General method of inclusion Stueckelberg fields:
Covariant counterpart of Batalin-Fradkin conversion of Hamiltonian second class constraints.

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The talk is based on the articles

S.L. Eur.Phys.J.C 81 (2021) 5, 472;
V.Abakumova and S.L. Phys.Lett.B 820 (2021) 136552

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Typical pattern of "Stueckelberg trick"

K. Stueckelberg 1938, modern interpretation and review N. Boulanger, C. Deffayet, S. Garcia-Saenz and L. Traina, 2018

- Action splits into invariant and "symmetry breaking" terms:

$$S(\phi) = S_{gauge}(\phi) + S_{nongauge}(\phi)\,,$$

$$\delta_\epsilon \phi^i = R^i_\alpha \epsilon^\alpha\,, \quad \delta_\epsilon S_{gauge} \equiv 0,\, \forall \epsilon\,; \quad \delta_\epsilon S_{nongauge} \neq 0\,.$$

- Stueckelberg fields ξ^α are introduced to shift the original fields,

$$\phi^i \mapsto \tilde{\phi}^i(\phi,\xi) = \phi^i + R^i_\alpha \xi^\alpha + \dots\,; \quad \delta_\epsilon \xi^\alpha = \epsilon^\alpha + \dots\,, \quad \delta_\epsilon \phi^i = R^i_\alpha \epsilon^\alpha + \dots\,;$$

$$S_{St} = S_{gauge}(\phi) + S_{nongauge}(\tilde{\phi}(\phi,\xi))\,, \quad \delta_\epsilon S_{St} \equiv 0\,.$$

The split onto S_{gauge} and $S_{nongauge}$, and choice of symmetry is ambiguous. Even for Proca action various options are possible,

$$\mathcal{L} = -\frac{1}{4}F^2_{\mu\nu} + \frac{m^2}{2}A^2_\mu = -\frac{1}{2}(\partial_\mu A^\mu)^2 + \frac{1}{2}A^\mu(\Box + m^2)A_\mu;$$

$$(i)\,\delta_\epsilon A_\mu = \partial_\mu \epsilon,\ \delta_\epsilon F^2_{\mu\nu} = 0\,; \quad (ii)\delta_\epsilon A^\mu = \partial_\nu \epsilon^{\mu\nu},\ \delta_\epsilon(\partial_\mu A^\mu)^2 = 0.$$

Batalin-Fradkin conversion of Hamiltonian 2nd class constraints.

Conversion begins with Hamiltonian action for the 2nd class system

$$S[\phi,\lambda] = \int dt\big(\rho_i(\phi)\dot{\phi}^i - H(\phi)\lambda^\alpha\theta_\alpha(\phi)\big)\,, \quad \det\{\theta,\theta\} \neq 0$$

Conversion idea: phase space is extended by extra variables ξ^α, $\psi = (\phi,\xi)$ with new Poisson brackets $\det\{\psi,\psi\} \neq 0$. Number of conversion variables = number of 2nd class constraints. All the constraints are converted to first class:

$$\theta_\alpha \mapsto \mathcal{T}_\alpha = \theta_\alpha + \Delta_{\alpha\beta}\xi^\beta + \dots\,, \quad \{\mathcal{T},\mathcal{T}\} \sim \mathcal{T}\,.$$

$$H(\phi) \mapsto \mathcal{H}(\phi,\xi) = H + \dots\,, \quad \{\mathcal{H},\mathcal{T}\} \sim \mathcal{T}\,,$$

Procedure works well for any θ's. No assumptions are made that there is the first class piece in this action, or any subset of constraints is first class.

First works: I.A. Batalin, E.S. Fradkin, 1986;

Abelian conversion – E. S. Egorian, R. P. Manvelyan, + I.A. Batalin, E.S. Fradkin 1989;

Existence theorem + local construction – I.A. Batalin, I.V. Tyutin 1991;

Non-abelian conversion, global construction and $*$-product for 2nd class systems – I.A. Batalin, M.A. Grigoriev, S.L. Lyakhovich, 2005

Involutive closure – the starting point for conversion into gauge system.

For the second class system equations read

$$\dot{O}(\phi) = \{O, H_T\}\,, \quad H_T = H(\phi) + \lambda^\alpha \theta_\alpha(\phi)$$

$$\theta_\alpha(\phi) = 0\,, \quad \lambda^\alpha\{\theta_\alpha, \theta_\beta\} + \{\theta_\beta, H\} = 0\,.$$

Key observations:

- The equations are ***involutive***, since λ's are fixed – no other zero order consequences. From the perspective of general PDE theory fixing λ's means to take involutive closure;
- There are gauge identities between equations since differential consequences $\dot{\theta} \approx \theta$ reduce to equations that define λ;
- There are no gauge symmetry related to these gauge identities.
- Conversion of the constraints to the first class involves the new variables ξ^α such that the consequences $\dot{\mathcal{T}} \approx 0$ become the part of **Noether identity**. Hence the system gets gauge symmetry;
- Degree of freedom is under control before and after conversion, and it is unchanged by construction.

Batalin-Fradkin conversion of Hamiltonian 2nd class constraints: similarity with and distinctions from the Stuckelberg pattern

Stueckelberg trick

- Begins with the covariant action;
- Assumes split of the action into invariant and non-invariant parts;
- Involutive closure is out of interest, gauge identities of the involutive closure are ignored;
- Introduces Stueckelberg fields by the fixed ansatz to make the system gauge invariant;
- Degree of freedom is not counted before and inclusion of Stueckelberg fields.

Batalin-Fradkin conversion

- Begins with Hamiltonian second class system;
- It is not assumed to split the action into first class part, and the remnants;
- The system is involutive by construction. Gauge identities of the system are explicit;
- Introduces the conversion variables by iterative procedure, with proven existence;
- Degree of freedom is counted before and after conversion, and it remains unchanged.

Covariant inclusion of Stuckelberg fields by Batalin-Fradkin conversion.

The guide without tricks, and with existence theorem.

1. Take involutive closure of Lagrangian equations by complete set of the lower order consequences;
2. Find gauge identities between the equations in the involutive closure;
3. Count degree of freedom;
4. Introduce the Stueckelberg field for every consequence, and the gauge parameter for every identity;
5. Iteratively construct the Stueckelberg formalism with the first order of action defined by the added consequences, while the first order of gauge symmetry generators is determined by the generators of gauge identities;
6. Degree of freedom remains unchange upon inclusion of Stueckelberg fields;

Existence of the inclusion procedure is proven by the tools of cohomological perturbation theory in BV formalism. Specifics of the proof: the Stueckelberg fields are assigned with positive resolution degree (aka "anti-ghost number"), though they have zero ghost number.

Involutive closure and degree of freedom count

Involutive system includes all the **lower order consequences.** Any system can be brought into involution by inclusion of the differential consequences. **Gauge algebra of involutive system and DoF count.** Assume the system is involutive, but not necessarily variational. Then, there is no pairing between gauge symmetries and gauge identities

$$T_a(\phi, \partial\phi, \partial^2\phi \ldots) = 0;$$

$$\delta_\epsilon \phi^i = R^i_\alpha \epsilon^\alpha, \quad \delta_\epsilon T_a = V^b_a T_b \quad L^a_A T_a \equiv 0\,.$$

Covariant DoF count: D.S.Kaparulin, S.L., A.A.Sharapov, 2014

$$N_{DoF} = \sum_{n=0}^{\infty} n(t_n - r_n - l_n)$$

t_n – number of the equations of n-th order, r_n –number of gauge symmetries of of n-th order, l_n – number of gauge identities of n-th order.

Involutive closure of variational equations

Variational equations $\partial_i S(\phi)$ are **not necessarily involutive**. Let us take involutive closure by adding the **lower order consequences.** $\tau_\alpha(\phi) \equiv \Gamma^i_\alpha(\phi)\partial_i S(\phi)$, such that the system

$$\partial_i S(\phi) = 0 \,, \quad \tau_\alpha = 0 \,.$$

is involutive. Γ^i_α is termed **generator of the lower order consequences**
The set of the consequences τ_α is assumed complete and irreducible.
By construction, there are gauge identities in this system:

$$\Gamma^i_\alpha(\phi)\partial_i S(\phi) + \tau_\alpha(\phi) \equiv 0 \,.$$

Simplest example of involutive closure – Proca equations,

$$\frac{\delta S_{Proca}}{\delta A_\mu} \equiv -\partial_\nu F^{\mu\nu} + m^2 A_\mu = 0 \,, \quad \tau \equiv \partial_\mu \frac{\delta S_{Proca}}{\delta A_\mu} \equiv m^2 \partial_\mu A^\mu = 0$$

Gauge identity:

$$\partial_\mu \frac{\delta S_{Proca}}{\delta A_\mu} - \tau \equiv 0 \,.$$

Covariant degree of freedom count, $t_2 = 4$, $t_1 = 1$, $l_3 = 1$

$$N_{DoF} = \sum_{n=0}^{\infty} n(t_n - r_n - l_n) = 2 \cdot 4 + 1 \cdot 1 - 3 \cdot 1 = 6$$

Involutive closure of Hamiltonian 2nd class system and conversion to the first class: lesson to learn for the general variational equations.

Determination of Lagrange multipliers in the second class system is the **example** of involutive closure

$$\frac{\delta S}{\delta x^i} \equiv \omega_{ji}\dot{x}^i - \{x^i, H_T\} = 0, \quad \frac{\delta S}{\delta \lambda^\alpha} \equiv \theta_\alpha(x) = 0.$$

Zero order consequences are the equations that define Lagrange multipliers:

$$\tau_\alpha(x,\lambda) \equiv \frac{d}{dt}\frac{\delta S}{\delta \lambda^\alpha} - \{\theta_\alpha, x^i\}\frac{\delta S}{\delta x^i} \equiv \{\theta_\alpha, \theta_\beta\}\lambda^\beta - \{\theta_\alpha, H(x)\} = 0.$$

Gauge identity:

$$\frac{d}{dt}\frac{\delta S}{\delta \lambda^\alpha} - \{\theta_\alpha, x^i\}\frac{\delta S}{\delta x^i} - \tau_\alpha \equiv 0\,.$$

Key observation: Inclusion of conversion variables ξ^α, $\theta_\alpha \mapsto \mathcal{T}_\alpha = \theta_\alpha + \xi - \textit{terms}$ turns this non-variational identity into the Noether identity between variational equations,

$$\frac{d}{dt}\frac{\delta S_{BF}}{\delta \lambda^\alpha} - \{\mathcal{T}_\alpha, x^i\}\frac{\delta S_{BF}}{\delta x^i} - \{\mathcal{T}_\alpha, \xi^\beta\}\frac{\delta S_{BF}}{\delta \xi^\beta} \sim \frac{\delta S_{BF}}{\delta \lambda^\alpha}$$

Covariant conversion for variational system: a general guide.

1. Find the complete set of the lower order consequences $\tau_\alpha(\phi)$;
2. Find gauge identities in the involutive closure;
3. Introduce Stueckelberg fields ξ^α being dual to the consequences;
4. Iteratively construct the Stueckelberg gauge generators proceeding from the generators of the gauge identities and simultaneously iterate Stueckelberg action, proceeding from the original action, and the lower order consequences.

The Stueckelberg action is sought for as a power series in the fields ξ^α:

$$\mathcal{S}_{St}(\phi,\xi)=\sum_{k=0}^{\infty}\mathcal{S}_k\,,\qquad \mathcal{S}_k(\phi,\xi)=W_{\alpha_1\dots\alpha_k}(\phi)\,\xi^{\alpha_1}\cdots\xi^{\alpha_k}\,,$$

$\mathcal{S}_0(\phi)$ is the original action, and the first coefficient W_α is defined by τ_α

$$W_\alpha(\phi)=\frac{\partial\mathcal{S}_{St}(\phi,\xi)}{\partial\xi^\alpha}\Big|\xi=0=\tau_\alpha\,.$$

At $\xi=0$, the field equations for the Stueckelberg action reproduce the involutive closure of the original Lagrangian equations.

Iterative inclusion of Stueckelberg fields

The equivalence of the Stueckelberg theory to the original one is provided by the gauge symmetry such that the fields ξ^α can be gauged out, with $\xi^\alpha = 0$ being admissible gauge fixing condition. The gauge transformations are iteratively sought for order by order of the Stueckelberg fields

$$\delta_\epsilon \phi^i = R^i_\alpha(\phi,\xi)\epsilon^\alpha\,, \quad \delta_\epsilon \xi^\gamma = R^\gamma_\alpha(\phi,\xi)\epsilon^\alpha\,,$$

$$R^i_\alpha(\phi,\xi) = \sum_{k=0} \overset{(k)}{R}{}^i_\alpha, \quad \overset{(k)}{R}{}^i_\alpha(\phi,\xi) = R^i_{\alpha\beta_1\dots\beta_k}(\phi)\xi^{\beta_1}\dots\xi^{\beta_k}\,;$$

$$R^\gamma_\alpha(\phi,\xi) = \sum_{k=0} \overset{(k)}{R}{}^\gamma_\alpha, \quad \overset{(k)}{R}{}^\gamma_\alpha(\phi,\xi) = R^\gamma_{\alpha\beta_1\dots\beta_k}(\phi)\xi^{\beta_1}\dots\xi^{\beta_k}\,.$$

The gauge symmetry of Stueckelberg action means the Noether identities

$$\delta_\epsilon \mathcal{S}_{St} \equiv 0\,, \quad \forall\epsilon^\alpha \quad \Leftrightarrow \quad R^i_\alpha \partial_i \mathcal{S}_{St} + R^\gamma_\alpha \frac{\partial \mathcal{S}_{St}}{\partial \xi^\gamma} \equiv 0\,.$$

These identities can be iteratively solved for the expansion coefficients for the action and gauge generators.

Sequence of relations for gauge generators and Stueckelberg action

$$\left(R_\alpha^\gamma(\phi,\xi)\frac{\partial}{\partial\xi^\gamma}+R_\alpha^i(\phi,\xi)\frac{\partial}{\partial\phi^i}\right)\mathcal{S}_{St}\equiv$$

$$\sum_{k=0}\sum_{m=0}^{k}\left(\overset{(k-m)}{R}{}^{\gamma}_{\alpha}\,\frac{\partial\mathcal{S}_{(m+1)}}{\partial\xi^\gamma}+\overset{(k-m)}{R}{}^{i}_{\alpha}\,\frac{\partial\mathcal{S}_{(m)}}{\partial\phi^i}\right)\equiv 0\,.$$

Once the Noether identities are valid for every order in ξ, each term in the sum over k vanishes separately. This results in the sequence of relations

$$\sum_{m=0}^{k}\left(\overset{(k-m)}{R}{}^{\gamma}_{\alpha}\,\frac{\partial\mathcal{S}_{(m+1)}}{\partial\xi^\gamma}+\overset{(k-m)}{R}{}^{i}_{\alpha}\,\frac{\partial\mathcal{S}_{(m)}}{\partial\phi^i}\right)\equiv 0\,,\quad k=0,1,2,\ldots\,.$$

For $k=0$, given the boundary condition at $\xi=0$, this reads,

$$\overset{(0)}{R}{}^{\gamma}_{\alpha}\tau_\gamma+\overset{(0)}{R}{}^{i}_{\alpha}\partial_i S\equiv 0\,.$$

Any identity between τ_α and $\partial_i S$ reduces to definition of τ

$$\overset{(0)}{R}{}^{\gamma}_{\alpha}\left(\tau_\gamma+\Gamma^i_\gamma\partial_i S\right)\equiv 0\,,$$

$\overset{(0)}{R}{}^{\gamma}_{\alpha}$ can be any non-degenerate matrix. We choose it as δ^γ_α. Any other choice can be absorbed by change of gauge parameters.

Iterative solution of relations for gauge generators and action

Given the boundary condition for the action, we arrive at the first order of the action and zero order of gauge symmetry

$$\mathcal{S}_{St}(\phi,\xi) = S(\phi)+\tau_\alpha(\phi)\xi^\alpha+\dots\,, \quad \delta_\epsilon\phi^i = \Gamma^i_\alpha(\phi)\epsilon^\alpha+\dots\,, \quad \delta_\epsilon\xi^\alpha = \epsilon^\alpha+\dots\,,$$

To solve the relations for S_2 and $\overset{(1)}{R}$ we consider the gauge identity

$$\mathbf{I}_\alpha \equiv \sum_{m=0}^{k}\left(\overset{(k-m)}{R}{}^{\gamma}_{\alpha}\frac{\partial\mathcal{S}_{(m+1)}}{\partial\xi^\gamma} + \overset{(k-m)}{R}{}^{i}_{\alpha}\frac{\partial\mathcal{S}_{(m)}}{\partial\phi^i}\right) \equiv 0\,, \quad k=1,2,\dots\,.$$

at $k=1$. $\mathbf{I}_\alpha$ is linear in ξ^β. Symmetric in α,β coefficients contribute to the action, while the antisymmetric ones go to the gauge generators. This can be seen from the identity

$$\Gamma^i_\alpha(\phi)\partial_i\tau_\beta = W_{\alpha\beta} + R^i_{\alpha\beta}\partial_i S + R^\gamma_{\alpha\beta}\tau_\gamma\,, \quad W_{\alpha\beta} = W_{\beta\alpha},$$

being the differential consequence of the relation $\tau_\alpha + \Gamma^i_\alpha\partial_i S \equiv 0$. On shell, $W_{\alpha\beta} \approx \Gamma^i_\alpha\Gamma^j_\beta\partial^2_{ij}S$, $R_{\alpha\beta} \approx -R_{\beta\alpha}$.
Deducing higher order consequence of the identities of the involutive closure, one can iteratively find all the higher order structure coefficients in the expansion of the action and gauge generators.

BV master equation for Stueckelberg theory, grading and boundary.

Problem setup: Given the original action S, lower order consequences τ_α, and generators of consequences Γ^i_α, to construct the BV-master action for Stueckelberg theory iterating both in anti-fields and Stueckelberg fields. The set of fields, anti-fields, and anti-brackets

$$\begin{array}{llll} \epsilon(\phi^i)=\epsilon(\xi^\alpha)=0, & \epsilon(C^\alpha)=1, & gh(\phi^i)=gh(\xi^\alpha)=0\,, & gh(C^\alpha)=\ 1; \\ \epsilon(\phi^*_i)=\epsilon(\xi^*_\alpha)=1, & \epsilon(C^*_\alpha)=0, & gh(\phi^*_i)=gh(\xi^*_\alpha)=-1, & gh(C^*_\alpha)=-2. \end{array}$$

$$(A,B)=\frac{\partial^R A}{\partial\varphi^I}\frac{\partial^L B}{\partial\varphi^*_I}-\frac{\partial^R A}{\partial\varphi^*_I}\frac{\partial^L B}{\partial\varphi^I}\,,\quad \varphi^I=(\phi^i,\xi^\alpha,C^\alpha)\,,\quad \varphi^*_I=(\phi^*_i,\xi^*_\alpha,C^*_\alpha)\,.$$

Boundary condition for the Stueckelberg master action reads

$$S_{BV}(\varphi,\varphi*)=S(\phi)-\tau_\alpha(\phi)\xi^\alpha+C^\alpha\Gamma^i_\alpha(\phi)\phi^*_i+C^\alpha\xi^*_\alpha+\dots\,,$$

and the higher orders in the anti-fields **and** Stueckelberg fields have to be found from the master equation

$$(S_{BV},S_{BV})=0\,.$$

Resolution degree is introduced which differs from the antighost number

$$deg(\xi^\alpha)=deg(\xi^*_\alpha)=deg(\phi^*_i)=1,\quad deg(C^*_\alpha)=2,\quad deg(C^\alpha)=deg(\phi^i)=0\,.$$

BV master equation for Stueckelberg theory, the first iteration

The solution to the master equation is sough for as the expansion of the action $S_{BV}(\varphi,\varphi^*)$ w.r.t. the resolution degree,

$$S_{BV}(\varphi,\varphi^*) = \sum_{k=0} \overset{(k)}{S} , \qquad deg \ \overset{(k)}{S} = k \, .$$

Once the solution is found in all the orders resolution degree, the complete Stueckelberg action is extracted as zero order w.r.t. to the anti-ghost number (i.e. with switched off anti-fields), while the Stueckelberg gauge generators are defined by the first order of S_{BV} w.r.t. the anti-ghost degree (i.e. as the coefficients at ξ^*_γ and ϕ^*_i). Consider the master action up to the next order of the resolution degree after the boundary condition,

$$\begin{aligned} S_{BV}(\varphi,\varphi*) &= S(\phi) - \tau_\alpha\xi^\alpha + C^\alpha\left(\Gamma^i_\alpha(\phi)\phi^*_i + \xi^*_\alpha\right) + \\ &+ \frac{1}{2}W_{\alpha\beta}\xi^\alpha\xi^\beta + C^\alpha\left(R^\gamma_{\alpha\beta}\xi^\beta\xi^*_\gamma + R^i_{\alpha\beta}\xi^\beta\phi^*_i\right) + \\ &+ \frac{1}{2}C^\beta C^\alpha\left(U^\gamma_{\alpha\beta}C^*_\gamma + \phi^*_j\phi^*_i E^{ij}_{\alpha\beta} + \xi^*_\mu\phi^*_i E^{\mu i}_{\alpha\beta} + \xi^*_\mu\xi^*_\nu E^{\mu\nu}_{\alpha\beta}\right) + \ldots \, , \end{aligned}$$

Master equation for Stueckelberg action, the second order of solution.

Let us expand the l.h.s. of the master equation w.r.t. the resolution degree up to the first order. Notice that $\overset{(k)}{S}$, $k > 2$ cannot contribute to zero and first orders of the expansion, this is sufficient.

$$(S_{BV}, S_{BV})_0 = 2(\Gamma^i_\alpha \partial_i S + \tau_\alpha)C^\alpha \equiv 0\,,$$

$$\begin{aligned}(S_{BV}, S_{BV})_1 \;\; &= \;\; 2\xi^\gamma(\Gamma^i_\alpha \partial_i \tau_\gamma - R^i_{\alpha\gamma}\partial_i S - R^\beta_{\alpha\gamma}\tau_\beta - W_{\gamma\alpha})C^\alpha - \\ &- \;\; C^\alpha C^\beta\big(\phi^*_i(\Gamma^j_\alpha \partial_j \Gamma^i_\beta - \Gamma^j_\beta \partial_j \Gamma^i_\alpha - U^\gamma_{\alpha\beta}\Gamma^i_\gamma - R^i_{\alpha\beta} + R^i_{\beta\alpha} \\ &- \;\; E^{ji}_{\alpha\beta}\partial_j S - E^{i\gamma}_{\alpha\beta}\tau_\gamma) - \\ &- \;\; \xi^*_\mu(U^\mu_{\alpha\beta} - R^\mu_{\alpha\beta} + R^\mu_{\beta\alpha}) + E^{j\mu}_{\alpha\beta}\partial_j S - E^{\mu\nu}_{\alpha\beta}\tau_\nu)\big) = 0\,.\end{aligned}$$

Zero order relation is valid, given the original gauge identity. The first order of the master equation holds by virtue of identities upon identification of the structure coefficients in the expansion with corresponding structure functions in the differential consequences of the original identity.

Stueckelberg master action, cohomological perturbation theory.

Consider k-th order of the master equation w.r.t. resolution degree

$$(S_{BV}, S_{BV})_k = \delta \overset{(k+1)}{S} + B_k(S, \overset{(1)}{S}, \dots, \overset{(k)}{S}),$$

B_k involves only $\overset{(l)}{S}$, $l \leq k$, and the operator δ reads:

$$\delta O = -\frac{\partial^R O}{\partial \phi^*_i}\partial_i S - \frac{\partial^R O}{\partial \xi^*_\alpha}\tau_\alpha + \frac{\partial^R O}{\partial C^*_\alpha}\left(\phi^*_i \Gamma^i_\alpha + \xi^*_\alpha\right) + \frac{\partial^R O}{\partial \xi^\alpha} C^\alpha \,.$$

By virtue of the original identity, the operator δ squares to zero,

$$\delta^2 O = \frac{\partial^R O}{\partial C^*_\alpha}\left(\Gamma^i_\alpha \partial_i S + \tau_\alpha\right) \equiv 0,$$

so it is a differential. Obviously, δ decreases the resolution degree by one,

$$deg(\delta) = -1 \,.$$

Notice that δ is acyclic in the strictly positive resolution degree because the original identities are independent, i.e.

$$\delta X = 0, \; deg(X) > 0 \quad \Leftrightarrow \quad \exists\, Y : \, X = \delta Y \,.$$

By Jacobi identity $(S, (S, S)) \equiv 0$, $\forall S$. Expanding the identity w.r.t. the resolution degree, one can see that B_k is δ-closed,

Stueckelberg master action, cohomological perturbation theory.

Then, because of acilicity of δ, B_k is δ-exact,

$$\exists Y_{k+1} : \ B_k = \delta Y_{k+1}, \quad deg(Y_{k+1}) = k+1 .$$

This leads to the relation

$$\delta \left(\overset{(k+1)}{S} + Y_{k+1} \right) = 0 . \tag{1}$$

This provides solution for $\overset{(k+1)}{S}$

$$\overset{(k+1)}{S} = -Y_{k+1} + \delta Z_{k+2}, \qquad deg(Z_{k+2}) = k+2 .$$

The solution is unique modulo natural δ-exact ambiguity.
In this way, one can iteratively find the master action of the Stueckelberg theory, given the original action, generators Γ^i_α of consequences τ_α included into the involutive closure of Lagrangian system.
The solution is unobstructed at any order, so the Stueckelberg action can be always iteratively constructed.
□

Summary of results

The fine art of Sueckelberg tricks is converted into Batalin-Fradkin science for inclusion of the gauge symmetry.

Thank you for your attention!

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