What is quantum field theory in de Sitter space-time

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Motivation

- de Sitter (dS) space solves

$$R_{\alpha\beta} - \frac{1}{2} R\, g_{\alpha\beta} = \Lambda\, g_{\alpha\beta} + \langle T_{\alpha\beta} \rangle,$$

- Is $\langle T_{\alpha\beta} \rangle$ relevant or not? Common wisdom is that it is always not relevant. I will try to convince you that under certain circumstances this is a wrong intuition even for massive fields.
- There is UV divergence in $\langle T_{\alpha\beta} \rangle$. At leading order it is the same as in flat space $\langle T_{\alpha\beta} \rangle \propto g_{\alpha\beta}$. Leads to the renormalization of Λ.
- On top of that there also can be non-trivial fluxes in $\langle T_{\alpha\beta} \rangle$, because dS metric is time dependent — the situation is non–stationary. Everything depends on the initial state wrt which the average is done in $\langle T_{\alpha\beta} \rangle$.

Motivation

In time-dependent strong background fields:

- There are no asymptotic states: modes behave as $g(t) \sim A_\pm \, e^{-i\omega_\pm \, t} + B_\pm \, e^{i\omega_\pm \, t}$, as $t \to \pm\infty$;
- There is no energy conservation;
- As a result there is no factorisation in the vertexes of diagrams for soft modes and, hence, there is no cancellation of IR divergences;
- No such a notion as particle: free Hamiltonian cannot be diagonalized once and forever.

Motivation

- One should use Schwinger-Keldysh in-in diagrammatic technique rather than the Feynman in-out one;
- In non–stationary situations the quantities to consider are the correlation functions:

$$\langle O(t_1, \dots, t_n)\rangle = \left\langle st \left| U^+ \, T[O(t_1, \dots, t_n)\, U] \right| st \right\rangle ,$$

 rather than amplitudes:

$$A = \frac{\langle out \,| T[O(t_1, \dots, t_n)\, U]|\, in\rangle}{\langle out \,|U|\, in\rangle}.$$

 At least because there are no asymptotic states and due to the presence of uncontrolable IR divergences.
- Observables are correlation functions, such as e.g. $\langle T_{\alpha\beta}\rangle$, rather than cross-sections. They are defined in a geodesically incomplete space-time with an initial Cauchy surface;

Motivation

- In any interacting QFT on a strong background field for any initial state there are secular IR loop contributions to the correlation functions;
- There are several different types of secular effects;
- The secular growth in the loops, which is of interest for us is an IR effect. It does not affect UV physics;
- The presence of secular effects means that quantum corrections can be of the same order as classical contributions;
- IR effects are non–local. Hence, their contributions can depend on reference system and initial conditions.

Motivation

- dS has $SO(D, 1)$ isometry. Similar to the Poincaré invariance in Minkowski space;
- If dS isometry is respected, then all correlation functions depend only on geodesic distances and, e.g., $\langle T_{\alpha\beta} \rangle \propto g_{\alpha\beta}$ — No any flux! That is true even in the presence of the secular contributions;
- Is the dS isometry always respected? On tree–level and/or in the loops? For all initial states? For all patches of dS space?
- If there is such a ground state that respects dS isometry at all stages of quantization, is it stable under non–symmetric perturbations? Can a secular growth provide a destructive flux, $\langle T_{\alpha\beta} \rangle$?
- We will see that even for the massive scalar fields the situation is quite counterintuitive.

Setup

- We consider

$$S = \int d^D x \sqrt{|g|} \left[\frac{1}{2} g^{\alpha\beta} \partial_\alpha \phi \, \partial_\beta \phi \;\; + \; V(\phi) \right],$$

where $[sign(g) = (-, +, +, +)]$;

- We start with the consideration as the background the expanding Poincaré patch (EPP) $ds^2 = \frac{1}{\eta^2} \left[-d\eta^2 + d\vec{x}^2 \right]$, where $\eta = e^{-t}$;
- In the EPP the conformal time is ranging form $\eta = +\infty$ at past infinity $(t = -\infty)$ to $\eta = 0$ at future infinity $(t = +\infty)$;
- In the contracting Poincare patch (CPP) the conformal time is changing in the reverse direction;
- Global dS is the union of EPP and CPP;
- We set the radius of the dS spacetime to one. Our goal is to check whether the assumption of negligible backreaction is self-consistent or not for various sorts of initial conditions.

The Schwinger-Keldysh technique is causal

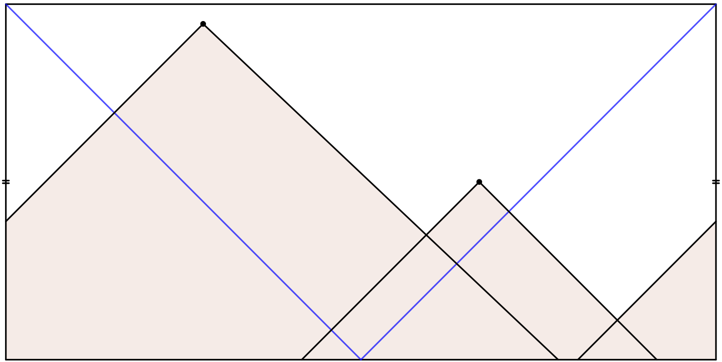

Figure: The union of past ligh cones of external points of a diagram on the Penrose diagram of **de Sitter space-time**; the blue line shows the boundary between **Expanding and Contracting Poincare Patches**. Within the framework of the **Schwinger-Keldysh** technique one integrates in the loop integrals over these past light-cones.

Setup

- Mode functions in the EPP are factorized as

$$g_p(\eta)\, e^{-i\,\vec{p}\,\vec{x}} = \eta^{(D-1)/2}\, h_\nu(p\eta)\, e^{-i\,\vec{p}\,\vec{x}},$$

  where $h_\nu(p\eta)$ is a solution of the Bessel equation of order $\nu = \sqrt{\left(\frac{D-1}{2}\right)^2 - m^2}$;
- $m > (D-1)/2$ — principal series. $m < (D-1)/2$ — complementary series;
- The field is expanded as:

$$\phi(\eta, \vec{x}) = \int d^{(D-1)}\vec{p}\, \left[a_{\vec{p}}\, g_p(\eta)\, e^{-i\,\vec{p}\,\vec{x}} \;+\; a^+_{\vec{p}}\, g^*_p(\eta)\, e^{i\,\vec{p}\,\vec{x}}\right].$$

  $a_{\vec{p}}$ and $a^+_{\vec{p}}$ obey proper commutation relations;
- This is just one of the possible ways of quantization that respects the dS isometry at least at tree-level. In this method at tree-level the initial Cauchy surface is not apparent.

Setup

- Any Bessel function of the order ν behaves as follows:

$$h_\nu(p\eta) = \begin{cases} a\,\frac{e^{i\,p\eta}}{\sqrt{p\eta}} + b\,\frac{e^{-i\,p\eta}}{\sqrt{p\eta}}, & p\eta \gg |\nu| \\ c\,(p\eta)^{\nu} + d\,(p\eta)^{-\nu}, & p\eta \ll |\nu|\,. \end{cases}$$

- E.g. the Bunch–Davies (BD) modes are as follows: $h_\nu(p\eta) \propto H_\nu^{(1)}(p\eta)$ — Hankel function.
- There are many more options for the choice of the mode functions that respects the Hadamard property of the correlation functions;
- IR limit implies that $p\eta \to 0$. It is in the future infinity of the EPP and in the past infinity of the CPP.

Secular effect specific for massless scalars in dS space

- The two–point function of massless field in EPP at coincident points wrt the BD state, $a_{\vec{p}}|\rangle = 0$:

$$\left\langle \phi^2(\eta, \vec{x}) \right\rangle_0 \propto \int \frac{d^3\vec{p}}{p^3} \left[1 + (p\eta)^2\right]$$

— divergent both in UV and IR.

- After UV and IR regularization:

$$\left\langle \phi_0^2(\eta, \vec{x}) \right\rangle_0 \propto \int_1^{1/\eta} \frac{dp}{p} \left[1 + (p\eta)^2\right] \propto \log(\eta) + \dots$$

— secular growth. ϕ_0 contains only modes for $1 \gg p\eta$.

- $\langle \phi(\eta_1, \vec{x}_1)\, \phi(\eta_2, \vec{x}_2) \rangle$ is not a function of the geodesic distance between $(\eta_1, \vec{x}_1)$ and $(\eta_2, \vec{x}_2)$, when $m = 0$. Violates dS isometry.

Secular effect specific for massless scalars in dS space

- In the loops the correlation function also receives secular corrections: $\left\langle \phi_0^2(\eta, \vec{x}) \right\rangle_n \propto \log(\eta) \left[\lambda \log^2(\eta)\right]^n$, where n grows with the number of loops.
- Even if λ is very small, after a long enough period of time quantum corrections become of the order of the classical contributions $\lambda \log^2(\eta) \sim 1$.
- Hence, to understand the physics in dS space one needs to resum at least the leading contributions from all loops.
- To do the resummation, one usually uses Starobinsky–Yokoyama method, which uses stochastic equation with a linear random source, $\dot{\phi}_0$, in a non–linear (self-interacting) theory.

Critique of the Starobinsky–Yokoyama method

- What is the approximation in which this method can be applied? For what initial conditions?
- Common wisdom is that this method can be used for any mass m and leads to the mass renormalization. But that is not true even for the principal series for a generic initial state.
- It is applicable only for small perturbations on top of the BD state. What if the perturbation is strong?
- It cannot be used in global dS space and in the CPP, where even IR divergences are present. It can be used only in EPP.

A universal secular growth in dS space

- Expanding Poincaré patch is spatially homogeneous. Hence, it is convenient to consider spatial Fourier transformation of the correlation functions:

$$D(\eta_1, \eta_2, p) \equiv \int d^{D-1}p \, e^{i\,\vec{p}\,\vec{x}} \left\langle \phi(\eta_1, \vec{x}) \;\; \phi(\eta_2, 0) \right\rangle$$

  Convenient form to trace the destiny of each mode with given physical momentum $p\eta$.

- In the massive $\lambda\,\phi^4$ theory, when $\eta_2/\eta_1 \to \infty$, there is a universal secular growth to any non–stationary situation:

$$D_n(\eta_1, \eta_2, p) \propto \left[\lambda^2 \;\; \log\left(\eta_1/\eta_2\right)\right]^n \mathrm{Re}\left[g_p(\eta_1) \;\; g_p^*(\eta_2)\right]$$

A universal secular growth in dS space

- Such a growth, with $t_2 - t_1 = \log \eta_2/\eta_1$, cannot be attributed just to IR limit. It can be also seen in the UV;
- In standard situations, the functional dependence of $\left[\lambda^2 \quad \log\left(\eta_1/\eta_2\right)\right]^n$ and of $\mathrm{Re}\left[g_p(\eta_1) \quad g_p^*(\eta_2)\right]$ is the same as $t_2 - t_2 \to \infty$;
- That means that the secular contributions are present simultaneously in the Retarded, Advanced and Keldysh propagators;
- As the result, after the resummation such a secular growth usually leads to the dispertion relation (complex) renormalization.

- In non–stationary situations any field is characterized by three propagators.
- Two of them are retarded and advanced propagators:

$$D_0^{\frac{R}{A}}\left(p\,|\eta_1,\eta_2\right) = \pm\theta\left(\mp\Delta\eta_{12}\right)\, 2\,\mathrm{Im}\left[g_p(\eta_1)\;\; g_p^*(\eta_2)\right].$$

They do not depend on the state, at least at tree–level.

- Another propagator is the Keldysh one:

$$D_0^K\left(p|\eta_1,\eta_2\right) = \left[\left(\frac{1}{2} + \left\langle a_{\vec{p}}^+\, a_{\vec{p}}\right\rangle_\Psi\right) g_p(\eta_1) g_p^*(\eta_2) + \right.$$
$$\left. + \left\langle a_{\vec{p}}\; a_{-\vec{p}}\right\rangle_\Psi g_p(\eta_1)\, g_p(\eta_2) + h.c.\right].$$

If the initial state $|\Psi\rangle$ respects the spatial translational invariance. It does depend on the (initial) state.

Yet another universal secular growth

- In the Gaussian approximation — $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle_\Psi = const$, and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle_\Psi = const$. All time dependence is gone into harmonic functions — $h(p\eta)$.
- If the initial state is the ground one: $|\Psi\rangle = |ground\rangle$ and $a_p\,|ground\rangle = 0$, we always have that $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle_g = \left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle_g = 0$.
- All the quasi–classical results are obtained with the use of the tree–level propagator:

$$D_0^K\left(\eta_1,\eta_2|p\right) = \mathrm{Re}\,\left[g_p(\eta_1)\, g_p^*(\eta_2)\right]$$

  E.g. Bunch–Davies's $\langle T_{\mu\nu}\rangle_0$ in de Sitter space and Hawking's flux in black hole collapse, and Schwinger's $\langle J_\mu\rangle_0$ in QED.
- However, in non-stationary situations $\left\langle a^+_{\vec{p}}\, a_{\vec{p}} \right\rangle$ and $\left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle$ start to depend on time. That may strongly modify quasi–classical flux.

Expanding Poincaré patch of de Sitter space

- In the expanding Poincaré patch for $m < (D-1)/2$: $g_p(\eta) \approx A_- \, \eta^{\frac{D-1}{2}} \, (p\eta)^{-\nu}$, as $p\eta \to 0$;
- Tree–level plus sunset second loop diagram contribution is:

$$D_0^K + \Delta_2 D^K \approx A_-^2 \, \eta^{D-1}/(p\eta)^{2\nu} \left[1 + b \, \lambda^2 \, \log\left(\frac{p\eta}{|\nu|}\right)\right],$$

Where b is an integral of a product of modes and $\eta = \sqrt{\eta_1 \, \eta_2}$. This result is obtained in the limit as $p\eta \to 0$ and $\eta_1/\eta_2 = const$;

- There are no secular contributions of the type that we consider in the Retarded and Advanced propagators;
- Such a secular growth cannot be absorbed into the self-energy renormalization and can be attributed only to the change of the state of the theory.

Contracting Poincare patch of de Sitter space–time

- The **contracting Poincare patch**, $ds^2 = dt^2 - e^{-2t}\, d\vec{x}^2$, is the time reversal of the **expanding Poincare patch**;
- Now **in the loops** one sees the secular divergences:

$$\begin{array}{ll} \lambda^2 \log\left(\frac{\eta}{\eta_0}\right) & p\,\eta < |\nu|, \\ \lambda^2 \log\left(\frac{|\nu|}{p\,\eta_0}\right) & p\,\eta > |\nu|. \end{array}$$

- η_0 is the position of the initial Cauchy surface. If η_0 is taken to past infinity, $\eta_0 = 0$, loop corrections are infinite even after the introduction of the UV cutoff;
- Loop corrected propagator is not a function of the **geodesic distance** anymore. For any initial state!
- **Global de Sitter** contains both **expanding and contracting patches** simultaneously. The situation there is similar to the one in contracting patch.

Resummation (general discussion)

To resum the leading corrections from all loops for the case of the last secular growth one has to

- Check that there are no leading corrections to the retarded and advanced propagators and the leading secular growth of last type is present only in the Keldysh propagator;
- Check that there are no leading corrections to the vertexes;
- Put in the system of the Dyson–Schwinger equations retarded and advanced propagators (and vertexes, if possible) to their tree–level values. Then this system reduces to the single equation for the Keldysh propagator. What remains to be checked what type of diagrams contribute leading corrections;
- The described way is to obtain the Boltzmann's kinetic equation in the standard situations.

Resummation (dS invariant case)

- If one takes exactly BD state at exactly past infinity of the expanding Poincaré patch, then one can show that dS isometry is respected at every loop order, if $m > 0$;
- Moreover, one can show that in this case leading contributions come from the summation of the bubble diagrams;
- That is the reason why in the Dyson–Schwinger equation one can put the exact Keldysh propagator only into one of the external legs. As a result in this case the Dyson–Schwinger equation reduces to a system of linear integro–differential equations;
- The result of the resummation for the principal series is that anomalous average $\kappa_p = \left\langle a_{\vec{p}}\, a_{-\vec{p}} \right\rangle$ is evolving from zero to such a value that after Bogolyubov rotation of a_p, a_p^+ and g_p, g_p^* correlation functions behave as in the out–state. This is not a mass renormalization, as Starobinsky–Yokoyama approach predicts.

Resummation for non–invariant perturbations

- We propose to consider an initial nonsymmetric density perturbation on top of the BD state.

- We cannot just put initial comoving density n_p^0 at past infinity of the EPP, because then the physical density will be infinite. Due to the symmetries of the EPP we put an initial comoving density at an initial value of the physical momentum $P_0 \equiv (p\eta)_0 \sim \nu$.

- However, if we cut the integration over the physical momentum at P_0 and put an initial value $n\,(P_0)$, for the comoving density of the exact modes, the internal legs also bring leading corrections of the type $\left|\lambda^2 \log(p\eta)\right|^n$.

- Then there follows a non–linear integro–differential equation.

Conclusions

For the explosive solutions the Keldysh propagator blows up at a finite proper time. Then, also the expectation value of the stress–energy tensor blows up (which would appear at the RHS of the Einstein equations due to the quantum fluctuations). That means that the backreaction is not negligible. One possibility is that that the cosmological constant is secularly screened because the expectation value of the stress–energy tensor under discussion does not respect the dS isometry. This is a subject of a separate study. Here we do not consider the backreaction issue.