Penrose theorem

no singularity $\Rightarrow$ NEC-violation





Bounce and genesis require NEC-violation

As well as wormhole-like solutions

NEC-violation

- NEC was considered robust for a long time

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

$$\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

  $\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

  quadratic action $\qquad L_\zeta^{(2)} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

  $\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

  quadratic action $\qquad L^{(2)}_{\zeta} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

  and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

  $\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

  quadratic action $\qquad L^{(2)}_\zeta = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

  and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

  stability requirement: $\qquad U > 0\,, \quad V > 0\,, \quad W \geq 0\,.$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory

- for all linear perturbations of each dynamical field in the theory

  $\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

  quadratic action $\qquad L_\zeta^{(2)} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

  and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

  stability requirement: $\qquad U > 0 \,, \quad V > 0 \,, \quad W \geq 0 \,.$

  (sub)luminality requirement: $\qquad V \leq U$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory unless one considers **higher derivatives in the action**

- for all linear perturbations of each dynamical field in the theory

$\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

quadratic action $\qquad L^{(2)}_{\zeta} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

stability requirement: $\qquad U > 0 \,, \qquad V > 0 \,, \qquad W \geq 0 \,.$

(sub)luminality requirement: $\qquad V \leq U$

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory unless one considers **higher derivatives in the action**

- for all linear perturbations of each dynamical field in the theory

  $\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

  quadratic action $\qquad L_\zeta^{(2)} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

  and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

  stability requirement: $\qquad U > 0 \, , \quad V > 0 \, , \quad W \geq 0 \, .$

  (sub)luminality requirement: $\qquad V \leq U$

- **Horndeski theory**

NEC-violation

- NEC was considered robust for a long time
- indeed, violation of NEC leads to pathologies in the theory unless one considers **higher derivatives in the action**

\- for all linear perturbations of each dynamical field in the theory

$\pi = \pi_0 + \chi, \qquad g_{\mu\nu} = \tilde{g}_{\mu\nu} + h_{\mu\nu}$

quadratic action $\qquad L_\zeta^{(2)} = \frac{1}{2} U \dot{\zeta}^2 - \frac{1}{2} V (\partial_i \zeta)^2 - \frac{1}{2} W \zeta^2$

and dispersion relation $\qquad U\omega^2 = V\mathbf{p}^2 + W$

stability requirement: $\qquad U > 0\,, \quad V > 0\,, \quad W \geq 0\,.$

(sub)luminality requirement: $\qquad V \leq U$

- **Horndeski theory** and beyond

beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$

$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Big[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}_{ik}^T \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h_{kl}^T \right)^2 + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \Big]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

A healthy and stable solution requires correct signs for kinetic and gradient terms as well as subluminal propagation:

$$\mathcal{G}_\mathcal{T} \geq \mathcal{F}_\mathcal{T} > \epsilon > 0, \quad \mathcal{G}_\mathcal{S} \geq \mathcal{F}_\mathcal{S} > \epsilon > 0$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}. \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a\left(\mathcal{G}_\mathcal{T} - \mathcal{D}\dot{\pi}\right)\mathcal{G}_\mathcal{T}}{\Theta}. \end{aligned} \tag{2}$$

Early Universe

- Different "successful" models of the early Universe:

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum
  - amplitude of tensor spectrum (r-ratio)

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum
  - amplitude of tensor spectrum (r-ratio)
  - anisotropies and non-gaussianities

Early Universe

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations
- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum
  - amplitude of tensor spectrum (r-ratio)
  - anisotropies and non-gaussianities
  - PTA experiments, neutrino, ...

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways

– geodesically incompliteness

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$
  - cuscuton

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$
  - cuscuton
  - spatial curvature

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$
  - cuscuton
  - spatial curvature
  - different geometry or formalism (torsion, Palatini)

Early Universe cosmology

- Another issue that arises in Horndeski cosmology is apparent no-go theorem

  If one tries to construct globally stable non-singular solution

- This issue was also resolved in many different ways
  - geodesically incompliteness
  - strong gravity in the past
  - $\theta = 0$
  - cuscuton
  - spatial curvature
  - different geometry or formalism (torsion, Palatini)
  - **consider beyond Horndeski or DHOST theory**

Static compact objects

- No-go for wormhole solutions in Horndeski theory

Static compact objects

- No-go for wormhole solutions in Horndeski theory
- "almost" resolved in beyond Hondeski

Static compact objects

- No-go for wormhole solutions in Horndeski theory
- "almost" resolved in beyond Hondeski
- Many different "successful" blackhole solutions (with or without hair)

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)
- There is an additional phenomenological restriction, if we study modern Universe (models of dark energy and dark matter)

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)
- There is an additional phenomenological restriction, if we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)
- There is an additional phenomenological restriction, if we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)
- There is an additional phenomenological restriction, if we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$

$$|\frac{c_{\mathcal{T}}}{c} - 1| < 10^{-15}$$

Modern Universe cosmology

- We consider Horndeski theory (and beyond Horndeski)
- There is an additional phenomenological restriction, if we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$
$$\left|\frac{c_{\mathcal{T}}}{c} - 1\right| < 10^{-15}$$

- In addition to Horndeski we consider U(1) vector field (EM = light)

- For trivial Maxwell electrodynamics (c=1) it means $c_{\mathcal{T}} = 1$ too.

- For trivial Maxwell electrodynamics (c=1) it means $c_{\mathcal{T}} = 1$ too.
- $c_{\mathcal{T}}^2 = \dfrac{\mathcal{F}_{\mathcal{T}}}{\mathcal{G}_{\mathcal{T}}}$

- For trivial Maxwell electrodynamics (c=1) it means $c_{\mathcal{T}} = 1$ too.
- $c_{\mathcal{T}}^2 = \dfrac{\mathcal{F}_{\mathcal{T}}}{\mathcal{G}_{\mathcal{T}}}$
- There is a possibility to constrain Lagrangian functions and/or background

- For trivial Maxwell electrodynamics (c=1) it means $c_{\mathcal{T}} = 1$ too.
- $c_{\mathcal{T}}^2 = \frac{\mathcal{F}_{\mathcal{T}}}{\mathcal{G}_{\mathcal{T}}}$
- There is a possibility to constrain Lagrangian functions and/or background
- Since speed are so close we assume $c_{\mathcal{T}} = c$ to be a natural property of the theory

(without constrains on the background)

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + XG_{5\phi} - 2HX\dot{\pi}G_{5X}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - XG_{5\phi} - 2X\ddot{\pi}G_{5X}$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + XG_{5\phi} - \cancel{2HX\dot{\pi}G_{5X}}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - XG_{5\phi} - \cancel{2X\ddot{\pi}G_{5X}}$$

  - $$G_{5X} = 0$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + XG_{5\phi}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - XG_{5\phi}$$

- $$G_{5X} = 0$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + XG_{5\phi}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - XG_{5\phi}$$

  - $G_{5X} = 0$
  - $2G_{4X} = G_{5\phi}$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4 \cancel{-4XG_{4X} + XG_{5\phi}}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 \cancel{-XG_{5\phi}}$$

  - $G_{5X} = 0$
  - $G_{4X} = 0$
  - $G_{5\phi} = 0$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $G_{5X} = 0$
- $G_{4X} = 0$
- $G_{5\phi} = 0$
- $G_4 = G_4(\phi)$
- $G_5 = const$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $G_{5X} = 0$
- $G_{4X} = 0$
- $G_{5\phi} = 0$

- $G_4 = G_4(\phi)$
- $G_5 = const$
- Beyond Horndeski

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + XG_{5\phi} - 2HX\dot{\pi}G_{5X} + 2F_4X^2 + 6HX^2\dot{\pi}F_5$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 - XG_{5\phi} - 2X\ddot{\pi}G_{5X}$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $G_{5X} = 0$
- $G_{4X} = 0$
- $G_{5\phi} = 0$

- $G_4 = G_4(\phi)$
- $G_5 = const$
- Beyond Horndeski

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} \cancel{+XG_{5\phi}} \cancel{-2HX\dot{\pi}G_{5X}} + 2F_4X^2 + \cancel{6HX^2\dot{\pi}F_5}$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4 \cancel{-XG_{5\phi}} \cancel{-2X\ddot{\pi}G_{5X}}$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $G_{5X} = 0$
- $G_{4X} = 0$
- $G_{5\phi} = 0$

- $G_4 = G_4(\phi)$
- $G_5 = const$
- Beyond Horndeski

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + 2F_4X^2$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- Horndeski:

$$\mathcal{G}_{\mathcal{T}} = 2G_4$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $G_{5X} = 0$
- $G_{4X} = 0$
- $G_{5\phi} = 0$
- $G_4 = G_4(\phi)$
- $G_5 = const$
- Beyond Horndeski

$$\mathcal{G}_{\mathcal{T}} = 2G_4 - 4XG_{4X} + 2F_4X^2$$

$$\mathcal{F}_{\mathcal{T}} = 2G_4$$

- $F_4 = \frac{2G_{4X}}{X}$
- $G_5 = const$

Modified Maxwell

- Let us consider $c \neq 1$

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$
- This, in principle, can be achieved by a proper modification of the Maxwell theory

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$
- This, in principle, can be achieved by a proper modification of the Maxwell theory

- How to find such modifications?

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$
- This, in principle, can be achieved by a proper modification of the Maxwell theory

- How to find such modifications?

- Couple photon to a disformally connected metric

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$
- This, in principle, can be achieved by a proper modification of the Maxwell theory

- How to find such modifications?

- Couple photon to a disformally connected metric
- Consider general scalar-photon couplings and solve the constraint $c_{\mathcal{T}} = c$

Modified Maxwell

- Let us consider $c \neq 1$
- The only requirement then is $c_{\mathcal{T}} = c$
- This, in principle, can be achieved by a proper modification of the Maxwell theory

- How to find such modifications?

- Couple photon to a disformally connected metric
- Consider general scalar-photon couplings and solve the constraint $c_{\mathcal{T}} = c$
- We propose a **natural** way to modify Maxwell in accordance with gravity

- **Kaluza-Klein compactification from higher dimensions**

KK compactification of Horndeski theory

$$R^5 \qquad \longrightarrow \qquad R^4 \times S^1$$

KK compactification of Horndeski theory

$$R^5 \quad \longrightarrow \quad R^4 \times S^1$$

* Generalized Galileons $\longrightarrow$ Generalized Galileons

KK compactification of Horndeski theory

$$R^5 \quad \longrightarrow \quad R^4 \times S^1$$

* Generalized Galileons $\longrightarrow$ Generalized Galileons

2nd derivatives in the action $\longrightarrow$ 2nd derivatives in the action

KK compactification of Horndeski theory

$$R^5 \quad \longrightarrow \quad R^4 \times S^1$$

* Generalized Galileons $\longrightarrow$ Generalized Galileons

2nd derivatives in the action $\longrightarrow$ 2nd derivatives in the action

no higher derivatives in EOMs $\longrightarrow$ no higher derivatives in EOMs

KK compactification of Horndeski theory

$$R^5 \longrightarrow R^4 \times S^1$$

* Generalized Galileons $\longrightarrow$ Generalized Galileons

2nd derivatives in the action $\longrightarrow$ 2nd derivatives in the action

no higher derivatives in EOMs $\longrightarrow$ no higher derivatives in EOMs

* Metric + scalar $\longrightarrow$ Metric + vector + scalar + scalar [U(1) gauge]

Horndeski theory

$$S = \int \mathrm{d}^5 x \sqrt{-g}\, (\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\ ,$$

$$\mathcal{L}_2 = F(\pi, X)\,,$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi\,,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right]\ ,$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{mn}\pi_{;mn} - \frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right]\ ,$$

where π is the Galileon field, $X = -\frac{1}{2}g^{mn}\pi_{,m}\pi_{,n}$,
$m, n = 0..4$

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix}$$

Horndeski theory

$$S = \int \mathrm{d}^5 x \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_6)\ ,$$
$$\mathcal{L}_2 = F(\pi, X)\,,$$
$$\mathcal{L}_3 = K(\pi, X)\Box\pi\,,$$
$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right]\ ,$$
$$\mathcal{L}_5 = G_5(\pi, X)G^{mn}\pi_{;mn} - \frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right]\ ,$$
$$\mathcal{L}_6 = G_6(\pi, X)\,\mathcal{O}\left[R^2\right] + G_{6X}(\pi, X)\,\mathcal{O}\left[R(\nabla^2\pi)^2\right] + G_{6XX}(\pi, X)\,\mathcal{O}\left[(\nabla^2\pi)^4\right]\ ,$$

where π is the Galileon field, $X = -\frac{1}{2}g^{mn}\pi_{,m}\pi_{,n}$,
$m, n = 0..4$

$$g_{mn} = \begin{pmatrix} g_{\mu\,\nu} + \phi^2\,A_\mu\,A_\nu & \phi^2\,A_\mu \\ \phi^2\,A_\nu & \phi^2 \end{pmatrix}$$

Horndeski theory

$$S = \int \mathrm{d}^5 x \sqrt{-g} \left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \cancel{\mathcal{L}_6}\right) ,$$

$$\mathcal{L}_2 = F(\pi, X) ,$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi ,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;mn}\pi^{;mn}\right] ,$$

$$\mathcal{L}_5 = G_5(\pi)G^{mn}\pi_{;mn} - \frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;mn}\pi^{;mn} + 2\pi_{;mn}\pi^{;ml}\pi_{;l}{}^{n}\right] ,$$

$$\mathcal{L}_6 = G_6(\pi, X)\,\mathcal{O}\left[R^2\right] + G_{6X}(\pi, X)\,\mathcal{O}\left[R(\nabla^2\pi)^2\right] + G_{6XX}(\pi, X)\,\mathcal{O}\left[(\nabla^2\pi)^4\right] ,$$

where π is the Galileon field, $X = -\frac{1}{2}g^{mn}\pi_{,m}\pi_{,n}$,
$m, n = 0..4$

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} + \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & \phi^2 \end{pmatrix}$$

$$
\begin{aligned}
&\phi(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4) + \mathcal{L}_{4A_\mu} + \mathcal{L}_{4\phi} = \\
&\int d^4x \sqrt{-g}\, \phi \left[\underline{G_2(\pi, X) + G_3(\pi, X) \Box\pi} + G_4(\pi, X) \left(\underline{R} - \frac{1}{4}\phi^2 F^2 - 2\frac{\Box\phi}{\phi} \right) \right. \\
&\left. + G_{4,X}(\pi, X) \left(\underline{(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2} + 2\,\frac{1}{\phi}\,\nabla_\mu\phi\nabla^\mu\pi\,\Box\pi - \frac{1}{2}\phi^2 F_\mu{}^\sigma F_{\nu\sigma}\nabla^\mu\pi\,\nabla^\nu\pi \right) \right]
\end{aligned}
$$

$$
\begin{aligned}
&\phi\mathcal{L}_5 + \mathcal{L}_{5A_\mu} + \mathcal{L}_{5\phi} = \int d^4x \sqrt{-g}\, \phi\, G_5(\pi) \left[\left(\underline{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R} \right) \nabla_\mu\nabla_\nu\pi \right. \\
&- \frac{1}{2\,\phi} R\, \nabla_\mu\phi\, \nabla^\mu\pi + \frac{1}{\phi}\left(\Box\phi\, \Box\pi - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\pi \right) + \frac{1}{2}\phi^2 F_{\mu\nu}\, \nabla_\sigma F^{\nu\sigma}\, \nabla^\mu\pi \\
&\left. + \frac{1}{8}\phi\, F^{\mu\nu} F^{\sigma\rho} \left(3\, g_{\nu\rho}(-4\, g_{\lambda\mu}\, g_{\beta\sigma} + g_{\lambda\beta}\, g_{\mu\sigma})\, \nabla^\lambda\pi\nabla^\beta\phi + \phi\, g_{\sigma\mu}\, (-4\, \nabla_\nu\nabla_\rho\pi + g_{\rho\nu}\Box\pi) \right) \right]
\end{aligned}
$$

Horndeski cosmology Speed of graviton

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} - \frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{S}_T = \int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\left[\frac{1}{2\,a^2}\left(\mathcal{G}_\tau\left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau(\partial_k\,h_{ij})^2\right)\right]$$

$$\mathcal{G}_T = 2\left[G_4 - 2XG_{4,X} - X\left(H\dot{\pi}\,G_{5X} - G_{5\phi}\right)\right]$$

$$\mathcal{F}_T = 2\left[G_4 - X\left(\ddot{\pi}\,G_{5X} + G_{5\phi}\right)\right]$$

Horndeski cosmology   Speed of graviton

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi)R + \cancel{G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right]},$$

$$\mathcal{L}_5 = \cancel{G_5(\pi, X)}G^{\mu\nu}\pi_{;\mu\nu} - \cancel{\frac{1}{6}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right]},$$

$$c_T^2 = c_v^2 = 1$$

$$\mathcal{G}_T = 2\left[G_4 - \cancel{2XG_{4,X}} - \cancel{X\left(H\dot{\pi}G_{5X} - G_{5\phi}\right)}\right]$$

$$\mathcal{F}_T = 2\left[G_4 - \cancel{X\left(\ddot{\pi}G_{5X} + G_{5\phi}\right)}\right]$$

Horndeski cosmology   Speed of graviton

$$S=\int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2+\mathcal{L}_3+\mathcal{L}_4+\mathcal{L}_5\right),$$

$$\mathcal{L}_2=F(\pi,X),$$

$$\mathcal{L}_3=K(\pi,X)\Box\pi,$$

$$\mathcal{L}_4=G_4(\pi,X)R+G_{4X}(\pi,X)\left[(\Box\pi)^2-\pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5=G_5(\pi)G^{\mu\nu}\pi_{;\mu\nu}-\frac{1}{6}G_{5X}\left[(\Box\pi)^3-3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu}+2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}^{\ \ \nu}\right],$$

$$c_T^2=c_v^2\neq 1$$

$$\mathcal{G}_T=2\left[G_4-2XG_{4,X}-X\left(H\dot{\pi}G_{5X}-G_{5\phi}\right)\right]$$

$$\mathcal{F}_T=2\left[G_4-X\left(\ddot{\pi}G_{5X}+G_{5\phi}\right)\right]$$

allowed Horndeski theory

$$S = \int \mathrm{d}^4 x \sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi)R$$

$$\mathcal{L}_5 = c\; G^{\mu\nu}\pi_{;\mu\nu}$$

allowed Horndeski theory

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4\right),$$
$$\mathcal{L}_2 = F(\pi, X),$$
$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = G_4(\pi)R$$
$$\mathcal{L}_5 = c\, G^{\mu\nu}\pi_{;\mu\nu}$$

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$
$$\mathcal{L}_2 = F(\pi, X),$$
$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$
$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$
$$\mathcal{L}_5 = G_5(\pi)G^{\mu\nu}\pi_{;\mu\nu}$$

allowed Horndeski theory

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi)R$$

$$\mathcal{L}_5 = c\, G^{\mu\nu}\pi_{;\mu\nu}$$

$$S = \int \mathrm{d}^4 x \sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi)G^{\mu\nu}\pi_{;\mu\nu}$$

Fab four, Vainstein

Let us consider spherically symmetrical dynamical background

Let us consider spherically symmetrical dynamical background

2 tensor modes	vector	2 scalars
$\swarrow \searrow$	$\swarrow \searrow$	$\Downarrow$
odd even	odd even	2 even

Let us consider spherically symmetrical dynamical background

2 tensor modes $\swarrow \searrow$ odd, even

vector $\swarrow \searrow$ odd, even

2 scalars $\Downarrow$ 2 even

odd (tensor) $\searrow$ odd (vector) $\swarrow$

Odd sector of the theory (graviton+photon)

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + \mathcal{Q}_{ij}\dot{v}^i v^{j\prime} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^i v^j + \dots$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}{v^i}'{v^j}' + \mathcal{Q}_{ij}\dot{v}^i{v^j}' - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^iv^j + \ldots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}{v^i}'{v^j}' + \mathcal{Q}_{ij}\dot{v}^i{v^j}' - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^iv^j + \dots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$c_r^2 - 2\sqrt{\frac{B}{A}}\frac{\mathcal{J}}{\mathcal{F}} \cdot c_r - \frac{\mathcal{G}}{\mathcal{F}} = 0$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + \mathcal{Q}_{ij}\dot{v}^i v^{j\prime} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^i v^j + \dots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$c_r^2 - 2\sqrt{\frac{B}{A}}\frac{\mathcal{J}}{\mathcal{F}}\cdot c_r - \frac{\mathcal{G}}{\mathcal{F}} = 0$$

$$c_r^{(\pm)} = \sqrt{\frac{B}{A}}\frac{\mathcal{J}}{\mathcal{F}} \pm \frac{1}{\mathcal{F}}\sqrt{\mathcal{Z}}$$

$$c^+_{r,Q} = c^+_{r,\mathcal{V}}, \quad c^-_{r,Q} = c^-_{r,\mathcal{V}}$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + \mathcal{Q}_{ij}\dot{v}^i v^{j\prime} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^i v^j + \dots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$c_r^2 - 2\sqrt{\frac{B}{A}}\frac{\mathcal{J}}{\mathcal{F}}\cdot c_r - \frac{\mathcal{G}}{\mathcal{F}} = 0$$

$$c_r^{(\pm)} = \sqrt{\frac{B}{A}}\frac{\mathcal{J}}{\mathcal{F}} \pm \frac{1}{\mathcal{F}}\sqrt{\mathcal{Z}}$$

$$c^+_{r,Q} = c^+_{r,\mathcal{V}}, \quad c^-_{r,Q} = c^-_{r,\mathcal{V}}$$

$$\left(c_\theta^2\cdot \mathbf{I}_{ij} - \frac{J^2}{A}\mathcal{K}^{-1}_{ik}\mathcal{M}_{kj}\right)\Big|_{Eigenvalues} = 0$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}{v^i}'{v^j}' + \mathcal{Q}_{ij}\dot{v}^i{v^j}' - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^iv^j + \ldots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$c_r^2 - 2\sqrt{\tfrac{B}{A}}\tfrac{\mathcal{J}}{\mathcal{F}}\cdot c_r - \tfrac{\mathcal{G}}{\mathcal{F}} = 0$$

$$c_r^{(\pm)} = \sqrt{\tfrac{B}{A}}\tfrac{\mathcal{J}}{\mathcal{F}} \pm \tfrac{1}{\mathcal{F}}\sqrt{\mathcal{Z}}$$

$$c^+_{r,Q} = c^+_{r,\mathcal{V}}, \quad c^-_{r,Q} = c^-_{r,\mathcal{V}}$$

$$\left(c_\theta^2\cdot\mathbf{I}_{ij} - \tfrac{J^2}{A}\mathcal{K}^{-1}_{ik}\mathcal{M}_{kj}\right)\Big|_{Eigenvalues} = 0$$

$$c^2_{\theta,Q} = c^2_{\theta,\mathcal{V}} = \tfrac{\mathcal{Z}}{\mathcal{F}\mathcal{H}}$$

$$\mathcal{L}^{(2)}_{h+\mathcal{V}} = \mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + \mathcal{Q}_{ij}\dot{v}^i v^{j\prime} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}v^i v^j + \ldots$$

$$\left(\omega^2\mathcal{K}_{ij} - k^2\mathcal{G}_{ij} - \omega k\mathcal{Q}_{ij} - \ell(\ell+1)\mathcal{M}_{ij(\ell^2)}\right)\Big|_{Eigenvalues} = 0$$

$$c_r^2 - 2\sqrt{\tfrac{B}{A}}\tfrac{\mathcal{J}}{\mathcal{F}}\cdot c_r - \tfrac{\mathcal{G}}{\mathcal{F}} = 0$$

$$c_r^{(\pm)} = \sqrt{\tfrac{B}{A}}\tfrac{\mathcal{J}}{\mathcal{F}} \pm \tfrac{1}{\mathcal{F}}\sqrt{\mathcal{Z}}$$

$$c^+_{r,Q} = c^+_{r,\mathcal{V}}, \quad c^-_{r,Q} = c^-_{r,\mathcal{V}}$$

$$\left(c_\theta^2\cdot\mathbf{I}_{ij} - \tfrac{J^2}{A}\mathcal{K}^{-1}_{ik}\mathcal{M}_{kj}\right)\Big|_{Eigenvalues} = 0$$

$$c^2_{\theta,Q} = c^2_{\theta,\mathcal{V}} = \tfrac{\mathcal{Z}}{\mathcal{F}\mathcal{H}}$$

$$\mathcal{F} = 2\phi\left[G_4 - G_{4X}\tfrac{\dot{\pi}^2}{A} - G_{5\pi}\left(X - \tfrac{\dot{\pi}^2}{A}\right)\right]$$

$$\mathcal{G} = 2\phi\left[G_4 - 2G_{4X}\left(X - \tfrac{\dot{\pi}^2}{2A}\right) + G_{5\pi}\left(X - \tfrac{\dot{\pi}^2}{A}\right)\right]$$

$$\mathcal{H} = 2\phi\left[G_4 - 2G_{4X}X + G_{5\pi}X\right]$$

$$\mathcal{J} = 2\phi\,\dot{\pi}\pi'(G_{4X} - G_{5\pi})$$

Horndeski theory $\rightarrow$ U(1) vector gauge field

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$\mathcal{L}_2$, $\mathcal{L}_3$ $\rightarrow$ no dynamical A_μ

Horndeski theory $\to$ U(1) vector gauge field

$$S = \int \mathrm{d}^4x\sqrt{-g}\,(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5)\,,$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$\mathcal{L}_4 \to$ non minimal coupling to A_μ

Horndeski theory $\rightarrow$ U(1) vector gauge field

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

$$\mathcal{L}_6 = G_6(\pi, X)\,\mathcal{O}\left[R^2\right] + G_{6X}(\pi, X)\,\mathcal{O}\left[R(\nabla^2\pi)^2\right] + G_{6XX}(\pi, X)\,\mathcal{O}\left[(\nabla^2\pi)^4\right],$$

$\mathcal{L}_5$, $\mathcal{L}_6$ $\rightarrow$ A_μ is a vector Galileon

$$\phi(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4) + \mathcal{L}_{4A_\mu} + \mathcal{L}_{4\phi} =$$

$$\int d^4x \sqrt{-g}\,\phi\, \Big[\underline{G_2(\pi, X) + G_3(\pi, X)\,\Box\pi} + G_4(\pi, X) \left(\underline{R} - \frac{1}{4}\phi^2 F^2 - 2\frac{\Box\phi}{\phi} \right)$$

$$+ G_{4,X}(\pi, X) \left(\underline{(\Box\pi)^2 - (\nabla_\mu\nabla_\nu\pi)^2} + 2\,\frac{1}{\phi}\,\nabla_\mu\phi\nabla^\mu\pi\,\Box\pi - \frac{1}{2}\phi^2\, F_\mu{}^\sigma\, F_{\nu\sigma}\,\nabla^\mu\pi\,\nabla^\nu\pi \right) \Big]$$

$$\phi\mathcal{L}_5 + \mathcal{L}_{5A_\mu} + \mathcal{L}_{5\phi} = \int d^4x \sqrt{-g}\,\phi\, G_5(\pi) \Big[\Big(\underline{R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R} \Big) \nabla_\mu\nabla_\nu\pi$$

$$- \frac{1}{2\phi} R\,\nabla_\mu\phi\,\nabla^\mu\pi + \frac{1}{\phi}\left(\Box\phi\,\Box\pi - \nabla_\mu\nabla_\nu\phi\nabla^\mu\nabla^\nu\pi\right) + \frac{1}{2}\phi^2\, F_{\mu\nu}\,\nabla_\sigma F^{\nu\sigma}\,\nabla^\mu\pi$$

$$+ \frac{1}{8}\phi\, F^{\mu\nu}\, F^{\sigma\rho} \Big(3\, g_{\nu\rho}(-4\, g_{\lambda\mu}\, g_{\beta\sigma} + g_{\lambda\beta}\, g_{\mu\sigma})\,\nabla^\lambda\pi\nabla^\beta\phi + \phi\, g_{\sigma\mu}\,(-4\,\nabla_\nu\nabla_\rho\pi + g_{\rho\nu}\Box\pi) \Big) \Big]$$

Conclusion and outlook

- Horndeski theory is OK for modern Universe

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

- To do:

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

- To do:

- KK of beyond Horndeski, DHOST

Conclusion and outlook

- Horndeski theory is OK for modern Universe
- There are gauge vector galileons
- To do:
  - KK of beyond Horndeski, DHOST
  - explicit Vainshtein calculation

Conclusion and outlook

- Horndeski theory is OK for modern Universe
- There are gauge vector galileons
- To do:
- KK of beyond Horndeski, DHOST
- explicit Vainshtein calculation
- Different compactifications
- . . . . .

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

- To do:

- KK of beyond Horndeski, DHOST
- explicit Vainshtein calculation
- Different compactifications
- . . . . .

Thank you for your attention!

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

- To do:

- KK of beyond Horndeski, DHOST
- explicit Vainshtein calculation
- Different compactifications
- . . . . .

Thank you for your attention!

Conclusion and outlook

- Horndeski theory is OK for modern Universe

- There are gauge vector galileons

- To do:

- KK of beyond Horndeski, DHOST
- explicit Vainshtein calculation
- Different compactifications
- . . . . .

Thank you for your attention!

	$c_g = c$	$c_g \neq c$
Horndeski	General Relativity quintessence/k-essence [46] Brans-Dicke/$f(R)$ [47, 48] Kinetic Gravity Braiding [50]	quartic/quintic Galileons [13, 14] Fab Four [15] de Sitter Horndeski [49] $G_{\mu\nu}\phi^{\mu}\phi^{\nu}$ [51], $f(\phi)\cdot$Gauss-Bonnet [52]
beyond H.	Derivative Conformal (19) [17] Disformal Tuning (21) quadratic DHOST with $A_1 = 0$	quartic/quintic GLPV [18] quadratic DHOST [20] with $A_1 \neq 0$ cubic DHOST [23]
	Viable after GW170817	Non-viable after GW170817

$$
\begin{aligned}
\mathcal{L}_j &= \sqrt{-g}V_j(\phi)G^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi, \\
\mathcal{L}_p &= \sqrt{-g}V_p(\phi)P^{\mu\nu\alpha\beta}\nabla_\mu\phi\nabla_\alpha\phi\nabla_\nu\nabla_\beta\phi, \\
\mathcal{L}_g &= \sqrt{-g}V_g(\phi)R, \\
\mathcal{L}_r &= \sqrt{-g}V_r(\phi)\hat{\mathcal{G}},
\end{aligned}
$$

Vainshtein